Bilateral Teleoperation Experiments: Scattering Transformation and Passive Output Synchronization Revisited *

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Abstract: It is well known that the scattering variables transform the transmission delays into a passive virtual transmission line, hence, its interconnection with passive subsystems preserves passivity of the overall system. However, wave reflections may occur. Using a symmetric velocity controller, on the master and the slave, and by matching the impedances, the scattering transformation reduces to a passive output synchronization scheme. In this paper we revisit this relation and perform some experiments on this line.

Keywords: Telerobotics, Time-delay, Robot control, Lyapunov functions.

1. INTRODUCTION

Most bilateral teleoperators use a communication channel that imposes a time-delay between data transfers. It is well known that this time-delay affects the overall stability of the teleoperator. The control of these systems has become an highly active research field amongst engineering scientists. The ground–breaking work of Anderson and Spong [1989] has ever since dominated this field. They proposed to send the scattering signals to transform the transmission delays into a passive virtual transmission line. The transmission line is then interconnected with the master and slave robots, which define passive force to velocity operators, while the human operator and the contact environment constitute the terminations to the transmission line. Since power–preserving interconnection of passive systems is again passive $L_2$–stability of the overall system is ensured under the reasonable assumption that the human operator and the environment define passive (force to velocity) maps. Since then, the use of scattering theory has been widely extended. The reader is referred to Hokayem and Spong [2006] and Arcara and Melchiorri [2002] for two detailed surveys regarding the control of teleoperators.

Recently, Chopra and Spong have presented an interesting control architecture, based on the passive output synchronization of $n$-agents. In their work they have achieved delay-independent output synchronization of the agents, for any constant time-delay (Chopra and Spong [2007], Chopra and Spong [2006]). They have shown that using a symmetric controller on the teleoperator and by matching their impedances with the virtual transmission line, the scheme reduces to the one used for the passive output synchronization. The stability for both schemes is analyzed using the passivity property of the transmission line, for the scattering transformation, and with a Liapunov-Krasovskii functional, respectively. In this work we revisit this analysis and present some teleoperation simulations and experiments that show the stable behavior of the overall system.

The paper is arranged as follows: modeling the $n$-DOF teleoperator is shown in Section 2; Section 3 and Section 4 analyze the stability of the scattering transformation and the passive output synchronization schemes, respectively; some simulations are shown in Section 5 with the experiments in Section 6; finally, the conclusions and future work are outlined in Section 7.

2. MODELING THE $n$–DOF TELEOPERATOR

Before continuing, let us introduce the following notation. $R := (−∞, ∞), R^+ := (0, ∞), R^+_0 := [0, ∞)$. $\lambda_m[A]$ and $\lambda_M[A]$ represent the minimum and maximum eigenvalue of matrix $A$, respectively. $|·|$ stands for the Euclidean norm and $∥·∥_2$ for the $L_2$ norm. In order to keep equations as clear as possible, the argument of all time dependent signals will be omitted (e.g. $\dot{q}(t) \equiv \dot{q}$), except for those which are time-delayed (e.g. $\dot{q}(t − τ)$). Following this reasoning, the argument of the signals inside the integrals will be omitted, and it is supposed that is equal to the variable on the differential, unless otherwise noted (e.g. $\int_0^t x(\sigma) d\sigma \equiv \int_0^t x(\sigma) d\sigma$).
The master and the slave are modeled as a pair of n-degree of freedom (DOF) serial links with revolute joints. Their corresponding nonlinear dynamics are described by
\[
\begin{align*}
M_m(q_m)\ddot{q}_m + C_m(q_m, \dot{q}_m)\dot{q}_m + g_m(q_m) &= \tau^*_m - \tau_h \\
M_s(q_s)\ddot{q}_s + C_s(q_s, \dot{q}_s)\dot{q}_s + g_s(q_s) &= \tau_e - \tau_s
\end{align*}
\]
where \(\dot{q}_m, \dot{q}_s, q_m, q_s \in \mathbb{R}^n\) are the acceleration, velocity and joint position, respectively. \(M_s(q_s) \in \mathbb{R}^{n \times n}\) are the inertia matrices, \(C_s(q_s, \dot{q}_s) \in \mathbb{R}^{n \times n}\) the coriolis and centrifugal effects, \(g_s(q_s) \in \mathbb{R}^n\) represent the vectors of gravitational forces, \(\tau^*_m \in \mathbb{R}^n\) are the control signals and \(\tau_h \in \mathbb{R}^n\), \(\tau_e \in \mathbb{R}^n\) are the forces exerted by the human operator and the environment interaction, respectively. \(i = m\) represents the master and \(i = s\) the slave.

These robot dynamic models have some important properties:

P1. Due to the fact that all joints are revolute then \(M_i(q_i)\) is lower and upper bounded, i.e.
\[
0 < \lambda_m(M_i(q_i))I \leq M_i(q_i) \leq \lambda_M(M_i(q_i))I < \infty
\]

P2. The Coriolis matrix \(C_i(q_i, \dot{q}_i)\) is given by
\[
C_{ij}^{kl}(q_i, \dot{q}_i) = \sum_{k=1}^{n} \frac{1}{2} \left[ \frac{\partial M_{ik}^{kl}}{\partial q_j} + \frac{\partial M_{jk}^{kl}}{\partial q_i} - \frac{\partial M_{ij}^{kl}}{\partial q_k} \right] \dot{q}_j
\]
where \(\Gamma_i^{kl}(q_i)\) are the Christoffel symbols of the first kind with the symmetric property that \(\Gamma_i^{kl}(q_i) = \Gamma_i^{lk}(q_i)\), hence, the matrix \(M_i(q_i) - 2C_i(q_i, \dot{q}_i)\) is skew-symmetric, i.e.
\[
M_i(q_i) = C_i(q_i, \dot{q}_i) + C_i^{T}(q_i, \dot{q}_i)
\]

2.1 General Assumptions

A1. Following standard considerations, we assume the human operator and the environment define passive (force to velocity) maps, that is, there exists \(\kappa_i \in \mathbb{R}^+_0\) s.t.
\[
\int_0^t q_m^T \tau_h ds \geq -\kappa_m, \quad -\int_0^t \dot{q}_m \cdot \tau_e ds \geq -\kappa_s
\]
for all \(t \geq 0\).

A2. In order to simplify some calculations and to focus on the main idea of this article, we will assume that the gravitational forces are precompensated by the controllers \(\tau^*_m, \tau^*_s\) (i.e. \(\tau_m = \tau_m + g_m(q_m)\) and \(\tau_s = \tau_s - g_s(q_s)\)). Hence, the dynamical models (1) are reduced to
\[
\begin{align*}
M_m(q_m)\ddot{q}_m + C_m(q_m, \dot{q}_m)\dot{q}_m + g_m(q_m) &= \tau^*_m - \tau_h \\
M_s(q_s)\ddot{q}_s + C_s(q_s, \dot{q}_s)\dot{q}_s + g_s(q_s) &= \tau_e - \tau_s
\end{align*}
\]

A3. We assume that the time-delay imposed by the communication channel is constant on each direction, but it may differ from one to another. The total round trip time-delay is equal to \(T_m + T_s \geq 0\).

3. SCATTERING TRANSFORMATION

The scattering transformation is given by
\[
V_m = \frac{1}{\sqrt{2b}} [\tau_{md} - b \dot{q}_{md}] \quad V_s = \frac{1}{\sqrt{2b}} [\tau_{sd} - b \dot{q}_{sd}]
\]
where \(b\) is the virtual impedance of the transmission line. The master and the slave are interconnected, for constant time-delays in the forward and the backward paths (\(T_m\) and \(T_s\), respectively), as
\[
\begin{align*}
u_m &= u_m(t - T_m) \quad \nu_m = v_s(t - T_s)
\end{align*}
\]

Proposition 1. Consider the teleoperator given by (4) controlled by
\[
\begin{align*}
\tau_m &= \tau_{md} - B_m \dot{q}_m \\
\tau_s &= \tau_{sd} + B_s \dot{q}_s
\end{align*}
\]
with the communications governed by (5) and (6).

Proof. Let us propose the following Liapunov function
\[
V(q_i, \dot{q}_i) = \frac{1}{2} q_i^T M_i(q_i) q_i + \frac{1}{2} \dot{q}_i^T C_i(q_i, \dot{q}_i) \dot{q}_i + \frac{1}{2} \dot{q}_i^T C_i^{T}(q_i, \dot{q}_i) \dot{q}_i
\]

relating on the assumption A1, the property P1 of robot manipulators, and the well known fact that
\[
\int_0^t |\dot{q}_{md}^T \tau_{sd} - \dot{q}_m^T \tau_{sd} - \dot{q}_{sd}^T \tau_{md}| ds \geq 0
\]
which is the main contribution of the scattering transformation, we show that the Lyapunov candidate (9) is positive definite and radially unbounded.

Using the property P2, the resulting time derivative of \(V\) is given by
\[
\dot{V} = -B_m |\dot{q}_m|^2 - B_s |\dot{q}_s|^2 - K_{dm} |\dot{q}_m - \dot{q}_{md}|^2 - K_{ds} |\dot{q}_s - \dot{q}_{sd}|^2
\]

The use of scalar gains is made for simplicity. When these gains are positive definite diagonal matrices can be treated with slight modifications to the proof.
From which can conclude that the system velocities $\dot{q}_m$ and $\dot{q}_s$ asymptotically converge to the origin (part i, of the proposition). Moreover, invoking LaSalle Theorem for time-delayed systems [Hale (2006)], all solutions of (4), (7) and (6) converge to $M$, the largest invariant set in $S$. Where $S = \{\tilde{q}_i \in \mathbb{R}^n, i = \{m, s\} : \dot{V} = 0\}$ s.t. $\{\tilde{q}_m = \tilde{q}_{md}, \tilde{q}_s = \tilde{q}_{sd}\}$. In order to prove part ii, from (7) and (8) we can find that

$$
\dot{q}_{md} = \frac{K_{ds}}{b + K_{dm}} \dot{q}_d(t - T_s) + \frac{K_{dm}}{b + K_{dm}} \dot{q}_m + \frac{b - K_{ds}}{b + K_{dm}} \dot{q}_{ad}(t - T_s)
$$

$$
\dot{q}_{sd} = \frac{K_{ds}}{b + K_{ds}} \dot{q}_m(t - T_m) + \frac{K_{dm}}{b + K_{dm}} \dot{q}_s + \frac{b - K_{dm}}{b + K_{ds}} \dot{q}_{ad}(t - T_m)
$$

(12)

note that, by matching the impedances, the terms $\dot{q}_{ad}(t - T_i)$ on (12) disappear, and so do wave reflections. Using these new expressions the controllers become

$$
\tau_m = \frac{b}{2} \dot{q}_m(t - T_s) - \dot{q}_m - B_m \dot{q}_m
$$

$$
\tau_s = \frac{b}{2} \dot{q}_s(t - T_m) + B_s \dot{q}_s
$$

(13)

and the time derivative of $V$ given by (11) transforms into

$$
\dot{V} = -B_m |\dot{q}_m|^2 - B_s |\dot{q}_s|^2 - \frac{b}{4} |\dot{q}_m - \dot{q}_s(t - T_s)|^2 - \frac{b}{4} |\dot{q}_s - \dot{q}_m(t - T_m)|^2
$$

(14)

Similarly by LaSalle, $\dot{q}_m - \dot{q}_s(t - T_s) \to 0$ as $t \to \infty$. \qed

4. OUTPUT SYNCHRONIZATION

The control laws for the teleoperator are given by

$$
\tau_m = K[\dot{q}_m(t - T_s) - \dot{q}_m] - B_m \dot{q}_m
$$

$$
\tau_s = K[\dot{q}_s(t - T_m) - \dot{q}_s] + B_s \dot{q}_s
$$

(15)

where $K > 0$ is the controller gain. $T_i \geq 0, i = \{m, s\}$, is the time delay in the forward and backward paths, respectively. It is assumed that the initial conditions $\dot{q}_i(\theta) \in C_n^e$ for $\theta \in [-T, 0]$.

Proposition 2. (Chopra and Spong (2006)). Consider the teleoperator system (4) controlled by (15). Assume that $\tau_h$ and $\tau_e$ verify (3). Then all signals are bounded and the system velocity error, defined by $e = \dot{q}_m - \dot{q}_s(t - T_s)$, asymptotically converges to the origin, for any $K > 0$ and all $T_i \geq 0$.

Proof. Consider the following Lyapunov-Krasovskii functional, $V(t, \dot{q}_m, \dot{q}_s(t - T_s))$, given by

$$
V = \dot{q}_m^T M_m (q_m) \dot{q}_m + \dot{q}_s^T M_s (q_s) \dot{q}_s + \int_0^t \dot{q}_m^T \tau_h - \dot{q}_s \tau_e d\sigma + \kappa_m + \kappa_s + K \int_0^t |q_m(\theta)|^2 d\theta + K \int_t^{t + T_m} |q_s(\theta)|^2 d\theta
$$

(16)

Taking the time derivative of $V$ and using the property P2 of robot manipulators, we obtain

$$
\dot{V} = 2 \dot{q}_m^T \tau_m - 2 \dot{q}_s^T \tau_s + K[\dot{q}_m(t - T_m)]^2 - K[\dot{q}_m(t - T_m)]^2 + + K[\dot{q}_s(t - T_s)]^2
$$

(17)

The last four terms come from the Krasovskii-functional, and can be written as:

$$
K[\dot{q}_m(t - T_m)]^2 [\dot{q}_m + \dot{q}_s(t - T_s)] + + K[\dot{q}_s(t - T_m)]^2 [\dot{q}_m + \dot{q}_s(t - T_m)]
$$

(18)

using the expressions (15) and some algebraic manipulations, it is easily seen that (17) becomes

$$
\dot{V} = -2 B_m |\dot{q}_m|^2 - 2 B_s |\dot{q}_s|^2 - K|\dot{q}_m - \dot{q}_s(t - T_s)|^2 - K|\dot{q}_s - \dot{q}_m(t - T_m)|^2
$$

(19)

$$
\leq 0
$$

Towards this end, the Liapunov-Krasovskii functional only ensures uniform stability, but, using an extension of LaSalle’s Invariance principle for FDE, we can obtain uniform asymptotic stability of the $\dot{V}$ signals.

Note that, the function $V(t, \dot{q}_m, \dot{q}_s(t - T_s))$ is positive definite, and $\dot{V} \leq 0$ any arbitrary initial level set is positively invariant, hence, all signals in (4) are bounded.

Now, consider the set $S = \{\tilde{q}_i \in \mathbb{R}^n, i = \{m, s\} : \dot{V} = 0\}$ characterized by all trajectories s.t. $\{\dot{q}_s(t - T_s) = \dot{q}_m, \dot{q}_m(t - T_m) = \dot{q}_s\}$. Let $M \subset S$ be the largest invariant set in $S$. Using LaSalle Theorem (Hale [2006]), all solutions of (4) and (15) converge to $M$ as $t \to \infty$. This implies that the velocity error $e$ asymptotically converges to the origin for all $T_i \geq 0$ and all positive definite gain $K$. \qed

5. SIMULATIONS

This section presents a simulation of the teleoperator system. The master and slave are two link manipulators with revolute joints. Their corresponding nonlinear dynamics follow (1). The inertia matrix $M_i(q_i)$ is given by

$$
M_i(q_i) = \begin{bmatrix}
\alpha_i + 2\beta_i \cos(q_{i2}) & \delta_i + \beta_i \cos(q_{i2}) \\
\beta_i \cos(q_{i2}) & \delta_i
\end{bmatrix}
$$

$q_{i1}, k \in \{1, 2\}$ is the articular position of each link, $\alpha_i = l^2_{i2} m_{i2}, \beta_i = l_{i1} l_{i2} m_{i2}$, and $\delta_i = l^2_{i2} m_{i2}$, The lengths for both links $l_{i1}$ and $l_{i2}$, in each manipulator, are 0.38m. The mass of each link correspond to $m_{i1} = 3.9473kg, m_{i2} = 0.6232kg, m_{i2} = 3.2409kg$ and $m_{i2} = 0.3185kg$, respectively. These values are the same of those used by in Lee and Spong [2006]. Coriolis and centrifugal forces are modeled as the vector $C_i(q_i, \dot{q}_i, \ddot{q}_i)$, which are

$$
C_i(q_i, \dot{q}_i, \ddot{q}_i) = \begin{bmatrix}
-\beta_i \sin(q_{i2}) \dot{q}_{i1}^2 - \beta_i \sin(q_{i2}) \dot{q}_{i1} \dot{q}_{i2} \\
\beta_i \sin(q_{i2}) \dot{q}_{i1}^2
\end{bmatrix}
$$

$\dot{q}_{i1}$ and $\dot{q}_{i2}$ are the respective revolute velocities of the two links. The gravity effects $(g_i(q_i))$ for each manipulator are represented by

$$
g_i(q_i) = \begin{bmatrix}
g_1 \delta_i \cos(q_{i1}) + q_{i2} \\
\frac{1}{l_{i1}} (\alpha_i - \delta_i) \cos(q_{i1})
\end{bmatrix}
$$
Note that this vector is precompensated following Assumption 2. $\tau_h$ and $\tau_e$ are the operator and environmental torques. At this point, it should be addressed that the human exerts a force $f_h$ on the master manipulator’s tip, and the slave interaction force with the environment $f_e$ is also measured at the manipulator’s tip. Hence, for the simulations the following expressions are used $\tau = J_m^\top(q_m)f_h$ and $\tau = J_s^\top(q_s)f_e$, where $J_m^\top(q_m)$ is the Jacobian transposed of the robot manipulator. The controllers for these simulations are given by (15), with $K = 5$ and $B_i = 0.5$. The time delay is $T_m = 2.5s$ and $T_s = 3.5s$. The simulation has been carried out using MatLab Simulink™.

Two simulations were carried out: on the first, the human exerts a force on the master and the slave moves freely on the environment (Fig. 1); on the second, the human moves the master and the slave touches a high-stiffness virtual wall on the environment (Fig. 2). This virtual wall has been located in cartesian coordinates at $y = 0.25m$. The stiffness of the wall was set to 20000 $N/m$ with 10 $N/m$ of damping. We assume that the system is frictionless. On Fig. 1 the system moves freely, velocity error converges to the origin and it is clearly seen that the teleoperator is stable. Moreover, if touching a high-stiff wall, around 6s on Fig. 2, the system remains stable, but, position drift arises.

6. EXPERIMENTS

The experimental test-bed mainly consists of two direct-drive two DOF nonlinear manipulators. These manipulators are made of aluminium and are actuated by two pairs of Compumotor DM1015-B brushless DC motors. Optical encoders are used to measure the joint position, the joint velocity is digitally estimated and filtered. Two JR3 force-torque sensors, located at the manipulators end-effectors, are used to measure the force interaction with the human operator and environment, respectively. The controllers are implemented using WinCom 3.3 that enables Simulink™ models to interact with external hardware in real time. The sampling time is set to 4ms. An aluminium wall is located at one side of the slave in order to test the stability while interacting with an stiff environment. This setup is located at the CSL, UIUC, and is depicted in figure 3.

There are two experiments, the teleoperator moves freely on Fig. 4 and touches an aluminium wall around 12s on Fig. 5. The static friction has caused the teleoperator to exhibit a position error, although moving freely ($q_1, q_2$ on Fig. 4). However, the system is stable despite touching the aluminium wall. In order to set the control gain $K$ it can be used a model-based tuning method such as the one on pp. 213 of Kelly et al. [2005]. The master and slave controllers follow (15) and the gains were set to $K = 10$, $B_i = 2$, for both experiments. The two 2 DOF manipulators move on a parallel plane tangential to the earth surface, hence, the gravity vector is zero.

7. CONCLUSIONS AND FUTURE WORK

Chopra and Spong [2006] introduced the passive output synchronization, which has been the base platform for this paper. Using a symmetric controller with the scattering
transformation and matching the impedances, one can reduce the system to a passive output synchronization scheme. Both schemes ensure stability for any arbitrary constant time-delay. However, under velocity control, both schemes do not provide position synchronization. Future steps on this line are the study of the effects of variable time-delays and the analysis of position drift free schemes.

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REFERENCES

Fig. 4. Experiments with $T_m = 2.5\text{ s}$ and $T_s = 3.5\text{ s}$.

Fig. 5. Experiments touching the aluminium wall, with $T_m = 2.5\text{ s}$ and $T_s = 3.5\text{ s}$. 