A Self Tuning Suspension Controller for Multi-body Quarter Vehicle Model

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Abstract: In this paper we derive both LTI (Linear Time Invariant) and LPV (Linear Parameter Varying) controllers according to the $H_{\infty}$ methodology, based on a simple two degree-of-freedom quarter vehicle model using an industrial criterion to handle the compromise between comfort and suspension deflection. As such a model is very simplified, a validation of these control designs is performed on a multi-body dynamical model of the quarter vehicle, much closer to a realistic car which makes the solution interesting for implementation issues.

Keywords: Suspension, Multi-body dynamical vehicle modeling, LTI-LPV/$H_{\infty}$ control, Co-simulation.

1. INTRODUCTION

Suspension’s aim is to isolate the vehicle chassis to an uneven ground and to provide a good road-holding to ensure passenger safety, especially in driving manoeuvres. Many suspension control design have been studied in the past few years: Skyhook (Poussot-Vassal et al., 2006), $H_{\infty}$ (Samier et al., 2000; Zin et al., 2005), mixed $H_{\infty}/H_2$ (Abdellahi et al., 2000; Gáspár et al., 1998; Lu and DePoyster, 2002; Takahashi et al., 1998), LQ (Hrovat, 1997), MPC (Canale et al., 2006) and LPV (Fialho and Balas, 2002; Gáspár et al., 2004; Zin et al., 2006). Most of these controllers are designed and validated using a two degree of freedom nonlinear model that only catches vertical behavior. But it is well known that suspension systems have a specific geometry that involve other forces and moments than the vertical one (Gillespie, 1992).

The contribution is to provide an LPV controller achieving either comfort or road-holding objectives according to a rough rule evaluated thanks to an industrial performance criterion. Then the efficiency of this methodology is shown using simulations on a dynamical multi-body based quarter-car model.

The paper is organized as follow: in Section 2, presentation and comparison of the models, either 2-DOF (Degree Of Freedom) and multi-body, are done. In Section 3, different control strategies (LTI and LPV) are derived according to comfort and/or road-holding specifications (evaluated through an industrial criterion). Section 4 presents simulation results performed on the multi-body dynamical quarter model and validate the approach of the control design methodology. Concluding remarks and perspectives are given in Section 5.

2. SUSPENSION MODELING

2.1 Quarter vehicle simplified model

The simplified quarter vehicle model involved here includes the sprung mass ($m_s$) and the unsprung mass ($m_{us}$). The caught motions by this model are the vertical displacement of the chassis ($z_s$) and of the unsprung mass ($z_{us}$). As the damping coefficient of the tire is negligible, it is simply modeled by a spring linked to the road ($z_r$) where a contact point is assumed. The passive suspension, located between $m_s$ and $m_{us}$, is modeled by a damper and a spring (Figure 1, left) and the active one, by a spring and a force (Figure 1, right).

![Diagram](image)

Fig. 1. Passive (left) and Active (right) quarter vehicle 2-DOF model.

The nonlinear reference model (that represents a passive suspension model designed by the car manufacturer) is given by:

$$
\begin{align*}
    m_s \ddot{z}_s &= -F_k(z_{def}) - F_c(\dot{z}_{def}) \\
    m_{us} \ddot{z}_{us} &= F_k(z_{def}) + F_c(\dot{z}_{def}) - k_t(z_{us} - z_r) \\
    z_{def} &\in [\ddot{z}_{def}, -\ddot{z}_{def}]
\end{align*}
$$

and the nonlinear active model, when control is applied (see next Section), is given by:
\[
\begin{align*}
    m_s \ddot{z}_s &= -F_k(z_{def}) + u \\
m_{us} \ddot{z}_{us} &= F_k(z_{def}) - u - k_t(z_{us} - z_r) \\
    z_{def} \in \left[ z_{def}^-, z_{def}^+ \right]
\end{align*}
\]

where \( F_k(\cdot) \), \( F_r(\cdot) \), are nonlinear functions of \( z_{def} = z_s - z_{us} \) and \( z_{def} = z_s - z_{us} \) respectively (see Figure 2), \( u \) is the control input, and \( k_t \) is the tire stiffness (Table 1 gives identified parameters obtained on a Renault Scenic car, Zin et al. (2004)).

![Fig. 2. Nonlinear spring \( F_k \) (left) and damper \( F_c \) (right) characteristics.](image)

If the linear system is considered, \( F_k(z_{def}) = k(z_s - z_{us}) \) and \( F_r(\dot{z}_{def}) = c(\dot{z}_s - \dot{z}_{us}) \).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_s )</td>
<td>315kg</td>
<td>sprung mass</td>
</tr>
<tr>
<td>( m_{us} )</td>
<td>37.5kg</td>
<td>unsprung mass</td>
</tr>
<tr>
<td>( k )</td>
<td>29500N/m</td>
<td>suspension linearized stiffness</td>
</tr>
<tr>
<td>( c )</td>
<td>1500N/m/s</td>
<td>suspension linearized damping</td>
</tr>
<tr>
<td>( k_t )</td>
<td>208000N/m</td>
<td>tire stiffness</td>
</tr>
<tr>
<td>( z_{def} )</td>
<td>[-0.09; 0.05]m</td>
<td>suspension deflection bounds</td>
</tr>
</tbody>
</table>

Table 1. Linearized Renault Scenic parameters

2.2 Quarter vehicle multi-body dynamical model

The suspension can also be modeled using multi-body dynamic modeling software dedicated to vehicle simulation and analysis. Such a model is much more complex than the 2-DOF model described below. It takes into account the material properties, the geometry of the suspension and the type of joints between each mechanical elements. It models the vertical displacement of the suspended mass, the wheel, the moment created between the car and the wheel and the lateral forces. Concerning the tire, a contact path is also considered (Figure 3).

![Fig. 3. Multi-body 3D quarter car model built for simulation.](image)

3. CONTROL DESIGN

In order to derive a consistent controller, achieving different performances objectives, we first introduce an industrial criteria that allows to clearly specify the desired performances (comfort, road-holding). Then, in the LPV framework presented thereafter, it allows the designer to derive a scheduling strategy.

3.1 Performance criteria

In order to estimate the comfort, the vertical motion (\( z_s \)) and acceleration (\( \dot{z}_s \)) of the chassis have to be studied. The wheel vertical motion (\( z_{us} \)) and the suspension deflection (\( z_{def} \)) are related to road-holding specifications (Zin, 2005). In the following, four performance objectives are derived from industrial control specifications (Sammier et al., 2003) that are consistent with the one given in (Gillespie, 1992):

1. Comfort at high frequencies:
   The vibration isolation between \([4 - 30]Hz\) is evaluated by the transfer function \( \ddot{z}_s/\dot{z}_r \). The vertical acceleration of the chassis has to be limited in order to obtain good comfort at high frequencies \((> 5Hz)\), although the human body is not sensitive to vertical accelerations at high frequencies \((> 10Hz)\).

2. Comfort at low frequencies:
   The vibration isolation between \([0 - 5]Hz\) is evaluated by the transfer function \( z_s/z_r \). Ideally, the vertical displacement of the chassis should be the same as that of the road for low frequencies (lower than around 1Hz) and null for high frequencies (higher than around 1Hz). In practice, for low disturbances (\( z_r < 3cm \)), the maximal gain occurring between 1 and 5Hz of \( z_s/z_r \) has to be bounded by 1.8.

3. Road-holding:
   As indicated before, it is evaluated with the transfer function of \( z_{us}/z_r \). For a good road-holding, the maximal gain, in the range \([0 - 20]Hz\), of the considered transfer function has to be limited to 1.8 (for low disturbance).

4. Suspension constraints:
   The transfer \( z_{def}/z_r \) is a road-holding indicator and also a constraint on the deflection of the actuator evaluated between \([0 - 20]Hz\) in order to preserve its life cycle.

In each case the issue is to perform better than a passive suspension does. Therefore, to compare the control approach proposed thereafter with the passive one, the power spectral density (PSD) measure of each of these signals along the frequency and magnitude space of interest is used as the following formula:

\[
I_{f_1 \rightarrow f_2}(x) = \sqrt{\int_{f_1}^{f_2} x^2(f) df} \quad (1)
\]
where $f_1$ and $f_2$ are the lower and higher frequency bounds respectively and $x$ is the signal of interest.

Based on this PSD formulation, we derive the following criterion (which is a linear combination of the performance objectives described before):

$$J_0(Y) = v_1 \max_{z_1} I_{4-30}(z_1/z_{r1}) + \max_{z_2} I_{0-5}(z_2/z_{r2})$$
$$+ v_2 \max_{z_{u1}} I_{0-20}(z_{u1}/z_{r1})$$

where $Y \in S$ is the set of bounded degree of freedom (or parameters) of the control design involved, $v_i$ ($i = \{1, 2\}$) are weights according to defined objectives (comfort, road-holding) with the achieved $\gamma_\infty$. Note that $v_1$, $v_2$ are related to comfort specifications and $v_1$, $v_2$ to road-holding performances. Then, the problem is to find $Y^* \in S$ s.t. $\forall Y \in S, J_0(Y^*) \leq J_0(Y)$. In (Poussot-Vassal et al., 2006), authors use the same criteria with $Y = \{\alpha, c_{sky}\}$ to tune in an optimal way the Skyhook parameters. A contribution in this paper is also to extend such a criteria to $H_\infty$ design approach where the definition of the optimal weighting functions often is a complex engineer problem.

3.2 LPV/$H_\infty$ control, a polytopic approach

Even if it is not the only way, $H_\infty$ control is often used to tackle frequency specifications and ensure robustness. But performances are fixed (Chen and Guo, 2005). Then, LPV control is used either to enforce robustness by scheduling the controller according to measured varying parameters (Zin et al., 2006) or to change the desired performances by scheduling the controller objectives using exogenous parameters (Fialho and Balas, 2002; Poussot-Vassal et al., 2007). Here we aim at synthesizing a controller scheduled according to the deflection level of the nonlinear suspension spring. Such a controller aims at achieving either comfort and deflection limitation adapting the performance objectives to the deflection of the suspension spring, which can be viewed as an image of the road disturbance, adjusting the performances with respect to the driving conditions. The two degree of freedom control law applied is $u = w_{\infty}^T \phi$, where $w_{\infty}$ can be viewed as the linearized damping coefficient of the controlled damper used for the synthesis and $u_{\infty}$, the added energy to achieve the varying performances, obtained by $H_\infty$ synthesis. In (Zin et al., 2006) the $c_0$ parameter is also used as a varying parameter to change the vehicle behavior. In (Gáspár et al., 2007), this parameter is also used to prevent rollover situations. Consider the following LPV generalized plant,

$$\begin{bmatrix}
    \dot{x} \\
    z_{\infty} \\
    y
\end{bmatrix} =
\begin{bmatrix}
    A(\rho) & B(\rho) & B \\
    C(\rho) & D_{cw}(\rho) & D_{cw}(\rho) \\
    0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    x \\
    \omega_{\infty} \\
    u
\end{bmatrix} \quad (2)
$$

where $x = [x_{\text{system}}, x_{\text{weight}}]$ is the states of the system and weight functions, $z_{\infty} = [W_z z_\infty, W_{z_{\text{def}}}, W_{z_u} u]$ are the so-called controlled or performance outputs, $\omega_{\infty} = [W_z z_\infty, W_{z_{\text{def}}}, W_{z_u} u]$ are the exogenous outputs and $\rho \in [\rho_1, \rho_2]$ the varying parameters (here, $\rho_1 \in [0, 1.0, 9]$ and $\rho_2 \in (1 - \rho_1)$). In order to achieve a parameterized generalized problem, the considered weighting functions and block diagram scheme (Figure 4) are assumed.

$$\begin{cases}
    W_z(\rho_1) = 10\rho_1 \frac{1}{s/(2\pi f_{z_1}) + 1} \\
    W_{z_{\text{def}}}(\rho_2) = 20\rho_2 \frac{1}{s/(2\pi f_{z_{\text{def}}}) + 1} \\
    W_z = 5.10^{-2} \\
    W_{z_{\text{def}}} = 7.10^{-3} \\
    W_u = 10^{-4}
\end{cases}
$$

where $f_z = 6Hz$, $f_{z_{\text{def}}} = 1Hz$. $W_z$ and $W_{z_{\text{def}}}$ are shaped in order to reach the requirements previously described. $W_z$ is given in order to limit control signal, $W_{z_{\text{def}}}$ and $W_u$ model road and additive noise respectively. Note that $W_z$ and $W_{z_{\text{def}}}$ are both parameterized by $\rho$. Later, it will be used either to schedule the closed-loop performances or to minimize the above presented criteria.

![Fig. 4. Generalized plant and weighting functions.](image)

Then, the LPV controller to be synthesized is given by,

$$S(\rho) := \begin{bmatrix}
    \dot{x}_c \\
    u
\end{bmatrix} = \begin{bmatrix}
    A(\rho) & B(\rho) & B \\
    C(\rho) & D_{cw}(\rho) & D_{cw}(\rho) \\
    0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    x \\
    \omega_{\infty} \\
    u
\end{bmatrix} \quad (3)
$$

and the resulting closed-loop is given by,

$$CL(\rho) := \begin{bmatrix}
    \dot{x} \\
    z_{\infty}
\end{bmatrix} = \begin{bmatrix}
    A(\rho) & B(\rho) & B \\
    C(\rho) & D_{cw}(\rho) & D_{cw}(\rho) \\
    0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    x \\
    \omega_{\infty}
\end{bmatrix} \quad (4)
$$

Then the corresponding $H_\infty$ synthesis consists of, imposing $T_{\omega} = ||w_{\infty}/w_{\infty}\| < \gamma_\infty$. This problem can be solved thanks to so-called Bounded Real Lemma, extended to LPV systems, which consists in minimizing $\gamma_\infty$ subject to $K > 0$ and (5) (Apkarian and Gahinet, 1995).

$$\begin{bmatrix}
    A(\rho)^T K + K A(\rho) & K B(\rho) C(\rho)^T \\
    B(\rho)^T K & C(\rho) \xi D(\rho) - I
\end{bmatrix} < 0 \quad (5)
$$

As the previous inequality (5) is $\rho$ parameterized, it results in an infinite set of BMI (Bilinear Matrix Inequality) to solve, not tractable for SDP (Semi Definite Programming) solvers. Hence it is solved by relaxing it into a parameterized LMI (Linear Matrix Inequality) (Scherer et al., 1997), and in order to have to solve a finite set of inequalities, the polytopic approach is used. Such an approach consists of finding a common Lyapunov candidate $K$ and a $\gamma_\infty$ that solves the previous LMI problem at each $n$-vertex of the polytope (defined by the number of varying parameters). Then the control to apply is a convex combination of theses $n$-controllers expressed as follows,
\[ S(\rho) = \sum_{i=1}^{2^i} \alpha_n(\rho) \begin{bmatrix} A_{c_n} & B_{c_n} \\ C_{c_n} & D_{c_n} \end{bmatrix} \]

where,
\[ \alpha_n(\rho) = \frac{\prod_{j=1}^{i} |\rho(j) - C(\Theta_n)_j|}{\prod_{j=1}^{i} (\rho(j) - \rho(j))} \]
and \( \sum_{i=1}^{2^i} \alpha_n(\rho) = 1 \)

where \( i \) is the number of varying parameters and \( n = 2^i \), the number of corners of the polytope. Let note \( \mathcal{P} \) and \( \rho \) the upper and lower bounds of a parameter respectively. Finally, \( C(\Theta_n)_j \) represents the complementary of \( \Theta_n \), which is simply the \( n^{th} \) corner of the polytope (Biannic, 1996; Zin, 2005).

3.3 Optimal \( \rho \) parameter (LTI) and scheduling strategy (LPV)

Performance specifications, for the \( \mathcal{H}_\infty \) case, are given using the weight functions. We first use the criteria expressed at the beginning of this Section, with \( \Upsilon = \{\rho_1, \rho_2\} \). Then, we derive a scheduling strategy so that both comfort and deflection performances can be handled according to road disturbance conditions and make the controller varying. On Figure 5, we plot the criteria \( J_v(\rho_1, \rho_2) \) for two different weight parameters: \( v = \{10, 10, 1, 1\} \) (that improves comfort) and \( v = \{1, 1, 10, 10\} \) (improving road-holding).

Fig. 5. Criteria \( J_v(\rho_1, \rho_2) \) evaluation for \( v = \{10, 10, 1, 1\} \) and \( v = \{1, 1, 10, 10\} \).

Closed-loop frequency behavior of the optimal comfort (resp. road-holding) \( \mathcal{H}_\infty \) configuration is achieved with \( \{\rho_1 = 1, \rho_2 = 0.1\} \) (resp. \( \{\rho_1 = 0.3, \rho_2 = 1\} \)) and given on Figure 6 (resp. Figure 7). Note that these results are consistent with the weight interpretation given below.

Even if both obtained configuration provide good results and clearly improve passive behavior, a compromise between comfort and deflection is done. A smart controller would provide comfort, in normal cruise situations, when suspension deflection is small, and limit deflection in emergency cases, when deflection reaches the boundaries. Then, our LPV strategy consists of giving more importance to comfort weight when the suspension is in the linear part (far from deflection limits), and conversely, give more importance to deflection weight when suspension reaches its bounds. According to the results obtained thanks to Figure 5, an LPV controller and a scheduling strategy are build in order to achieve different objectives according to the situation. Bode diagrams are given on Figure 8.

Fig. 6. Passive (dashed) and LTI/\( \mathcal{H}_\infty \) comfort oriented closed-loop (solid) Bode diagrams. \( z_s/z_r \) (left) and \( z_{def}/z_r \) (right).

Fig. 7. Passive (dashed) LTI/\( \mathcal{H}_\infty \) road-holding oriented closed-loop (solid) Bode diagrams. \( z_s/z_r \) (left) and \( z_{def}/z_r \) (right).

4. MULTI-BODY MODEL BASED SIMULATION RESULTS

Validation of the LTI-LPV/\( \mathcal{H}_\infty \) controllers is done using the multi-body dynamical quarter vehicle model (Figure...
3). The co-simulation is performed using ADAMS software to model the quarter car model in the MATLAB environment, where the controllers are synthesized. At \( t = 5s \), a \(-4cm\) step bump affects the system. On Figures 10, 11 and 12 all proposed strategies are compared.

Fig. 9. Nonlinear suspension stiffness parameter (up) and scheduling \( \rho \) strategy (down).

Fig. 10. Suspended mass displacement (\( z_s [m] \)).

Fig. 11. Suspended mass acceleration (\( \ddot{z}_s [m/s^2] \)).

Fig. 12. Suspension deflection (\( z_{\text{def}} [m] \)), solicitation.

LTI road-holding controller considerably deteriorates the chassis displacement and acceleration while reducing suspension deflection (hence its solicitation). The LPV control strategy shows a good compromise improving road-holding (reducing suspension deflection) when the bump occurs, and improving comfort the rest of the time (see scheduling on Figure 14).

Fig. 13. Interface between MATLAB and ADAMS software.

Finally, Figure 13 shows the interface between MATLAB and ADAMS software.

5. CONCLUSION AND FUTURE WORKS

In this paper we investigate an LPV/\( \mathcal{H}_\infty \) control strategy that tunes the controller objectives according to some driving situations by varying the weight functions (function of the spring deflection). We use a performance criteria to evaluate the optimal parameters for comfort and/or deflection objectives and apply it to the \( \mathcal{H}_\infty \) methodology. This criteria was also used to find a good scheduling strategy for the LPV controller (which is a key point in all adaptive strategies). Validation of the controllers have been performed in co-simulation using a multi-body dynamical model of the quarter vehicle that involves complex kinetic and dynamical phenomenons. Such tests make the validation closer to the reality than the simple 2-DOF model.

Future works will consists in extending the multi-body dynamical model to the full vehicle and to develop a global attitude control strategy, using the four suspensions. Then, implementation on a real suspension system is an issue.
REFERENCES


