Fuzzy logic application to drying kinetics modelling

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Abstract:

A Fick’s model that includes a Takagi-Sugeno fuzzy model to estimate the effective diffusivity was analyzed. The modelling of drying kinetics on mango trough this diffusional-fuzzy model was compared with the theoretical Fick’s model and the empirical Peleg and Weibull models. The identification and validation was performed from experimental drying curves of ripe mango slices (Mangifera indica L. cv. Tommy Atkins) at constant air velocity (4 m/s) and different drying temperatures (40, 50, 60 and 70 °C). The fuzzy sets for the antecedent of Takagi-Sugeno system were identified by the Gustafson-Kessel clustering algorithm and approximated by membership functions of piecewise exponential form. On identification and validation, the diffusional-fuzzy model showed best results than the Fick’s model, whereas it showed little difference with the Weibull and Peleg models. The diffusional-fuzzy model keeps the interpretability of Fick’s model, improves the process simulation and avoids phenomenon and property considerations which require additional experimental and modelling work.

1. INTRODUCTION

Over the last years, dehydration food processes have advanced through merging of new technologies that make possible a more deep and complete study of processes. On this way, the modelling techniques have been applied for process design, control and optimization, as well as the study and research of involved phenomena (Mujumdar, 2006a).

These efforts has been realized on hot-air drying, one of the most ancient, common and diverse processes of conservation. Also, it is one of the industrial activities that demand more energy resources (Mujumdar, 2006b), in consequence, it causes considerable environmental effects, at the same time that affects significant parameters of commercial acceptance as organoleptic and nutritional quality of products (Jayaraman & Das Gupta, 2006, Raghavan & Orsat, 2006).

In order to advance on drying technologies, R&D approach leads to techniques that offer quality products and reduce the environmental impact by means of industrial energy efficiency. This approach includes, in relevant way, the establishment of mathematical models to study, develop or improve new or existing process and equipments (Mujumdar, 2006a).

The fuzzy modelling uses artificial intelligence techniques to model non-linear systems by sub-models integration. It is an alternative methodology on some complex processes that do not enable an effective application of traditional modelling techniques and require approximated and fast solutions (Diez, 2003, Labadini & Baker, 2006, Ross, 2004).

The mango (Mangifera indica L.) performs an important economic role for several nations world-wide. According to levels of production and importation, mango is ranked as the third tropical fruit and occupies the fifth place on all fruits on the world (FAO, 2005). Its dehydration is an alternative in order to utilize production surplus and increase the diversification and offer of added-value products (De La Cruz Medina & García, 2003).

The heterogeneity on solid-soluble and fiber content within a mango fruit could influence on the mechanisms that govern the drying process and difficult the representation by the diffusion model when the material properties are considered constant.

In this study the modelling of drying kinetics on mango trough a Fick’s model that includes a Takagi-Sugeno fuzzy model to estimate the effective diffusivity was analyzed. Thus, an alternative model to improve the process simulation was obtained. The resulting estimations were compared with the theoretical Fick’s model and the empirical Peleg and Weibull models.

2. PRELIMINARIES

In the case of foods and other biological materials, the convective drying normally describes the water removal process from a solid material by exposition to a hot-air flux. It is an operation realized to minimize the chemical and enzymatic reactions or prevent the microbial growing and degradation by reducing to a given level the moisture content. This technology occupies an important place on the foodstuff transformation and post-harvest treatments. Also, it entails the diminishing of volume and weight of products that, as fruits and vegetables, minimize packaging, storage and
transport costs (Jayaraman & Das Gupta, 2006, Raghavan & Orsat, 2006).

The food responses during the drying depend on product and hot-air properties of mass and heat transfer. The knowledge of the temperature and moisture distribution and its evolution through time is essential to process and equipment design, quality control and selection of suitable storage conditions and handling practices (Özilgen & Özdemir, 2001).

On the drying process, the variables of interest can be estimated by mathematical models that represent the mass and heat transport phenomena within the product. In this sense, the diffusion model based on the Fick’s second law is one of the most used and cited for agro-food applications. This theoretical model relates the experimental results to physic laws (García-Pascual et al., 2006).

The variation of moisture content across an infinite slab of material as function of time using the Fick’s second law, considering that moisture transport from interior to surface is mostly performed by liquid diffusion and the material is homogeneous and isotropous, can be expressed as (Crank, 1975, García-Pascual et al., 2006):

\[
\frac{\partial M(r,t)}{\partial t} = \frac{\partial}{\partial r} \left( D_e \frac{\partial M}{\partial r} \right) .
\]

(1)

\(M(r,t)\) is the local moisture content at time \(t\) (kg water / kg dried solid), \(t\) is the time (s), \(D_e\) is the effective diffusivity \((m^2/s)\) and \(r\) is the distance at slab centre in thickness direction \((m)\).

The formulation of the mass transport process on the slab proceeds defining its initial and boundary conditions. The initial moisture content in the slab is uniform \((2)\). The condition of system symmetry for the geometry is assumed \((3)\). The moisture content at surface corresponds to equilibrium moisture content with hot air \((4)\) when the effect to external resistance mass transfer is negligible (Simal et al., 2003).

\[M(r,0) = M_0.\]

(2)

\[\frac{\partial M(0,t)}{\partial t} = 0 \iff t = 0.\]

(3)

\[M(L,t) = M_e.\]

(4)

\(M(L,t)\) is the moisture content in the surface at time \(t\) \((kg_w/kg_d)\), \(M_e\) is the equilibrium moisture content \((kg_w/kg_d)\), \(L\) is the slab half-thickness \((m)\).

In the empirical models the proposed by Peleg, to describe the sorption curves, also has been used on the dehydration and rehydration modelling on diverse foodstuffs (Ruíz-Díaz et al., 2003, Simal et al., 2003, García-Pascual et al., 2006). The Peleg model applied to dehydration as shown in \((5)\), where \(\bar{M}\) is the average moisture content \((kg_w/kg_d)\), \(M_0\) is the initial moisture content and, \(k_{p1}\) and \(k_{p2}\) are the model parameters.

\[\bar{M} = M_0 - \frac{t}{k_{p1} + k_{p2}t}.\]

(5)

Other important empirical model is the probabilistic Weibull model, which describe the degradation kinetic of foodstuff submitted to stress conditions by a given time. This has been employed to describe the drying on diverse products in the form of Page’s model (García-Pascual et al., 2006):

\[\bar{M} = M_e + (M_0 - M_e) \exp \left( -\left( \frac{t}{k_{w2}} \right)^{k_{w1}} \right).\]

(6)

Here, \(k_{w1}\) and \(k_{w2}\) were the shape and scale parameters respectively.

3. METODOLOGY

From experimental information, three kinds of models were used to describe the drying kinetic: the theoretical Fick’s model, the empirical models of Peleg and Weibull, and the Fick’s model that integer the fuzzy model to calculate the effective diffusivity.

Matlab® was used as computational platform to program the algorithms of fuzzy-model identification, theoretical and empirical model identification, and numerical solving of partial differential equations of Fickian models.

3.1 Experimental information

Physical and chemical product properties are significant on movement moisture mechanisms that govern the mango drying process. The mango fruit commonly has fibrous pulp, with high content of soluble solids and water. Its heterogeneous ripening process is more pronounced at the exterior and the sunlight incidence zones. This characteristics influence on the existence of zones with different solid-soluble and fiber content, fiber orientation and consistency within a mango fruit. Likewise, on drying process they cause a product response of difficult representation by conventional theoretical models. Particularly, in the models based on the diffusion theory, the influence of those characteristics on the shrinking and internal resistance to transport phenomena should be assumed, raising in a substantial manner the complexity without to improve the solving accuracy.

The identification and validation of the considered models was performed from experimental data for drying curves of ripe mango slices \((Mangifera indica\) L. cv. Tommy Atkins, °Brix = 13.01 ± 0.93, acidity = 0.516 ± 0.078 % malic acid, thickness = 5 mm), dried by hot air at constant conditions during the process. The experiments was realized four times at 40, 50, 60 and 70 °C and 4 m/s hot air conditions until a average moisture content around of 0.25 kg_w/kg_d (Fig. 1). Three of the temperature series were used to fuzzy sets and model parameters identification, while the last one was used to model validations.

Through the discrete differentiation of average moisture content as function of time the drying rate was calculated. The drying rate and the average moisture content (Fig. 2) were selected as input variables to identify the fuzzy sets in the Takagi-Sugeno model, since the diffusion usually depends on moisture content and symbolizes the transport
coefficient in the diffusional mass transfer phenomena that govern the drying mechanisms (Saravacos, 2005).

\[ z_j^* = \frac{z_j - \bar{z}}{\sigma}. \]  

(7)

Here, \( z_j^* \) is the normalized datum of \( z_j \), \( \bar{z} \) and \( \sigma \) are the mean and the standard deviation of data respectively. The asterisk denotes the normalized data.

Fig. 1. Drying curves: average moisture content vs. time. Experimental data for one temperature series.

Fig. 2. Drying curves: drying rate vs. average moisture content. Experimental data for one temperature series.

3.2 Diffusional-fuzzy model formulation

In this work the diffusional-fuzzy model concerns to Fick’s model that integer a Takagi-Sugeno model for estimating the effective diffusivity.

The Takagi-Sugeno model is a base-rules fuzzy system that combines three fuzzy sets in the antecedent and associates each of them with effective diffusivity coefficients in the consequent. It is used to estimate the effective diffusivity as function of its membership to high, medium and low moisture ranges.

The identification of fuzzy sets was carried out by clustering through application of the Gustafson-Kessel algorithm on the normalized input variables (average moisture content and drying rate) (Fig. 3). Thus, antecedent fuzzy sets were established from the fuzzy partition projected on average moisture content.

The Zscore normalization (7) was applied on input variables before clustering because distance norm can be sensitive to variations in the numerical ranges of the different data (Chen et al., 2001). This normalization translates and scales the experimental data so that all the values have zero mean and unit variance.

The rule-base for the Takagi-Sugeno model were established as (8), where \( A \) is the fuzzy partition projected on \( \bar{M} \), \( A_i \) is the antecedent fuzzy set for each class \( i(i = 1, 2, 3) \), \( D_i \) is the effective diffusivity coefficient (m\(^2\)/s) in the consequent associated to each antecedent fuzzy set.

\[ R_i : \text{If } \bar{M} \text{ is } A_i \text{ then } D_i. \]  

(8)

The fuzzy partitions were approximated by membership functions of piecewise exponential form (Fig. 4) to represent mathematically each antecedent fuzzy set.

Fig. 3. Fuzzy clustering of normalized drying rate and average moisture content by Gustafson-Kessel algorithm. Data at 40ºC.

The denormalized membership functions are shown in (9), (10) and (11).

\[ A_1(\bar{M}, \alpha_1, \alpha_1, \beta) = \begin{cases} 1 - A_2 - A_3 & \text{if } \bar{M} < \alpha_1, \\ 1 - A_2 & \text{if } \bar{M} > \alpha_1. \end{cases} \]  

(9)
A2(\mathbf{M}, \alpha_1, \alpha_2, \beta) = \begin{cases} 
\left(\frac{\mathbf{M} - \alpha_1}{\beta}\right)^2 & \text{if } \mathbf{M} < \alpha_1, \\
\left(\frac{\mathbf{M} - \alpha_2}{\beta}\right)^2 & \text{if } \mathbf{M} > \alpha_2, \\
1 & \text{if } \alpha_1 < \mathbf{M} < \alpha_2.
\end{cases}

(10)

A3(\mathbf{M}, \alpha_1, \alpha_2, \beta) = \begin{cases} 
1 - A_2 & \text{if } \mathbf{M} < \alpha_2, \\
1 - A_1 - A_2 & \text{if } \mathbf{M} > \alpha_2.
\end{cases}

(11)

The parameters of membership functions for the antecedent of Takagi-Sugeno model were defined from averages values (Table 1) since the parameters exhibit similar values to different drying temperatures (Fig. 5).

<table>
<thead>
<tr>
<th>Sample</th>
<th>Drying temp. (°C)</th>
<th>(\alpha_1)</th>
<th>(\alpha_2)</th>
<th>(\beta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
<td>2.090</td>
<td>3.174</td>
<td>0.296</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>2.054</td>
<td>3.102</td>
<td>0.252</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>1.963</td>
<td>2.979</td>
<td>0.304</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>2.167</td>
<td>3.091</td>
<td>0.398</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>2.131</td>
<td>2.881</td>
<td>0.384</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>2.254</td>
<td>2.743</td>
<td>0.422</td>
</tr>
<tr>
<td>7</td>
<td>60</td>
<td>1.980</td>
<td>2.961</td>
<td>0.282</td>
</tr>
<tr>
<td>8</td>
<td>60</td>
<td>2.253</td>
<td>3.106</td>
<td>0.432</td>
</tr>
<tr>
<td>9</td>
<td>60</td>
<td>2.229</td>
<td>2.989</td>
<td>0.256</td>
</tr>
<tr>
<td>10</td>
<td>70</td>
<td>1.971</td>
<td>2.988</td>
<td>0.318</td>
</tr>
<tr>
<td>11</td>
<td>70</td>
<td>2.298</td>
<td>3.199</td>
<td>0.388</td>
</tr>
<tr>
<td>12</td>
<td>70</td>
<td>2.191</td>
<td>3.182</td>
<td>0.378</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>2.153</td>
<td>3.038</td>
<td>0.342</td>
</tr>
</tbody>
</table>

The identification and validation of the models for each drying temperature was done. The parameters identified are shown in Table 2, the models’ validation in Fig. 6 and the root mean square deviations and the explained variances in Table 3. The effective diffusivity calculated by Takagi-Sugeno fuzzy model is represented in Fig 7.

The effective diffusivity coefficients identified for the Takagi-Sugeno model were inversely proportional to moisture content of the sample: 8.99e-11, 3.84e-10 and 5.19e-10 m²/s at 40°C; 9.31e-11, 4.57e-10 and 7.15e-10 m²/s.
at 50ºC; 1.60e-10, 6.73e-10 and 1.03e-9 m²/s at 60ºC; 1.17e-10, 1.08e-9 and 1.65e-9 m²/s at 70ºC; for moisture ranges of high (3.3 to 6 kg_w/kg_ds), medium (1.8 to 3.3 kg_w/kg_ds) and low (0 to 1.8 kg_w/kg_ds) respectively (Table 2).

On identification and validation, the diffusional-fuzzy model showed best results than the traditional Fick’s model, whereas it showed little difference with the empirical Weibull and Peleg models, such as is indicated by root mean square deviations and explained variances in Table 3.

In contradistinction to empirical models that lack of meaning as regards heat and mass transfer phenomena, the diffusional-fuzzy model keeps the interpretability of Fick’s model and presents an alternative to effective diffusivity estimation that improves the process simulation. Likewise, it enables to estimate the moisture profiles inside the solid and estimates its evolution through time (Fig. 8), interesting advantages on study, design and optimization of some drying process.
Fig. 7. Effective diffusivity estimated by Takagi-Sugeno fuzzy model.

Fig. 8. Estimated moisture profiles by Fick and diffusional-fuzzy models. Data at 40ºC.

From obtained results, it is deduced that fuzzy logic integration to effective diffusivity within the Fick’s model improves the simulation of drying process on *Mangifera indica* L.

4. ACKNOWLEDGMENTS

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5. REFERENCES


