Design of Robust Power System Stabilizer using Genetic Algorithm-based Fixed-Structure $H_\infty$ Loop Shaping Control


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Abstract: This paper proposes a genetic algorithm (GA)-based fixed-structure $H_\infty$ loop shaping technique to design a robust power system stabilizer (PSS). The fixed-structure of designed PSS is a 2nd-order lead-lag compensator. In the design, system uncertainties are modeled by a normalized coprime factor. The performance and robust stability conditions of the designed system satisfying the $H_\infty$ loop shaping are formulated as the objective function in the optimization problem. The GA is applied to solve an optimization problem and to achieve control parameters of PSS. The performance and robustness against system uncertainties of the designed PSS are investigated in the single-machine infinite bus system in comparison with a conventional PSS and a PSS designed by $H_\infty$ loop shaping. Simulation results show that the robustness and damping effect of the proposed PSS are almost the same as those of the PSS with high-order controller designed by $H_\infty$ loop shaping method.

Keywords: Control system design, modeling, operation and control of power systems, robust control applications.

1. INTRODUCTION

The lack of damping of the electromechanical oscillation modes usually causes severe problems of low frequency oscillations in power systems. To solve this problem, a power system stabilizer (PSS) has been selected as a cost effective device to provide the additional damping via the excitation system (DeMello et al., 1969), (Larsen et al., 1981). Several approaches based on modern control theories have been successfully applied to design PSSs, such as eigenvalue assignment (Zhou et al., 1992), linear quadratic regulator (Aldeen et al., 1995) etc. Since these techniques do not take the presence of system uncertainties e.g. system nonlinear characteristics, variations of system configuration due to unpredictable disturbances, loading conditions etc. into consideration in the system modelling, the robustness of these PSSs against uncertainties can not be guaranteed.

To overcome these problems, $H_\infty$ control has been applied to design of robust PSS (Chen et al., 1995), (Yan, 1997) etc. In these works, the designed $H_\infty$ PSS via mixed sensitivity approach have confirmed the significant performance and high robustness. In this approach, however, due to the trade-off relation between sensitivity function and complementary sensitivity function, the weighting functions in $H_\infty$ control design can not be selected easily. Moreover, the order of $H_\infty$ controller depends on that of the plant. This leads to the complex structure PSS which is different from the conventional lead/lag PSS. Despite the significant potential of control techniques mentioned above, power system utilities still prefer the conventional lead/lag PSS structure. This is due to the ease of implementation, the long-term reliability, etc.

On the other hand, much research on a conventional lead/lag PSS design has paid attentions to tuning of PSS parameters. The parameters of a lead/lag PSS are optimized under various operating conditions by heuristic methods such as tabu search (Abdel-Magid et al., 2001), genetic algorithm (Abdel-Magid et al., 1999), and simulated annealing (Abido, M. A. 2000). In these studies, however, the uncertainty model is not embedded in the mathematical model of the power system. Furthermore, the robust stability against system uncertainties is not taken into consideration in the optimisation process. Therefore, the robust stability margin of the system in these works may not be guaranteed in the face of several uncertainties.

To solve this problem, this paper proposes the robust PSS design by the $H_\infty$ loop shaping technique and GA. The configuration of PSS is a fixed structure with a conventional 2nd-order lead/lag PSS. The normalized coprime factor (NCF) is used to model system uncertainties (Mcfarlane D.C. and K. Glover, 1990). By the advent of NCF approach, the selection of weighting function is significantly simplified. To optimize the control parameters, the performance and robust stability conditions in the $H_\infty$ loop shaping technique are formulated as the objective function. Then, the GA is applied to solve the optimization problem. Simulation study in a single machine infinite bus system is carried out to evaluate the robustness of the designed PSS in comparison with the PSS with high-order designed by $H_\infty$ loop shaping method.

This paper is organized as follows. First, system modelling is explained in section 2. Next, section 3 presents the proposed design procedure for optimization of PSS parameters by GA. Subsequently, section 4 shows the simulation results. Finally, the conclusion is given.
2. SYSTEM MODEL

Fig. 1: System configuration of SMIB

A single machine infinite bus system (SMIB) is shown in Fig. 1. The generator is fitted with the automatic voltage regulator (AVR), an excitation system, and the PSS. A linearized system in Fig. 1 is represented by the Heffron-Phillips model as shown in Fig. 2 (De Mello et al., 1969). This system is represented by a forth-order model with the small deviation of the power angle $\Delta \delta$, the rotor speed $\Delta \omega$, the internal voltage of generator and the field voltage $\Delta E_{fd}$ as the state variables. The initial condition used as the design condition of the proposed PSS is $P_e = 0.8$ p.u., $x_e = 0.2$ p.u. from (Rao, et al. 1999). The state equation of system in Fig. 2 can be expressed as

$$\begin{align*}
\dot{\Delta X} &= A\Delta X + B\Delta u_{pss} \\
\dot{\Delta Y} &= C\Delta X + D\Delta u_{pss} \\
\Delta u_{pss} &= K(s)\Delta \omega
\end{align*}$$

Where the state vector $\Delta X = [\Delta \delta \ \Delta \omega \ \Delta e' \ \Delta E_{fd}]^T$, the output vector $\Delta Y = [\Delta \omega]$, $\Delta u_{pss}$ is the control output signal of the PSS ($K(s)$), which uses only the angular velocity deviation ($\Delta \omega$) as a feedback input signal. Note that the system (1) is a single-input single-output (SISO) system. The proposed GA-based fixed-structure $H_\infty$ loop shaping is applied to design a robust PSS $K(s)$. The system (1) is referred to as the nominal plant $G$.

**3. GENETIC ALGORITHM-BASED FIXED-STRUCTURE $H_\infty$ LOOP SHAPING CONTROL DESIGN**

In this section, the design procedure of a fixed-structure controller using $H_\infty$ loop shaping and GA is explained. The flow chart of the proposed design is shown in Fig. 3.

**Step 1. Selection of Weighting functions $W_1$ and $W_2$**

**Step 2. Formulate the shaped plant $G_s$**

**Step 3. Evaluate the robust stability margin of the system**

**Step 4. Generate the objective function for GA**

**Step 5. Initialize the search parameters for GA**

**Step 6. Randomly generate the initial solutions**

**Step 7. Evaluate Objective function of each individual**

**Step 8. Select the best individual in the current generation**

**Step 9. Gen=Gen+1**

**Step 10. Genetic operator create the new population by selection, cross over and mutation.**

Fig. 3: Flow chart of the proposed design

**Step 1 Selection of weighting functions**

As in the conventional $H_\infty$ loop shaping design, the shaped plant is established by weighting functions. Because the nominal plant is an SISO system, the weighting functions $W_1$ and $W_2$ are chosen as

$$W_1 = K_w \frac{s + a}{s + b} \quad \text{and} \quad W_2 = I \quad (4)$$
Where $K_\alpha$, $a$ and $b$ are positive values. Because, the low frequency oscillation is in the vicinity of 1-2 Hz, $W_i$ is set as a high-pass filter ($a < b$).

**Step 2** Formulate the shaped plant $G_s$.

As shown in Fig. 4, a pre-compensator $W_i$ and a post-compensator $W_2$, are employed to form the shaped plant $G_s = W_i G W_2$, which is enclosed by a solid line. The designed robust controller $K = W_i K_\alpha W_2$ is enclosed by a dotted line where $K_\alpha$ is the $H_\infty$ controller.

![Fig. 4: Shaped plant $G_s$ and designed robust controller $K$](image)

**Step 3** Evaluate the robust stability margin of the system

A shaped plant $G_s$ is expressed in form of normalized left coprime factor $G_s = M_+^{-1} N_+$, when the perturbed plant $G_\Delta$ is defined as

$$G_\Delta = (M_+ + \Delta M_+)^{-1} (N_+ + \Delta N_+) : \|[\Delta N_+ \Delta M_+]\|_\infty \leq 1/\gamma$$

(5)

Where $\Delta M_+$ and $\Delta N_+$ are stable unknown transfer functions which represent uncertainties in the nominal plant model $G$. Based on this definition, the $H_\infty$ robust stabilization problem can be established by $G_\Delta$ and $K$ as depicted in Fig. 5. The objective of robust control design is to stabilize not only the nominal plan $G$ but also the family of perturbed plant $G_\Delta$. In (5), $1/\gamma$ is defined as the robust stability margin. The maximum stability margin in the face of system uncertainties is given by the lowest achievable value of $\gamma$, i.e. $\gamma_{\min}$. Hence, $\gamma_{\min}$ implies the largest size of system uncertainties that can exist without destabilizing the closed-loop system in Fig. 4.

The value of $\gamma_{\min}$ can be easily calculated from

$$\gamma_{\min} = \sqrt{1 + \lambda_{\max}(XZ)}$$

(6)

Where $\lambda_{\max}(XZ)$ denotes the maximum eigenvalue of $XZ$. For minimal state-space realization ($A$, $B$, $C$, $D$) of $G_s$, the values of $X$ and $Z$ are unique positive solutions to the generalized control algebraic Riccati equation

$$A - BS^{-1} D^T C Y + X(A - BS^{-1} D^T C) - XBS^{-1} B^T X + C^T R^{-1} C = 0$$

(7)

and the generalized filtering algebraic Riccati equation

$$(A - BS^{-1} D^T C) Z + Z(A - BS^{-1} D^T C)^T - ZC^T R^{-1} C Z + BS^{-1} B^T = 0$$

(8)

where $R = I + DD^T$ and $S = I + D^T D$. Note that no iteration on $\gamma$ is needed to solved for $\gamma_{\min}$. To ensure the robust stability of the nominal plant, the weighting function is selected so that $\gamma_{\min} \leq 4.0$ (Skogestad, 1996). If $\gamma_{\min}$ is not satisfied, then go to step 1, adjust the weighting function.

**Step 4** Generate the objective function for GA optimization.

In this study, the performance and robust stability conditions in $H_\infty$ loop shaping design approach is adopted to design a robust PSS. The conventional PSS with a 2nd-order lead-lag controller is represented by

$$K(s) = K_\alpha \left( \frac{s T_1 + 1}{s T_1 + 1} \right)$$

(9)

The control parameters $K_\alpha$, $T_1$, $T_2$, $T_3$ and $T_4$ are optimized by GA based on the following concept. As shown in Fig. 4, the designed robust controller $K(s)$ can be written as

$$K(s) = W_i K_\alpha W_2$$

(10)

Because $W_i = I$, $K_\alpha$ controller can be written as

$$K_\alpha = W_i^{-1} K(s)$$

(11)

As given in (Skogestad, 1996), the necessary and sufficient condition of the robust control $K(s)$ is

$$\begin{bmatrix} I & \left[I - G_s K_\alpha \right]^{-1}\left[I - G_s \right] \end{bmatrix} K_\alpha \leq \gamma$$

(12)

By substituting (11) into (12), the robust controller can be written as shown in (13).

$$\begin{bmatrix} I & \left[I - G_s \left( W_i^{-1} K(s) \right) \right]^{-1}\left[I - G_s \right] \end{bmatrix} \leq \gamma$$

(13)

This condition can be formulated as the objective function in the optimization problem as

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Minimize \[
\begin{bmatrix}
\frac{I}{W_1^3 K(s)} (I - G_i W_1^3 K(s))^{-1} (I - G_i)
\end{bmatrix},
\tag{14}
\]
Subject to
\[
\begin{align*}
\zeta & \geq \zeta_{\text{spec}} \\
\sigma & \leq \sigma_{\text{spec}} \\
K_{c,\text{min}} & \leq K_c \leq K_{c,\text{max}} \\
T_{i,\text{min}} & \leq T_i \leq T_{i,\text{max}} \\
& \text{for } i = 1, 2, 3, 4.
\end{align*}
\tag{15}
\]

Where \(\zeta\) and \(\zeta_{\text{spec}}\) are actual and desired damping ratio, respectively, \(\sigma\) and \(\sigma_{\text{spec}}\) are actual and desired real part of the electromechanical mode, \(K_{c,\text{max}}\) and \(K_{c,\text{min}}\) are minimum and maximum gains of PSS, \(T_{i,\text{min}}\) and \(T_{i,\text{max}}\), \(i = 1, \ldots, 4\) are minimum and maximum time constants of PSS. The optimization problem is solved by GA.

**Step 5** Initialize the search parameters for GA. Define genetic parameters such as population size, crossover, mutation rate, and maximum generation.

**Step 6** Randomly generate the initial solution.

**Step 7** Evaluate objective function of each individual in (14).

**Step 8** Select the best individual in the current generation. Check the maximum generation.

**Step 9** Increase the generation.

**Step 10** While the current generation is less than the maximum generation, create new population using genetic operators and go to step 7. If the current generation is the maximum generation, then stop.

### 4. PERFORMANCE SIMULATION AND RESULTS

In this section, simulation studies in SMIB system are carried out. Based on (4), the weighting functions are selected as:

\[
W_1 = 168 \frac{s + 16}{s + 19}, \quad W_2 = I
\tag{16}
\]

Fig. 6 shows the weighting function \(W_1\). Accordingly, the shaped plant \(G_c\) can be established. As a result, \(\gamma_{\text{max}} = 2.35\).

In the optimization, the ranges of search parameters and GA parameters are set as follows: \(K_c \in [1, 60]\), \(T_i\), \(i = 1, 2, 3\), and \(T_4 \in [0.0001, 1]\), \(\zeta_{\text{spec}} = 0.4\), \(\sigma_{\text{spec}} = -0.5\), arithmetic crossover, uniform mutation, population size is 100 and maximum generation is 100. Consequently, the convergence curve of the objective function can be shown in Fig. 7.

As a result, the designed PSS is

\[
K(s) = 48.02 \begin{pmatrix}
0.7010 & 0.0808 \\
0.2761 & 0.0003
\end{pmatrix}
\tag{17}
\]

![Fig. 6 : Weighting function \(W_1\)](image)

![Fig. 7 : Objective function versus iteration](image)

![Fig. 8 : Bode diagram of CH_PSS and proposed PSS](image)
The eigenvalues corresponding to the electromechanical mode without PSS and the proposed PSS are listed in Table 1. Clearly, the desired damping ratio and the desired real part of the oscillation mode are achieved by the proposed PSS. In simulation studies, the performance and robustness of the proposed controllers are compared with those of the PSS designed by conventional $H_\infty$ loop shaping method with weighting function in (16), that is

$$ (CH\_PSS)K(s) = \frac{0.00022s^5 + 0.0189s^4 + 0.574s^3}{0.000145s^5 + 0.0094s^3 + 0.364s} $$

and the conventional lead-lag controller (CPSS) obtained from (Rao P.S and Sen I, 1999), that is

$$ (CPSS)K(s) = 5.5 \frac{(1 + 0.1732s)^2}{(1 + 0.0577s)^2} $$

Fig. 8 shows the bode diagram of the proposed PSS and CH_PSS. In the vicinity of oscillation mode frequency (1-2 Hz), the magnitude and phase plots of both PSSs have almost the same characteristic.

<table>
<thead>
<tr>
<th>Table 2 Operating Conditions.</th>
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<tbody>
<tr>
<td>System Parameters</td>
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<tr>
<td>$P$(p.u)</td>
</tr>
<tr>
<td>$Q$(p.u)</td>
</tr>
<tr>
<td>$x_e$(p.u)</td>
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</tbody>
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Fig. 9 depicts the bode plots of the systems without PSS, with CH_PSS and with the proposed PSS. Without PSS, the peak resonance of the oscillation mode occurs at frequency about 1 Hz. For system with each PSS, the peak resonance is reduced significantly. This signifies the stabilizing effects of both PSSs. In simulation studies, the limit on each PSS output ($\Delta u_{pss}$) is $\pm 0.05$ p.u and the limit on $\Delta E_{ref}$ is $\pm 6.0$ p.u. The system responses with PSSs are examined under three case studies as in Table 2, while a small disturbance of 5% (0.05 p.u.) step response of $\Delta V_{ref}$ is applied to the system at $t = 0$ s.

Fig. 10 shows the responses of electrical power output deviation in case a. CPSS, CH_PSS and the proposed PSS are able to damp power oscillations. Nevertheless, the overshoot and setting time of power oscillations in cases of CH_PSS and the proposed PSS are much lower than those of CPSS. In case b as shown in Fig. 11, the damping effect of CPSS is deteriorated by the increase in transmission line reactance. On the other hand, the power oscillations are effectively...
stabilized by CH_PSS and the proposed PSS. Both PSSs are rarely sensitive to the weak line condition. In addition to the weak line condition in case b, the electrical power output is increased in case c. Fig. 12 shows that the CPSS fails to damp power system. The power oscillation gradually increases and diverges. In contrast, the CH_PSS and the proposed PSS can tolerate this situation. The power oscillations are significantly damped.

In this study, a robust GA-based fixed-structure controller design of PSS using $H_{\infty}$ loop shaping technique has been proposed. The performance and stability conditions of $H_{\infty}$ loop shaping technique have been applied as the objective function in the optimization problem. The GA has been used to tune the control parameters of PSS. The designed PSS is based on the conventional $2^{nd}$-order lead-lag compensator. Accordingly, it is easy to implement in real systems. The damping effects and robustness of the proposed PSS have been evaluated in the SMIB system. Simulation results confirm that the proposed PSS is very robust against various uncertainties. With lower order, the stabilizing effect and robustness of the proposed PSS are almost the same as those of the PSS with high-order designed by $H_{\infty}$ loop shaping technique. For future development, the proposed method will be applied to design PSSs in a multi-machine power system.

**5. CONCLUSION**

REFERENCES


