Stability Analysis - Multiconvexity
Approach

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Abstract: A new stability condition in terms of LMIs is studied in this paper, continuous- and discrete-time fuzzy systems treated in a unified manner. Based on a premise-dependent Lyapunov function and multiconvexity, we release the conservativeness that commonly exists in the common P approach.

1. INTRODUCTION

Recently a large number of literature on fuzzy control are TS model-based control where, mostly, the common P approach searching for a single Lyapunov function remains active Wang et al. [1996], Tanaka et al. [1998], Kim and Lee [2000], Blanco et al. [2000], Tuan et al. [2001]. Another category emphasizes parameter-dependent functions with multiple P matrices as a candidate of Lyapunov function Johansson et al. [1999], Kiriakidis [2001], Chadli et al. [2000], Tanaka et al. [2001a, b], Feng and Ma [2001], Cao et al. [1996]. To remove the time-derivative dependence, progress has been made recently in obtaining less conservative results using non-quadratic approach (multiple Lyapunov functions) Morel et al. [1999], Guerra and Perruquetti [2001], Guerra and Vermeiren [2001], Tanaka et al. [2003] where a fuzzy Lyapunov candidate is used for a discrete- and continuous-time T-S fuzzy model and the resulting stability condition is shown to be more relaxed than the condition derived from the common P approach. In this paper, multiconvexity property from Apkarian and Tuan [2000] is combined with premise-dependent Lyapunov function to derive sufficient conditions for stability test of TS fuzzy systems.

The paper is organized as follows. Section II rehearses some useful results which forms the foundation for later developments. Section III derives the stability conditions for continuous- and discrete-time systems via a premise-dependent Lyapunov in conjunction with multiconvexity property. Two examples are illustrated in Section IV and conclusion is drawn in Section V.

2. PRELIMINARIES

To begin with, we introduce the following definitions and corollaries which serve as the entry point to this paper.

A polytope II in R^n is defined as the compact set

II := \{ \sum_{i=1}^{r} \mu_i v_i : \sum_{i=1}^{r} \mu_i = 1, \mu_i \geq 0, v_i \in R^n \} = co V \ (1)

which constitutes the convex hull of the set V = \{v_1, \ldots, v_r\}. We denote the set of vertices of II as vert II := V.

The following corollary is a useful tool permitting us to convert maximization of a function over a polytope II into exploring maximum of a function over vert II. The corollary below clarifies this fact Apkarian and Tuan [2000]:

Corollary 1. (Multiconvexity). Consider a polytope II and the directions d_1, \ldots, d_q determined by the edges of II. f has a maximum over II in vert II if the following is satisfied

\frac{\partial^2 f(v + \lambda d_l)}{\partial \lambda^2} \geq 0 \ \forall v \in II, l = 1, \ldots, q \ (2)

where v + \lambda d_l is a direction vector paralleling the edges of II.

To find applications of the Corollary 1 to Lyapunov theory, it is instructive to consider the case in which f is a quadratic function, f(\mu) = \mu^T Q \mu + \mu^T \mu + a. In particular f(\mu) will be the time derivative function of a Lyapunov candidate function.

3. STABILITY ANALYSIS

In this section, we derive a stability condition for an open-loop fuzzy system that is displayed below:

\delta x = \sum_{i=1}^{r} \mu_i A_i x = A_\mu x \ (3)

where A_i is a system matrix of each rule i and \mu_i \geq 0 is the firing strength of rule i. \delta is a derivative operator for continuous-time systems, (\delta x = \dot{x}(t)) and a delay operator for discrete-time systems, (\delta x = x(k + 1)).

1 This work was supported in part by the National Science Council of the ROC under grant NSC-95-2221-E-008-046.
Theorem 2. (Continuous Stability). The open loop system (3) is stable if there exist symmetric, positive definite matrices \( \mu \) and upper bounds \( |\tilde{\mu}_j| \leq \phi_j \) satisfying the following LMIs:

\[
\sum_{\rho=1}^{r} \phi_{\rho} X_{\rho} + \chi (X_{\rho} A_{\rho}^T + A_{\rho} X_{\rho} < 0, \quad 1 \leq j \leq r
\]

\[
X_{i} A_{i}^T + A_{i} X_{i} + \chi (X_{i} A_{i}^T + A_{i} X_{i} - (X_{i} A_{i}^T + A_{i} X_{i}) \geq 0, \quad 1 \leq i < j \leq r
\]

Proof:

Consider a quadratic function \( V(x(t)) = x^T(t)X_{\mu}^{-1}x(t) \), where \( X_{\mu} = \sum_{j=1}^{r} \mu_{h} X_{j} \) and \( X_{j} \)'s are symmetric, positive definite matrices such that for all \( t \) and \( \dot{X}_{\mu} = \sum_{\rho=1}^{r} \mu_{h} X_{\rho} \), the time derivative of \( V(x(t)) \) along the state trajectories is

\[
\dot{V}(x) = \dot{x}^T X_{\mu}^{-1} x + x^T X_{\mu}^{-1} \dot{x} + x^T \frac{dX_{\mu}^{-1}}{dt} x
\]

\[
= x^T (A_{\mu}^T X_{\mu}^{-1} + X_{\mu}^{-1} A_{\mu} + \frac{dX_{\mu}^{-1}}{dt}) x.
\]

Based on Lyapunov theory, a sufficient condition is

\[
A_{\mu}^T X_{\mu}^{-1} + X_{\mu}^{-1} A_{\mu} + \frac{dX_{\mu}^{-1}}{dt} < 0.
\]

Pre- and post-multiplying the inequality above by \( X_{\mu} \) yields

\[
X_{\mu} A_{\mu}^T + A_{\mu} X_{\mu} + X_{\mu} \frac{dX_{\mu}^{-1}}{dt} X_{\mu} < 0.
\]

Since

\[
\frac{dX_{\mu}^{-1}}{dt} = -X_{\mu}^{-1} \dot{X}_{\mu} X_{\mu}^{-1}
\]

we have

\[
X_{\mu} A_{\mu}^T + A_{\mu} X_{\mu} - \dot{X}_{\mu} < 0
\]

yielding

\[
X_{\mu} A_{\mu}^T + A_{\mu} X_{\mu} - \sum_{\rho=1}^{r} \mu_{h} X_{\rho} < 0
\]

(6)

By the virtue of the bounded \( \tilde{\mu}_j \) assumption, an upper bound expression is given below:

\[
LHS(6) \leq \sum_{\rho=1}^{r} \phi_{\rho} X_{\rho} + \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{i} \mu_{j} M_{ij}
\]

\[
= \sum_{\rho=1}^{r} \phi_{\rho} X_{\rho} + \mu^T M \mu
\]

\[
= f(\mu) < 0
\]

(7)

where

\[
|\tilde{\mu}_j| \leq \phi_j, \quad \phi_j \geq 0 \quad \text{and} \quad \mu = [\mu_1 \cdots \mu_r]'
\]

\[
M = \begin{bmatrix}
\begin{array}{cccc}
M_{11} & \cdots & M_{1r} \\
\vdots & \ddots & \vdots \\
M_{r1} & \cdots & M_{rr}
\end{array}
\end{bmatrix}, \quad M_{ij} = (X_{i} A_{j}^T + A_{i} X_{j})
\]

and \( M_{ij} \) are real symmetric, matrix-valued and linear functions of decision variables (multiple Lyapunov matrices) \( X_{i} \). Note that the problem arisen with (7) involves infinitely many LMIs associated with each value of the parameter \( \mu \) and is known to be intractable Apkarian and Tuan [2000]. By enforcing some constraints of geometric structure on the functional dependence in \( \mu \), it is possible to reduce the problem to a feasibility problem of solving a finite number of LMIs. To this end, we note that the parameter vector \( \mu = [\mu_1 \cdots \mu_r]' \), known as the firing strengths, evolves in the simplex defined below

\[
\Gamma := \{ \mu : \sum_{i=1}^{r} \mu_i = 1, \mu_i \geq 0 \}.
\]

Recalling (1), we have the polytope \( \Gamma \) shown in Figure 1:

![Figure 1: Firing strength in the three-dimension space](image)

Fig. 1. Firing strength in the three-dimension space

structure becomes tangible and the vertices of \( \Gamma \) can be found to be \( v_1 = [1, 0, 0]' \), \( v_2 = [0, 1, 0]' \), \( v_3 = [0, 0, 1]' \) (see Figure 1) and the sufficient condition (7) for \( r = 3 \) becomes

\[
f(\mu) = \sum_{\rho=1}^{3} \phi_{\rho} X_{\rho} + \mu^T M \mu
\]

\[
< 0
\]

(8)

Equation (8) being a quadratic function of \( \mu \), Corollary 1 says that \( f(\mu) \) is negative whenever it is multiconvex along lines parallelizing the edges of \( \Gamma \) and furthermore \( f(\mu) \) is negative over \( vert \ \Gamma \). The remaining of the proof follows in two phases: (A) showing the second derivative condition (2) is satisfied so that the negativeness of \( \dot{V} \) is assured by (B) checking the vertexes of \( \Gamma \).

(A) To check the multiconvexity along the edges of \( \Gamma \).

The directions \( d_l, l = 1, \cdots, q \) is determined by vectors with all but two zero coordinates, the nonzero coordinates having opposite signs.

Along the direction \( d_1 := [1, -1, 0]' \) of Figure 1, we get

\[
f(\mu + \lambda d_1) = \sum_{\rho=1}^{r} \phi_{\rho} X_{\rho} + \mu^T M \mu + \mu^T \lambda d_1
\]

\[
= \begin{bmatrix}
\mu_1 + \lambda \\
\mu_2 - \lambda \\
\mu_3
\end{bmatrix}^T
\begin{bmatrix}
M_{11} & M_{12} & M_{13} \\
M_{21} & M_{22} & M_{23} \\
M_{31} & M_{32} & M_{33}
\end{bmatrix}
\begin{bmatrix}
\mu_1 + \lambda \\
\mu_2 - \lambda \\
\mu_3
\end{bmatrix}
\]

\[
\leq 0
\]
which yields
\[ \frac{\partial^2 f}{\partial \lambda^2} = M_{11} + M_{22} - M_{12} - M_{21}. \]
Along the direction \( d_2 := [0, 1, -1]' \), we get
\[ f(\mu + \lambda d_2) = \sum_{\rho=1}^{r} \phi_\rho x_\rho + [\mu_1 \mu_2 + \lambda \mu_3 - \lambda] \]
which yields
\[ \frac{\partial^2 f}{\partial \lambda^2} = M_{22} + M_{33} - M_{23} - M_{32}. \]
Similarly, along the direction \( d_3 := [-1, 0, 1]' \) we get
\[ \frac{\partial^2 f}{\partial \lambda^2} = M_{11} + M_{33} - M_{13} - M_{31}. \]
For \( r = 3 \), the multiconvexity is assured if
\[
\begin{align*}
M_{11} + M_{22} - M_{12} - M_{21} &\geq 0 \\
M_{22} + M_{33} - M_{23} - M_{32} &\geq 0 \\
M_{11} + M_{33} - M_{13} - M_{31} &\geq 0
\end{align*}
\]
are satisfied. Arguing in the same fashion as \( r = 3 \) case, we have the following results for the general case
\[
\begin{align*}
M_{ij} + M_{jj} - M_{ij} &\leq 0, \quad 1 \leq i < j \leq r.
\end{align*}
\]
This proves inequality (5). What follows is to (B) check the vertices
At the vertex \([1, 0, 0]'\)
\[
f(v_1) = \sum_{\rho=1}^{r} \phi_\rho x_\rho + [1 0 0] \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \sum_{\rho=1}^{r} \phi_\rho x_\rho + M_{11}.
\]
At the vertex \([0, 1, 0]'\)
\[
f(v_2) = \sum_{\rho=1}^{r} \phi_\rho x_\rho + [0 1 0] \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \sum_{\rho=1}^{r} \phi_\rho x_\rho + M_{22}.
\]
Similarly, at the vertex \([0, 0, 1]'\) we have
\[
f(v_3) = \sum_{\rho=1}^{r} \phi_\rho x_\rho + M_{33}.
\]
For \( r = 3 \), to ensure negativeness, we need
\[
\begin{align*}
\sum_{\rho=1}^{r} \phi_\rho x_\rho + M_{11} &< 0 \\
\sum_{\rho=1}^{r} \phi_\rho x_\rho + M_{22} &< 0 \\
\sum_{\rho=1}^{r} \phi_\rho x_\rho + M_{33} &< 0.
\end{align*}
\]
Paralleling the argument for \( r = 3 \), we arrive at the following form for the general case.
\[
\begin{align*}
\sum_{\rho=1}^{r} \phi_\rho x_\rho + M_{jj} &< -\sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{ij} Z_{ij} \\
\sum_{\rho=1}^{r} \phi_\rho x_\rho + X_j A'_j + A_j X_j &< 0, \quad 1 \leq j \leq r.
\end{align*}
\]
This proves inequality (4).

**Remark 1**: The assumption of boundedness in the rate of state-dependent firing strength \( \mu \) is removed by using an idea of piecewise differential quadratic (PDQ) Lyapunov function and linear systems with jump Ma and Feng [2003].

**Remark 2**: By strengthening the condition in (4), one can slightly relax the multiconvexity requirement in (5). As an example, the feasibility problem to inequality (7) can be equivalently recast into the following problem: There exist matrices \( Z_{ij} \) such that the following inequality is feasible
\[
\sum_{\rho=1}^{r} \phi_\rho x_\rho + \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{ij} Z_{ij} < -\sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{ij} z_{ij}
\]
where \( \forall \mu \in \Gamma \) and
\[
Z = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1r} \\
Z_{21} & Z_{22} & \cdots & Z_{2r} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{r1} & Z_{r2} & \cdots & Z_{rr} \end{bmatrix} \geq 0.
\]

**Proof**: Similar lines to those in Apkarian and Tuan [2000].

Arguing as in Theorem 1, the associated solvability conditions are easily obtained as \( (1 \leq j \leq r, 1 \leq i < j \leq r) \)
\[
\sum_{\rho=1}^{r} \phi_\rho x_\rho + X_j A'_j + A_j X_j < -Z_{jj}
\]
\[
X_j A'_j + A_j X_j < -Z_{jj}
\]
\[
\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1r} \\
Z_{21} & Z_{22} & \cdots & Z_{2r} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{r1} & Z_{r2} & \cdots & Z_{rr} \end{bmatrix} \geq 0.
\]

Analogously, we will derive the stability condition for the discrete-time open-loop system (3), demonstrating that the multiconvexity property can be applied as well, constituting a unified treatment for multiconvexity stability analysis.
Theorem 3. (Discrete Stability). The open loop system (3) is stable if there exist symmetric, positive definite matrices $X_j$ satisfying the following LMIs (1 ≤ $i < j ≤ r$):

$$
\begin{bmatrix}
-X_j & X_j A' \\
A_j X_j & -X_j
\end{bmatrix} < 0, \ 1 \leq j \leq r
$$

(12)

$$
\begin{bmatrix}
0 & 2 \mu \\
A_j X_j + A_j X_j - (A_j X_j + A_j X_j) * 0
\end{bmatrix} \geq 0
$$

(13)

Proof: Consider a quadratic function $V(x(k)) = x'(k) X^{-1}_j x(k)$ where $X_j = \sum_{j=1}^r \mu_j X_j$. The time difference of $V(x(k))$ is displayed below:

$$
\Delta V = V(x(k+1)) - V(x(k))
$$

$$
= x'(k+1) X^{-1}_j x(k+1) - x'(k) X^{-1}_j x(k)
$$

$$
= x'(k) A'_j X^{-1}_j A_j x(k) - x'(k) X^{-1}_j x(k)
$$

$$
= x'(A'_j X^{-1}_j A_j - X^{-1}_j) x(k) < 0.
$$

A sufficient condition is

$$
A'_j X^{-1}_j A_j - X^{-1}_j < 0
$$

yielding

$$
X_j A'_j X^{-1}_j A_j X_j - X^{-1}_j < 0.
$$

Schur complement gives

$$
\begin{bmatrix}
-X_j & X_j A' \\
A_j X_j & -X_j
\end{bmatrix} < 0
$$

Rewriting the matrix inequality yields

$$
0 > \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j X_{ij} - \sum_{j=1}^r \mu_j X_{ij}
$$

$$
= -\sum_{j=1}^r \mu_j \begin{bmatrix} X_j & 0 \\ 0 & X_j \end{bmatrix} + \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \begin{bmatrix} 0 & X_j A' \\ A_j X_j & 0 \end{bmatrix}
$$

$$
= -\mu' M + \mu' M \mu
$$

(14)

where $\mu = [\mu_1 \cdots \mu_r]'$ and

$$
\hat{M} = \begin{bmatrix} M_1 & \cdots & M_r \\ M_r & \cdots & M_1 \end{bmatrix}, \quad M_j = \begin{bmatrix} X_j & 0 \\ 0 & X_j \end{bmatrix}
$$

Notice that the matrices $M_j$ and $M_{ij}$ are real symmetric, matrix-valued and linear functions of decision variables (multiple Lyapunov matrices) $X_j$.

For $r = 3$, we have (14) displayed below.

$$
f(\mu) = -\begin{bmatrix} \mu_1 & \mu_2 & \mu_3 \\ \mu_2 & \mu_1 & 0 \\ \mu_3 & 0 & \mu_1 \end{bmatrix} \begin{bmatrix} M_1 & M_2 & M_3 \\ M_2 & M_1 & M_3 \\ M_3 & M_3 & M_1 \end{bmatrix} \begin{bmatrix} \mu_1 & 0 & \mu_2 \\ 0 & \mu_1 & \mu_3 \\ \mu_2 & \mu_3 & \mu_1 \end{bmatrix}
$$

Arguing in the same fashion as in the proof of continuous case, we

(A) check multiconvexity condition: Along the direction $d_1 := [1, -1, 0]'$ (Figure 1), we get

$$
f(\mu + 0d_1) = -\begin{bmatrix} \mu_1 + \lambda & \mu_2 - \lambda & \mu_3 \\ \mu_2 - \lambda & \mu_1 + \lambda & 0 \\ \mu_3 & 0 & \mu_1 \end{bmatrix}
$$

Then

$$
\frac{\partial^2 f}{\partial \lambda^2} = M_{11} + M_{22} - M_{12} - M_{21}.
$$

Similar to the first direction just shown, along the direction $d_2 := [0, 1, -1]'$ we get

$$
\frac{\partial^2 f}{\partial \lambda^2} = M_{22} + M_{33} - M_{23} - M_{32}.
$$

and along the direction $d_3 := [-1, 0, 1]'$, we get

$$
\frac{\partial^2 f}{\partial \lambda^2} = M_{11} + M_{33} - M_{13} - M_{31}.
$$

For $r = 3$, the multiconvexity condition is satisfied if

$$
M_{11} + M_{22} - M_{12} - M_{21} \geq 0
$$

$$
M_{22} + M_{33} - M_{23} - M_{32} \geq 0
$$

$$
M_{11} + M_{33} - M_{13} - M_{31} \geq 0.
$$

Arguing in the same fashion as $r = 3$ case, we have the following results for the general case (1 ≤ $i < j ≤ r$)

$$
M_{ii} + M_{jj} - M_{ij} - M_{ji}
$$

$$
= \begin{bmatrix} A_i X_i + A_j X_j - (A_i X_j + A_j X_i) * \\ 0 \end{bmatrix} \geq 0
$$

This proves inequality (13).

(B) Check vertexes condition:

At the vertex $[1, 0, 0]'$

$$
f(v_1) = -\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_1 & M_2 & M_3 \\ M_2 & M_1 & M_3 \\ M_3 & M_3 & M_1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = -M_1 + M_{11}.
$$

Similar to vertex $[1, 0, 0]'$, we arrive at the following expression for the vertex $[0, 1, 0]'$

$$
f(v_2) = -M_3 + M_{33}.
$$

For $r = 3$, to ensure negativity, we need

$$
-M_1 + M_{11} < 0
$$

$$
-M_2 + M_{22} < 0
$$

$$
-M_3 + M_{33} < 0.
$$

Paralleling the argument for $r = 3$, we arrive at the following form for $r > 3$
\[-M_j + M_{jj} = \begin{bmatrix} -X_j & 0 \\ 0 & -X_j \end{bmatrix} + \begin{bmatrix} 0 & X_j A'_j \\ A_j X_j & 0 \end{bmatrix} < 0\]

This proves inequality (12).

**Remark 3:** Likewise, by strengthening the condition in (12), one can slightly relax the multiconvexity requirement in (13). (See Remark 1)

**Proof:** Similar lines to those in Apkarian and Tuan [2000]. Arguing as in Theorem 2, the associated feasibility conditions are easily obtained as (1 ≤ j ≤ r, 1 ≤ i < j ≤ r)

\[-X_i + Z_{ij1} \begin{bmatrix} Z_{i1} & Z_{i2} & \cdots & Z_{ir} \\ Z_{i12} & Z_{i22} & \cdots & Z_{ir2} \\ \vdots & \vdots & \cdots & \vdots \\ Z_{i1r} & Z_{i2r} & \cdots & Z_{rr} \end{bmatrix} A_j X_j < 0\]

where ∀\(\mu \in \Gamma\) and

\[Z = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1r} \\ Z_{121} & Z_{122} & \cdots & Z_{12r} \\ \vdots & \vdots & \cdots & \vdots \\ Z_{1r1} & Z_{1r2} & \cdots & Z_{rr} \end{bmatrix} \geq 0, \quad \bar{Z}_{ij} = \begin{bmatrix} Z_{ij1} & Z_{ij2} \\ Z_{ij2} & Z_{ij3} \end{bmatrix} \geq 0\]

4. EXAMPLES

In order to appreciate the efficiency of the proposed method, we consider examples where the T-S fuzzy models are borrowed from existing papers.

4.1 Continuous fuzzy systems

A continuous fuzzy system, borrowed from Tanaka et al. [2003], composed of the following two rules

\[R_1 : \text{IF } x_1(t) \text{ is } M_1, \text{ THEN } \dot{x}(t) = A_1 x(t)\]

\[R_2 : \text{IF } x_1(t) \text{ is } M_2, \text{ THEN } \dot{x}(t) = A_2 x(t)\]

The fuzzy sets are described by the following two triangular membership functions:

\[\mu_1(x(t)) = \frac{1 + \sin(x_1(t))}{2}, \quad \mu_2(x(t)) = \frac{1 - \sin(x_1(t))}{2}\]

and

\[A_1 = \begin{bmatrix} -5 & -4 \\ -1 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2 & -4 \\ 20 & -2 \end{bmatrix}\]

the global T-S fuzzy model is:

\[\dot{x}(t) = (\mu_1(x(t)) A_1 + \mu_2(x(t)) A_2) x(t)\]

With \(\phi_1 = 0.85, \phi_2 = 0.85\) and solving (9)-(11) and the following matrices are obtained:

\[X_1 = \begin{bmatrix} 15.9551 & -13.0451 \\ -13.0451 & 19.0928 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 6.3777 & 0.9237 \\ 0.9237 & 27.7212 \end{bmatrix}\]

\[Z_{11} = \begin{bmatrix} 18.1033 & 5.6567 \\ 5.6567 & 2.2445 \end{bmatrix}, \quad Z_{12} = \begin{bmatrix} -52.5093 & -154.8967 \\ -154.8967 & 588.9001 \end{bmatrix}\]

\[Z_{21} = \begin{bmatrix} 0.1002 - 0.0334 \\ -0.0334 - 0.5345 \end{bmatrix} \times 10^{-16}\]

\[Z_{22} = \begin{bmatrix} 6.9587 - 1.3361 \\ -1.3361 - 17.0730 \end{bmatrix}\]

4.2 Discrete fuzzy systems

Consider a discrete-time fuzzy system borrowed from Feng [2004] in which the rule base listed below:

\[R_l : \text{IF } x_l(t) \text{ is } \mu_l, \text{ THEN } \dot{x}(t) = A_l x(t), \text{ for } l = 1, \ldots, 7\]

The system matrices are given as

\[A_1 = \begin{bmatrix} 1.0000 & 0.5000 \\ -0.3000 & 0.8000 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.0000 & 0.4875 \\ -0.2750 & 0.8000 \end{bmatrix}\]
The matrices $A_3$, $A_4$, $A_5$, and $A_7$ are given by:

$$A_3 = \begin{bmatrix} 1.0000 & 0.4750 \\ -0.2500 & 0.8000 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 1.0000 & 0.4500 \\ -0.2000 & 0.8000 \end{bmatrix}, \quad A_5 = \begin{bmatrix} 1.0000 & 0.4250 \\ -0.1500 & 0.8000 \end{bmatrix}, \quad A_7 = \begin{bmatrix} 1.0000 & 0.4000 \\ -0.1000 & 0.8000 \end{bmatrix}.$$ 

By using the MATLAB LMI toolbox, one can easily verify that for the common $P$ solution, there exists no positive definite matrix for the fuzzy system to guarantee its stability. In other words, the fuzzy does not admit a global quadratic Lyapunov function. By solving (15)-(17) and the following matrices are obtained:

$$X_1 = \begin{bmatrix} 12.6862 & -2.6269 \\ -2.6269 & 7.5386 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 12.6589 & -2.6219 \\ -2.6219 & 7.5225 \end{bmatrix},$$

$$X_3 = \begin{bmatrix} 14.6175 & -3.0920 \\ -3.0920 & 7.7062 \end{bmatrix}, \quad X_4 = \begin{bmatrix} 15.6588 & -3.3812 \\ -3.3812 & 7.2376 \end{bmatrix},$$

$$X_5 = \begin{bmatrix} 16.4814 & -3.6419 \\ -3.6419 & 6.5599 \end{bmatrix}, \quad X_6 = \begin{bmatrix} 16.8279 & -3.7749 \\ -3.7749 & 6.1639 \end{bmatrix},$$

$$X_7 = \begin{bmatrix} 9.0780 & -1.3718 \\ -1.3718 & 8.1157 \end{bmatrix}.$$

indicating a stable system.

5. CONCLUSION

In this paper, two stability conditions based on multiconvexity are developed for both continuous- and discrete-time T-S fuzzy systems. The proposed approach utilizes a premise-dependent Lyapunov function to prove Lyapunov stability of the underlying fuzzy systems, leading to a non-common $P$ method that releases the conservativeness of the common $P$ scheme. It is shown and demonstrated via examples that the stability can be determined by solving a set of LMIs.

REFERENCES


