Adaptive Feed-Forward Cancellation Control of a Full-Bridge DC-AC Voltage Inverter

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Abstract: Dc-ac inverters are needed in many applications. The full-bridge dc-ac inverter has been widely used in industry applications and through the literature. If the nature of the load is not known a priori, some considerations should be taken in order to assure the quality of service to be provided by the inverter. When nonlinear loads are fed by a full-bridge dc-ac inverter, odd harmonics of the fundamental ac frequency are introduced into the output voltage shape. For the purpose of producing a good sinusoidal output voltage signal, the control strategy must be able to reject periodic output disturbances. Adaptive Feed-forward Cancellation (AFC) is a control technique that has been successfully used to selectively reject periodic output disturbances in continuous-time mechanical systems. This paper deals with the use of AFC to control the output voltage of an electrical system, in this case, a dc-ac full-bridge inverter, to produce a standard European ac voltage signal, 230 Vrms and 50 Hz, accomplishing the design of the controller directly in the $z$-domain.

1. INTRODUCTION

Feeding an isolated load with a dc-ac inversion system is a challenging task, generally due to the lack of knowledge about the nature of the load (linear or nonlinear), and the necessity of keeping the desired output voltage characteristics (frequency, amplitude, THD, etc.) within certain limits that determine the quality of service to be provided. Non-linear loads, such as diode rectifiers and fluorescent lights with electronic ballasts, introduce odd harmonics of the fundamental ac frequency (Pileggi et al. (1993)), generally 50 Hz or 60 Hz, into the output voltage shape of the dc-ac inversion system. Adaptive Feed-forward Cancellation (AFC) is a control technique that allows the designer to reject or attenuate, in a selective manner, specific harmonics of periodic disturbance signals. AFC has successfully been used in mechanical systems (Ludwick (1999); Bodson (2005); Byl et al. (2005)), to reject periodic disturbances that are harmonics of some fundamental frequency. It is interesting to note that under certain circumstances (Bodson et al. (1992, 1994)), the AFC problem becomes equivalent to that of the Internal Model Principle (IMP) (Francis and Mohan (1976)), and therefore, perfect disturbance rejection of the selected harmonics can be expected. The IMP states that under such circumstances, the AFC model has to include the open-loop transfer function, requiring then, previous knowledge of the disturbance signal. The design and the equivalence of controllers between the AFC and IMP schemes were treated by Messner and Bodson (1994) and Bodson (2004), and a loop shaping approach is given by Byl et al. (2005) in order to help the designer in the selection of the design parameters.

2. DISCRETE-TIME AFC

The AFC approaches treated by Bodson et al. (1992, 1994); Messner and Bodson (1994); Ludwick (1999); Bodson (2004, 2005); Byl et al. (2005) assume continuous-time variables and controller design, although in practice, almost all controllers need to be implemented by means of digital computers, and hence, in discrete time. This work deals with the design of an AFC controller directly in the $z$-domain.

In the continuous-time framework, the advantage on the use of the AFC control structure to reject periodic disturbances lies in the placement of an infinite gain on the open-loop transfer function at the desired $\omega_k$ frequency by means of the $R_k(s)$ resonator. By following the IMP, perfect disturbance rejection at such frequency can be expected. In order to obtain a feasible discrete-time realization, it needs to be shown that the main characteristics of the continuous-time realization are preserved. Fig.1 shows the block diagram of the AFC closed loop. A proportional block with constant $K_0$ is added in order to help the reference tracking. Fig.2 shows block diagram for the proposed discrete-time resonator.

2.1 Construction of the Discrete-Time Resonator $R_k(z)$:

The main difference between the continuous-time and the discrete-time realizations of the AFC controller resides in the construction of the $R_k(z)$ resonator modules. The $H(z)$ block in Fig.2 can be any discrete-time integrator structure. The resonator shown in Fig.2 can be expressed by the $z$-domain transfer function (1).

$$R_k(z) = \frac{1}{2g_k} \left[ H(z e^{-j\omega_k T})e^{-j\phi_k} + H(z e^{j\omega_k T})e^{j\phi_k} \right] \quad (1)$$
In this work, a Backward Euler integrator has been chosen for the $H(z)$ block, as seen in (2).

$$H(z) = \frac{z}{(z-1)}$$  (2)

By replacing (2) into (1) and simplifying, the $z$-domain transfer function of the resonator becomes

$$R_k(z) = g_k \frac{\cos(\phi_k) z^2 - \cos(\omega_k T + \varphi_k) z}{z^2 - 2 \cos(\omega_k T) z + 1}$$  (3)

Equation (3) shows the resulting structure for the discrete-time resonator implementation by selecting the first order structure (2) as integrator. The parameters involved in the discrete-time resonator structure are: $g_k$, $\omega_k$, $\varphi_k$ and $T$. Where $g_k$ is a positive real gain, $\omega_k$ is the frequency, in rad/s at which the desired resonating gain is to be placed, $\varphi_k$ is the phase shift parameter for $k$th resonator in rads, and finally $T$ is the sampling period in seconds for the discrete-time system.

### 2.2 Phase of the resonator $R_k(z)$ at the resonating frequency $\omega_k$:

First, the phase and gain characteristics of the discrete time resonator at the $\omega_k$ frequency are explored. The behavior of the system around the resonating frequency $\omega_k$ can be studied departing from (1).

When the continuous-time frequency $\omega$ approaches $\omega_k$, the discrete time variable $z$ becomes $z = e^{j(\omega T + \epsilon)}$, for a sufficiently small value of $|\epsilon| \geq 0$. Having substituted this value in (1) and simplifying, the resonator structure becomes:

$$R_k(z) = \frac{1}{2} g_k [H(e^{j\epsilon})e^{-j\varphi_k} + H(e^{j(2\omega_k T + \epsilon)})e^{j\varphi_k}]$$  (4)

By substituting (2) in (4) the first half of (4) can be rewritten as:

$$\frac{1}{2} g_k [H(e^{j\epsilon})e^{-j\varphi_k}] = \frac{1}{4} g_k \left[ \frac{e^{j(\frac{|\epsilon|}{2})} - \frac{\sin(\epsilon)}{\sin(\frac{|\epsilon|}{2})} \varphi_k}{\sin(\frac{|\epsilon|}{2})} \right]$$  (5)

For the sake of simplicity, let us assume a unitary gain $g_k = 1$. As $\epsilon$ approaches zero from either right or left sides, the modulus of (5) goes to infinite, as the denominator $\sin(\epsilon/2)$ goes to zero. The effects of the second half of (4) are marginal at this point, and (5) dominates the behavior of the resonator structure $R_k(z)$.

When $\epsilon$ approaches zero from the left, which is equivalent to say that the input frequency approaches the resonating frequency from the left ($\omega \rightarrow \omega_k^-$), the phase shift of the system becomes

$$\angle R_k(e^{j\omega k T})_{\omega \rightarrow \omega_k^-} = -\varphi_k + \frac{\pi}{2}$$

When $\epsilon$ approaches zero from the right, the phase component of the resonator structure becomes

$$\angle R_k(e^{j\omega k T})_{\omega \rightarrow \omega_k^+} = -\varphi_k - \frac{\pi}{2}$$

Therefore, the phase shift of the resonator at the resonating frequency $\omega_k$ is the average of the phase shifts of the resonator when $\omega$ approaches $\omega_k$ from the right and from the left, hence

$$\angle R_k(e^{j\omega k T}) = -\varphi_k$$  (6)

### 2.3 Phase of the resonator $R_k(z)$ at $\omega = 0$ and $\omega = \pi/T$:

Let us now inspect the phase characteristics of the resonator structure at the dc frequency. When $\omega = 0$, then $z = e^{j \frac{\pi}{2 T}} = 1$ and (3) takes the form:

$$R_k(1) = g_k \frac{\cos(\varphi_k) - \cos(\omega_k T + \varphi_k)}{2 - 2 \cos(\omega_k T)}$$  (7)

Let us assume $g_k = 1$. All the values in the equation above are real, and therefore $R_k(1)$ is a real number. A deeper inspection on the values would be necessary to determine the phase of such quantity.

The resonating frequency $\omega_k$ lies within the operating band and therefore $0 < \omega_k < \pi/T$, and then the denominator $(2 - 2 \cos(\omega_k T))$ is always greater than zero, as the $\cos(\omega_k T)$ value is always lower than 1. In the numerator $(\cos(\varphi_k) - \cos(\omega_k T + \varphi_k))$, the $\omega_k$ frequency is fixed as the independent variable, as this value is determined by the disturbance frequency to be rejected. The sampling period $T$ is fixed as well. The sign change conditions can be examined by looking at the locations of the zero crossings of the numerator. From the assumptions above the following phase conditions are imposed over $R_k(1)$.

$$\angle R_k(1) = \begin{cases} \pi & \text{if } \varphi_k < -\omega_k T/2 \\ 0 & \text{if } \varphi_k > -\omega_k T/2 \end{cases}$$  (8)

At the maximum allowed frequency by the discrete-time system a similar analysis can be performed. When $\omega = \pi/T$, $z = -1$, (3) takes the form:

$$R_k(-1) = g_k \frac{\cos(\varphi_k) + \cos(\omega_k T + \varphi_k)}{2 + 2 \cos(\omega_k T)}$$  (9)

In this case the denominator $(2 + 2 \cos(\omega_k T))$ is always positive. When examining the numerator $(\cos(\varphi_k) + \cos(\omega_k T + \varphi_k))$...
Fig. 3. Phase shift boundaries for \( \omega = 0 \) and \( \omega = \pi/T \) for the discrete time resonator structure \( R_k(z) \) 

\[ \varphi_k \), the following conditions are derived from fixing \( \omega_k \) as independent variable, with \( T \) fixed by the hardware constraints:

\[
\mathcal{L} R_k(-1) = \begin{cases} 
0 & \text{if } \varphi_k < (\pi - \omega_k T)/2 \\
\pi & \text{if } \varphi_k > (\pi - \omega_k T)/2 
\end{cases} \quad (10)
\]

From the conditions above, it can be concluded that the resonator structure will have a phase drop of \(-\pi \) rad at the resonating frequency \( \omega_k \), and the shift will be centered at \(-\varphi_k \) as stated in (6). The phase value at the beginning (\( \omega = 0 \)) and at the end of the band (\( \omega = \pi/T \)) will be determined by (8) and (10).

A single resonator \( R_k(z) \) can show three different combinations for the phase value at the start (\( \mathcal{L} R_k(1) \)) and at the end of the band (\( \mathcal{L} R_k(-1) \)) depending on the selection of \( \omega_k \) and \( \varphi_k \). Fig. 3 shows the phase shift boundaries for the discrete time resonator structure \( R_k(z) \) as a function of the resonating frequency \( \omega_k \). The upper line shows the boundary condition imposed to \( \mathcal{L} R_k(-1) \) (\( \omega = \pi/T \)) by (10). The lower line shows the boundary condition imposed to \( \mathcal{L} R_k(1) \) (\( \omega = 0 \)) by (8). As can be seen, a single resonator can change its start and finish phase values depending on the selection of the \( \varphi_k \) parameter. If \( \varphi_k = A \) is chosen (see Fig. 3), the start and finish phase values will be \( \mathcal{L} R_k(1) = 0 \) and \( \mathcal{L} R_k(-1) = \pi \) respectively. If \( \varphi_k = B \) the start and finish angles would be, \( \mathcal{L} R_k(1) = 0 \) and \( \mathcal{L} R_k(-1) = 0 \). If \( \varphi_k = C \), then \( \mathcal{L} R_k(1) = \pi \) and \( \mathcal{L} R_k(-1) = 0 \).

Fig. 4 shows the effect of the change of the phase shift parameter \( \varphi_k \) above or below the boundaries of Fig. 3. In this case a resonating frequency of \( \omega_k = 10,000 \text{ rad/s} \) has been chosen, the sampling frequency is 20,000 Hz which produces a sampling period of \( T = 50 \mu s \), a unitary gain \( g_k \) has been used, and three different values for the parameter \( \varphi_k \) have been considered. The \( \varphi_k \) values considered were, \( \varphi_k = 1.396 \text{ rad lying in the A region of Fig. 3, producing the red trace in Fig. 4. The } \varphi_k = 0 \text{ rad value lies in the B region, and produces the green trace, and } \varphi_k = -0.349 \text{ rad lies in the C region producing the blue trace. It is interesting to note that for the lower frequency range, when the resonating frequency } \omega_k \text{ is farther from the end of the band value } (\omega = \pi/T), \text{ the phase shift conditions are equivalent to the continuous-time conditions given by Byl et al. (2005). Conversely, these conditions will be equivalent if the sampling period trends to zero, which will make the discrete-time system response trend to the continuous-time one.}

2.4 Gain characteristics of \( R_k(z) \) at \( \omega = 0 \) and \( \omega = \pi/T \):

From (7) the gain characteristics at \( \omega = 0 \) can be analyzed. By plotting the values of \( \omega_k \) and \( T \) are fixed by the conditions imposed by the problem to be solved. The phase shift parameter \( \varphi_k \) is the independent variable. The gain growth or decrease regions at \( \omega = 0 \) (\( |R_k(1)| \)) can be determined by examining the location of the sign changes (see Fig. 3) and the inflection points of (7) respect to the phase \( \varphi_k \).

Assuming \( g_k = 1 \), only one inflection point is found for \( R_k(1) \) (\( \omega = 0 \)) when varying \( \varphi_k \). This point is located at \((\pi - \omega_k T)/2 \) having positive slope before the inflection point and negative slope after it.

It is also necessary to take into account the phase change conditions expressed in Fig. 3 for \( R_k(1) \). These conditions have a straightforward interpretation respect to its continuous-time one. The relationship between the growth/decrease on the gain of the resonator \( R_k(z) \) in function of \( \omega_k \) at \( \omega = 0 \) can be summarized as follows:

- \( |R_k(1)| \) decreases if \( \varphi_k \) increases and \( \varphi_k > -\omega_k T/2 \).
- \( |R_k(1)| \) increases if \( \varphi_k \) increases and \( -\omega_k T/2 < \varphi_k < (\pi - \omega_k T)/2 \).
- \( |R_k(1)| \) decreases if \( \varphi_k \) increases and \( \varphi_k > (\pi - \omega_k T)/2 \).
Fig. 5. Gain growth/decrease boundaries for $\omega = 0$ and $\omega = \pi/T$ for the discrete time resonator structure $R_k(z)$

A similar analysis can be performed for the frequencies at the end of the allowed band, where $\omega = \pi/T$, $z = -1$. In this case the gain increase/decrease characteristics can be studied departing from (9). Again, $\omega_k$ and $T$ are fixed by the problem statement and $g_k = 1$. The gain increase/decrease characteristics can be examined by determining the location of the sign changes (see Fig. 3) and the inflection points and of (9) respect to $\varphi_k$. Only one inflection point is found at $-\omega_k T/2$, when varying $\varphi_k$, and the sign change condition is located at $(\pi - \omega_k T)/2$. In this case the increase/decrease relationship can be summarized as follows

- $|R_k(1)|$ increases if $\varphi_k$ increases and $\varphi_k < -\omega_k T/2$
- $|R_k(1)|$ decreases if $\varphi_k$ increases and $-\omega_k T/2 < \varphi_k < (\pi - \omega_k T)/2$
- $|R_k(-1)|$ increases if $\varphi_k$ increases and $\varphi_k > (\pi - \omega_k T)/2$

The relationships imposed over the growth/decrease of $|R_k(1)|$ ($\omega = 0$) and $|R_k(-1)|$ ($\omega = \pi/T$) are summarized in Fig. 5. The upper line shows the location of the inflection point of $R_k(1)$ when varying $\varphi_k$ as a function of $\omega_k$. The lower line shows the location of the inflection point of $R_k(-1)$ when varying $\varphi_k$ as a function of $\omega_k$. It is interesting to note that the location of the inflection points of $R_k(1)$ and $R_k(-1)$ as a function of $\omega_k$ coincides with the location of the phase change boundaries presented in Fig. 3. For a single resonator $R_k(z)$ with resonating frequency $\omega_k$, if $\varphi_k$ is chosen to lie in the $A$ area, an increase in the value of $\varphi_k$ will cause a decrease in $|R_k(1)|$ and an increase in the value of $|R_k(-1)|$. If $\varphi_k$ is chosen to lie in the $B$ area, an increase in its value will cause an increase in $|R_k(1)|$ and a decrease $|R_k(-1)|$. If $\varphi_k$ is chosen to lie in the $C$ area, an increase in its value will cause a decrease in $|R_k(1)|$ and an increase in $|R_k(-1)|$.

Fig. 6 shows the effect over the gain and phase characteristics for a single resonator $R_k(z)$ with resonating frequency $\omega_k = 100$ rad/s, sampling period $T = 50$ $\mu$s, unitary $g_k$ gain and changes applied to the $\varphi_k$ value within the $B$ region. As can be appreciated, with values of $\varphi_k$ lying within the $B$ region, the start $(\zeta R_k(1))$ and finish

3. THE DC-AC INVERTER PROBLEM

Isolated electric energy generation systems are often needed to supply electric loads where the electrical network is not available or suitable for the application. This could be caused due to geographic isolation, the necessity of load mobility, demanded values of voltage and current that are not compatible with the local networks, etc.

This makes the design and construction of stand-alone energy generation systems a must. It is very usual that the loads to be fed require ac voltages, and therefore, special attention must be taken when designing ac output stages. Often, stand alone energy generation systems generate the ac output signal departing from a dc power source, such as batteries, photovoltaic arrays or fuel cells. This work deals with the analysis, design and control of a dc-ac inversion stage, which will be in charge of providing a high quality ac output signal departing from a dc source. The loads to be fed are isolated, and therefore, the system itself must be capable of maintaining the quality of service indicators within acceptable levels.

A standard full-bridge dc-ac inverter topology has been chosen. This topology has been widely used through the literature and in industrial applications due to its high reliability and relatively simple structure.

3.1 Description of the plant:

Fig. (7) shows the plant to be controlled by the discrete-time AFC control scheme. The experimental prototype works using three-level single-update centered pulse PWM
(Pulse Width Modulation). This PWM technique offers some advantages such as reduction of the harmonic content of the switching frequency (Holmes and Lipo (2003)) and only one duty cycle command is generated to control the two legs that form the full-bridge inverter. Under this assumptions the averaged behavior of the system can be expressed in the following form:

\[
\begin{bmatrix}
\dot{v}_{C_f} \\
\dot{i}_{L_f}
\end{bmatrix}
= \begin{bmatrix}
0 & 1/C_f \\
-1/L_f & r_{L_f}/L_f
\end{bmatrix}
\begin{bmatrix}
v_{C_f} \\
i_{L_f}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
V_{dd}/L_f
\end{bmatrix}
u
\]

\[
v_o = \begin{bmatrix}
1 \\
0
\end{bmatrix}
\begin{bmatrix}
v_{C_f} \\
i_{L_f}
\end{bmatrix}
\text{with } u \in [-1,1]
\]

From (11) the following transfer function that models the behavior of the system from the averaged duty cycle \(u\) to the output voltage is obtained

\[
G(s) = \frac{V_o}{U} = \frac{V_{dd}}{C_f \frac{1}{L_f} s^2 + \frac{1}{s^2} + C_f r_{L_f} s + 1}
\]

(12)

4. CONTROLLER DESIGN

This new approach on the AFC controller design has been used to control the output voltage of a full-bridge dc-ac inverter in Byl et al. (2005). The output voltage must follow a sinusoidal reference with a fundamental frequency and the inner closed loop has been designed to provide as much bandwidth as possible.

The plant to be controlled is in this case the discrete-time transfer function \(G(z) = \sum \{ZOH(s)G(s)\}\). Where \(ZOH(s)\) is a zero-order hold used as holding device with a sampling period of \(T = 50 \mu s\). A controller \((C(z) = 0.0098(2^2 - 1.837z + 0.9129)/(z - 0.856)(z - 0.078))\) has been designed for the inner loop. The stability margins for the open-loop transfer function \(L(z) = C(z)G(z)\) show acceptable levels, having a gain margin of 19.3 dB and a phase margin of 47°. The norm \(\|S(z)\|_{\infty}\) lies below 5 dB with \(S_1(z) = 1/(1 + C(z)G(z))\), which represents a good robustness indicator for the inner loop (Doyle et al. (1990)). The inner closed-loop transfer function \(P(z) = C(z)G(z)/(1 + C(z)G(z))\) acts as the new plant to be controlled by the discrete-time AFC approach. As stated in section 2.2, the phase shift of the resonator \(R_k(z)\) at the \(\omega_k\) frequency will be centered at \(-\phi_k\). As in the continuous

\[
\phi_k = \angle P(z_k) + \angle z_k = e^{j\omega_k T} \text{will cause the AFC transmission loop to center all the phase shifts around 0°,}
\]

and therefore providing the system of the highest phase margin available.

The parameters \(g_k\) have been chosen by using a hyperbolic profile, giving more gain to the low frequencies and less gain to the high frequencies. This shape allows the control system to provide the highest energy levels to the region of the fundamental frequency and the lower harmonics where the highest levels of disturbance are usually located.

The feed-forward path \(P_{F}^{-1}(z)\) will reinforce the tracking of the reference signal. As in this case the reference signal is a fixed 50 Hz (fundamental frequency) sinusoid, a constant value of \(P_{F}^{-1}(z) = 1/(P_1(z))\) with \(z_1 = e^{j2\pi T}\) has been chosen, and finally the proportional path \(K_0\) has a value of 0.01. The closed-loop system shows gain and phase margins within good levels while keeping good rejection at the desired harmonics. An overall gain margin of 17.80 dB, and a phase margin of 82.58° are obtained and \(\|S_{AFC}(z)\|_{\infty}\) lies below 5 dB with \(S_{AFC}(z) = 1/(1 + C_{AFC}(z)P_1(z))\).

Fig. 8 shows the Bode plot for the AFC transmission loop \(L_{AFC}(z) = C_{AFC}(z)P_1(z)\) where

\[
C_{AFC}(z) = K_0 + \sum_{k=1}^{30} R_k(z)
\]

As can be seen from Fig. 8 there are 30 resonant peaks at 50 Hz and its respective harmonics. The phase shifts are all centered at 0°, which guarantees the maximum phase margin on the system.

5. EXPERIMENTAL RESULTS

Fig. 9 shows the output voltage and current waveforms for the switched dc-ac voltage inverter system when feeding a full-bridge diode rectifier with a \(C\) filter and a \(R\) load at full load (4.07 kVA), Fig. 10 shows that the total harmonic distortion (THD) of the output voltage does not exceed 0.2% while the output (rectifier) current has a rms value of 26.43 A and a peak value of 73.5A, having then a CF of 2.78 and a THD of 73.5%. Load change tests have been performed from 0 kVA up to 1 kVA output power and vice versa, in both cases the settling time for the amplitude of the output voltage is less than 140 ms. The experimentation was performed over the prototype, at 20 kHz, using pulse width modulation (PWM) with centered pulse. A 2 µs dead time was included at the PWM module complementary signals. The components used in

\[
\text{The THD is calculated with respect to the rms value of the signal being measured, therefore, the THD ranges from 0% to 100%}
\]
6. CONCLUSIONS

The rejection of each harmonic can be improved by selectively increasing the gain $g_k$ in the appropriate resonator $R_k(z)$. As the phase advance parameters have been selected according to the phase value in the inner closed loop ($P_l(z)$), the only design parameters that remain free to adjust are the gains $g_k$ of the resonators and the proportional parameter $K_0$. It is important to note that the differences between the behavior of the resonator in the continuous-time case and the discrete-time case will be higher as the resonant frequency $\omega_k$ approaches the end of the frequency band $\pi/T$. Although the diode rectifier demands large amounts of current from the inverter at the odd harmonics of the fundamental ac frequency, the low output voltage THD achieved at full-load condition confirms the suitability on the use of digital AFC for the solution of this problem. Future research will be performed regarding the generation of the $K_0$ and $g_k$ parameters by means of optimization methods taking into account the appropriate constraints and robustness indexes.

REFERENCES


