Gain-Scheduled Smith PID Controllers for LPV Systems with Time Varying Delay: Application to an Open-flow Canal

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Abstract: In this paper, a new approach to design gain scheduled robust linear parameter varying (LPV) PID controllers with pole placement constraints (through LMI regions) is proposed for LPV systems with second order structure and time-varying delay. The controller structure includes a Smith predictor, real time estimated parameters that schedule the controller (including the known part of the delay) and unstructured dynamic uncertainty which covers the unknown portion of the delay. Finally, the proposed control technique is validated in a real case study based on real a single reach canal: the Lunax Gallery at Gascogne (France).

1. INTRODUCTION

Although the control community has developed new and, in many aspects, more powerful control techniques during the last three or four decades, the PID controller is still used in many of the real world control applications. The reason is that controllers designed with the aid of modern control techniques are usually of high order, difficult to implement and virtually impossible to re-tune on line. Furthermore, if implementation issues have been overlooked, they can produce extremely fragile controllers (small perturbations of the coefficients of the controller destabilise the closed-loop control system). Besides, there are a great number of techniques to tune the PID gains that have been proposed in the literature for linear time invariant (LTI) systems. Since the 60’s, the empirical or classical gain-scheduling (GS) control has been used for controlling non-linear and time-varying systems. But, this control methodology achieves closed loop stability, without guarantees, for slowly varying parameters. In order to overcome this deficiency, linear parameter-varying gain-scheduling (LPV GS) controllers are introduced to allow arbitrarily smooth or discontinuous variations of plant dynamics. The LPV GS method guarantees closed loop stability based on the concept of quadratic stability (QS) (Becker&Packard,1994) for all real parameter trajectories inside a given region. This methodology allows multi-objective criteria ($H_2$, $H_{\infty}$, pole-placement) as well (Apkarian et al., 1995a,b). Under additional hypotheses, such kind of synthesis problems can be transformed to a convex optimisation problem involving linear matrix inequalities (LMI’s). This results in a well behaved and computationally tractable problem. For analysis, when the LMI conditions depend on the system parameter vector in a multi-affine way, it suffices to verify these conditions only at the vertices of the parameter polytope.

The main contribution of this paper is to design a Linear Parameter Varying PID Smith Predictor controller (LPV PID+SP) for second order plus time delay linear parameter varying systems, taking into account robust stability, performance, closed loop pole constraints and essentially the time varying nature of the plant to be controlled: a single reach open-flow canal. This is a system with such dynamical behavior, being the case study used in this paper. The varying parameters are measured (estimated) in real time and used to schedule PID parameters. A “delay scheduling” Smith predictor scheme is used to compensate most of the estimated delay. However, there is still a remaining delay due to the inaccuracy in its estimation that it will be represented as unstructured dynamic uncertainty in a robust control framework. For a general LPV system case, the design of a LPV PID controller should be formulated as an output feedback control that usually derives in solving a non-convex optimization problem based on BMI’s (Mattei, 2003). But, because of the special structure of the plant model considered (second order plus delay), the basic idea in our approach is to tune the PID controller reformulating it as a convex state-feedback problem.

The structure of the paper is the following: In Section 2, the background on LPV theory and on the design of a PID control problem as a state feedback is presented. The formulation, synthesis and implementation of the PID control of a second order plus delay LPV system in LPV framework using a “delay scheduling” Smith predictor (Smith LPV PID) are presented in Sections 3. In Section 4, to validate the proposed methodology this is applied to a real case study based on the control of a single reach canal (Lunax Gallery). In Section 5, final conclusions are drawn.

2. BACKGROUND ON LPV CONTROL

2.1 A LPV General Framework

Given an LPV system described by state-space equations of the form

\[
\begin{align*}
\dot{x}(t) &= A(\theta)x(t) + B(\theta)u(t) + B_w(\theta)w(t) \\
z(t) &= C_z(\theta)x(t) + D_{2u}(\theta)u(t) + D_{2w}(\theta)w(t) \\
gen(t) &= C_q(\theta)x(t) + D_{qu}(\theta)u(t) + D_{qw}(\theta)w(t)
\end{align*}
\]  

(1)
where : the state vector, : the control and disturbance input vectors, respectively, : the measured and controlled output vectors, respectively, : the system output, and : the control parameter vector. We assume the time varying parameters of the system to be measurable. The synthesis technique for LPV systems is based on the following results:

**Theorem 1. (Quadratic H∞ Performance)** (see Apkarian et al., 1995b). The LPV system given by Eq. (1) is QS and has quadratic H∞ performance if there exists a positive definite matrix -stabilizable and detectable over , respectively.

**Remark 1.** According to the self-scheduled H∞ control synthesis problem for LPV systems developed by (Apkarian et al., 1995a), a control design which guarantees the performance for the closed-loop system, should fulfill the following necessary and sufficient conditions:

(i) for all admissible values of the parameter .

(ii) and are parameter independent or equivalently

(iii) The pairs and are quadratically stabilizable and detectable over , respectively.

**Theorem 2 (Quadratic D stability)** (see Chilali et al., 1999). Consider the LPV system with parameter , when is a fixed value ("frozen time"). Its pole location in the LMI-Region of the system at time ,, can be described by:

where is a positive definite matrix, and are related by the following condition:

Then, the matrix is quadratic D stable if and only if there exists a symmetric positive definite matrix such that for all admissible values of the parameter .

\[ \text{E} = \left[ \begin{array}{c} \alpha_{ij} x + \beta_{ij} A(x) x + \beta_{jk} X A(x)^T \end{array} \right]_{1 \leq i,j \leq m}, \]

where is a positive definite matrix, and are related by the following condition:

\[ \left[ X, A(x) x, X A(x)^T \right] \leftrightarrow (1, z, \bar{z}). \]

Moreover, the matrix is quadratic D stable if and only if there exists a symmetric positive definite matrix such that for all admissible values of the parameter .

Based on the fact that a finite set of LMI can be solved in the multi-affine case when the parameters vary in a polytope, a computationally feasible solution to the problem exists, first formulated in (Becker and Packard, 1994), as follows.

**Theorem 3.** (Vertex Property) (see Apkarian et al., 1995b). Consider a polytopic linear parameter-varying plant as in Eq. (1), where

\[ \Theta = \{ \Theta \} \]

and assume that there are three varying parameters in (2) and (4) (Ge et al., 2002). Theorem 3 is fulfilled, Theorem 1 and 2 only should be verified on the vertices of the parameter polytope . This implies that the number of inequalities needed to test the analysis conditions of these theorems can be reduced to a finite one, which makes such an approach appealing.

**2.2 PID LPV Control as State Feedback Control Problem**

For a general LPV system case, the design of a LPV PID controller, , should be formulated as an output feedback control that usually derives in solving a non-convex optimisation problem based on BMI’s. However if the system to be controlled has the following structure,

\[ G(s, \theta) = \frac{b_0(\theta)}{s^2 + a_1(\theta)s + a_0(\theta)} \]

where and are varying-parameters, a convex state feedback problem can be formulated (see Ge et al., 2002) (Zheng et al., 2002) for details, leading to the following state space description:

\[ \dot{x} = A(\theta)x + B(\theta)u + B_r r, \]

\[ y = Cx, \]

where is the system output, the state with variables defined by , and is the reference input, and

**In this state-space model, the PID controller design becomes a static state feedback controller, and the static feedback gain simply contains all the PID controller parameters.** Note also that there are three varying parameters in (2) and (4) (Ge et al., 2002).
3. LPV GAIN_SCHEDULING CONTROL METHODOLOGY

3.1 Smith LPV PID Control Problem Set-up

Let us consider the following LPV system

\[ G(s, \theta) = D(s, \theta)e^{-\tau(\theta)} \]  

(5)

with \( D(s, \theta) \) equal to (2), whose parameters are fixed functions of some vector of varying parameters \( \theta(t) \) that can be measured on-line as in the case of general LPV systems presented in Section 2.1. The parameter range \( \Theta \) is a box defined by \([\theta_{\min}, \theta_{\max}]\) for the gain \( b(\theta) \), and \([a_{\min}, a_{\max}]\) \([\theta_{\min}, \theta_{\max}]\) for denominator coefficients \( a_q(\theta) \), \( a_r(\theta) \) and \([\tau_{\min}, \tau_{\max}] \) for the time delay \( \tau(\theta) \). Our objective is to design a gain-scheduling PID controller using LPV theory for the plant model described by (5) which is an usual representation of many industrial and environmental processes. By including the parameter measurements/estimations, this controller adjusts to the variations in the plant dynamics in order to maintain stability and high performance along all trajectories \( \theta(t) \). In other words, the controller is 'self-scheduled', that is automatically gain-scheduled with respect to \( \theta(t) \). The variable delay in (5) can be handled in two different ways: 1) As an LTI dynamic uncertainty covered conveniently by a weight \( W_d \) as in (Skogestad et al., 1997) (2) As a time-varying parameter which updates a Smith Predictor. The first approach could be conservative, and unnecessarily decrease the overall performance. On the other hand, the second approach could provide a far better performance, but it does not take into account the measurement error of the time-varying delay \( \tau(\theta) \). In this paper, it is proposed to combine both approaches by assuming that a real time estimation \( \hat{\tau}(\theta) \) of the delay is available, which will be used to update a Smith Predictor (Fig. 1). The difference between the actual and the estimated delay is considered as global dynamic uncertainty as in (Skogestad et al., 1997) (Sánchez-Peña et al., 1998) (Morari & Zafiriou, 1989) and is used in the design and robustness conditions. Therefore, we assume that the time delay dynamics has a time varying nature although its estimation error dynamics is time invariant, with a constant bound. The latter can be explained as follows: sensors are usually modelled as time invariant systems, with a bounded error provided by the manufacturer, as we have assumed here. The dependence of the delay with the operating point can be determined by physical modelling (Bolea et al., 2004) or identification and is measured (estimated) in real time. Proceeding in such a way, most of the delay is compensated and the remaining portion, denoted as

\[ \Delta \tau(\theta) = \tau(\theta) - \hat{\tau}(\theta) \]  

(6)

can be covered by LTI unstructured uncertainty. This measurement error is always smaller that the actual delay, therefore the uncertainty is less conservative, which in turn has a lower impact on performance. This uncertainty is handled here as multiplicative output uncertainty and the following weight "covers" the delay measurement error frequency response as tightly as possible (see Chapter 11, (Sánchez-Peña et al., 1998)).:

\[ W_d(s, \tau) = \frac{2.05 Ar_{\max}^s}{Ar_{\max}^s + 1} \quad \text{with} \quad \Delta \tau(\theta) \leq \Delta \tau_{\max}. \]  

(7)

Although the delay is time varying, by assuming that the delay measurement error is time invariant, the same robust stability analysis of the Smith predictor can be performed, following the approach proposed in (Sánchez-Peña et al., 1998) (Morari & Zafiriou, 1989) for the LTI case. This is due to the fact that the remaining system, after the cancellation of delay with the use of its estimation, can be considered as finite dimensional LTI, according to this assumption. Therefore, the delay scheduled Smith Predictor eliminates the infinite dimensional as well as the time varying nature of the delay, reducing it to a LTI dynamic uncertainty. This is one of the main contribution of this work as compared to previous approaches (Ge et al., 2002).

![Fig.1. “Delay scheduled” Smith Predictor scheme.](image)

3.2 Statement of the Smith LPV PID Controller

The LPV PID controller design of the system described by (5) will now be formulated as a state feedback problem and will be embedded in a self-scheduled LPV control problem as developed by (Apkarian, 1995a) (Becker&Packard, 1994) briefly summarised in Section 2.1. The control design specifications that will be considered are a mixture of performance and robustness objectives arranged as a MSP (Skogestad et al., 1997) (Sánchez-Peña et al., 1998) (Morari & Zafiriou, 1989) and is used in the design and robustness conditions. Therefore, we assume that the time delay dynamics has a time varying nature although its estimation error dynamics is time invariant, with a constant bound. The latter can be explained as follows: sensors are usually modelled as time invariant systems, with a bounded error provided by the manufacturer, as we have assumed here. The dependence of the delay with the operating point can be determined by physical modelling (Bolea et al., 2004) or identification and is measured (estimated) in real time. Proceeding in such a way, most of the delay is compensated and the remaining portion, denoted as

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Here \( S \) is the sensitivity and \( T \) is the complementary sensitivity functions. These transfer functions represent weighted tracking error (or disturbance rejection), weighted control action and robust stability, respectively. In order to limit the control energy and bandwidth of the controller, a weight \( W_s \) is included in the design. Such weight is a transfer function with a crossover frequency approximately equal to that of the desired closed-loop bandwidth. The weight for the complementary sensitivity, \( W_c \), captures the uncertainty of the plant model (in this case coming from the delay measurement error) and also limits the closed loop bandwidth. Typically, a disturbance in the system output is a low frequency signal, and therefore it will be successfully rejected if the minimum value of \( S \) is achieved over the same frequency band. This is performed by selecting a weight \( W_c \), with a bandwidth equal to that of the disturbance in the controller design specifications. Robustness is presented as an \( H_\infty \) bound and is related with the dynamic uncertainty coming from the real time delay estimation error.
Performance is a combination of weighted error and control action minimization measured in terms of the energy integrals of the input and output signals involved. A PID controller is a good approximation of a robust high order controller at low frequencies, especially because of the inclusion of the integral action. Then, the resulting PID controller is expected to preserve the disturbance rejection performance of a high-order controller. Furthermore, the time response is tuned via a selected closed loop pole placement (see Section 3.1). A MSP can always be formulated as a Linear Fractional Transformation (LFT), and solved recasting two previous theoretical results (see Section 2.1): 1) Quadratic $H_\infty$ performance. 2) Robust and Quadratic $D$–Stability.

The problem statement is as follows:

**Problem 1.** Given the system in Eq. (1), find a gain-scheduling PID controller using an augmented LPV plant, guaranteeing QS and an $H_\infty$ norm bound less than a positive number $\gamma$ on the w-z input-output channel $\forall \theta \in \Theta$, and pole placement requirements applied to the MSP in Eq. (8).

The control design scheme proposed for Problem 1, which combines measured (estimated) LPV parameters and unstructured output uncertainties is presented in Fig. 2, and it is represented as a LFT. In such a LFT representation, the second order LPV model in (2) and (5) is represented by (1). To achieve a PID controller as a state feedback, it is necessary to consider the following issues:

1. The performance and control effort weight functions need to be constants ($W_c = D_c$, $W_u = D_u$), so that the order of the augmented model is the same as the one in Section 2.2, and a PID controller can be designed.
2. In order not to increase the augmented model’s order, the uncertainty weight in (8) is modified as follows:

$$ W_A(s, \Delta T) = 2.05 \Delta T \tau_{max}, $$

so that $W_A = \tilde{W}_A G_f$.

Then, using a Smith Predictor scheme (Fig. 1) and the uncertainty weight introduced in (9) bounding the delay measurement error in (6), the following LFT LPV system representation is achieved:

$$ q(t) = C_q(\theta)x(t) + D_{qu}(\theta)u(t) + D_{q\theta}(\theta)u_A(t) $$

with $x = [x_1 \ x_2 \ x_3]^T = [y \ x_d \ x_f]^T$, $u = [u_A \ u]^T$ , $z = [y_A \ u \ e]^T$, and

$$ A(\theta) = \begin{bmatrix} -a_u(\theta) & -a_q(\theta) & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad B(\theta) = \begin{bmatrix} D_A & C_A & 0 \\ 0 & 0 & 0 \end{bmatrix}, $$

$$ B_{u_A}(\theta) = \begin{bmatrix} 0 & 0 & 0 \\ D_u & 0 & 0 \end{bmatrix}, $$

$$ C_q(\theta) = \begin{bmatrix} -1 & 0 \\ 0 & D_f \end{bmatrix}, \quad D_{qu_A}(\theta) = -1, \quad D_{qu}(\theta) = 0. $$

(11)

### 3.3 Implementation of the Smith LPV PID Controller

Since the gain, $b_0(\theta)$ of the system in (5)-(6) varies with parameter $\theta$, to fulfill hypothesis (ii) associated to Remark 1 in Section 2.1, the time varying gain of the system can be compensated in the following way. First, the LPV gain-scheduling PID controller $\tilde{K}(\theta) = K[s, a_0(\theta), a_1(\theta)]$ is designed taking into account only the variation of the parameters $a_0(\theta)$ and $a_1(\theta)$, and assuming that the parameter $b_0(\theta)$ has a nominal value $b_{0nom}$. Finally, keeping the same inner loop through equation

$$ \tilde{K}[s, a_1(\theta), a_1(\theta), b_1(\theta)] = K[s, a_1(\theta), a_1(\theta)] b_{0nom}(\theta) $$

(12)

the variation of parameter $b_0(\theta)$ is considered in the design of the controller. Due to the fact that the time varying parameters enter affinely in the augmented model equations (see (10) and (11)), the parameter region is polytopic and since condition (ii) is fulfilled through the transformation introduced by (12), the model of the LPV system can be represented by:

$$\begin{bmatrix} A(\theta) & B(\theta) \\ C(\theta) & D(\theta) \end{bmatrix} = \sum_{i=1}^{\tau} A_i B_i C_i D_i. $$

The delay $\tau(\theta)$ has already been considered as a scheduled (time varying) parameter in the Smith Predictor implementation, and the delay estimation error bounded by a multiplicative uncertainty in the design process, as explained in Section 3.1. Next compute a static time varying state feedback controller, which satisfies QS and the quadratic $H_\infty$ performance specifications. Such a controller can be transformed by the equivalence introduced in Section 2.2, in a PID controller as in (3). This controller schedules the parameters $a_0(\theta)$, $a_1(\theta)$ and by means of the transformation in (12), the scheduling of parameter $b_0(\theta)$ is added. This controller guarantees QS and Quadratic $H_\infty$ Performance, as well as (“frozen”) closed loop pole location inside the desired LMI region. Since the plant is polytopic, the controller $K(s, \theta)$ is designed as a polytopic model and implemented according to:
\( K(\theta) \in \text{Co}\{K(v_1), K(v_2), \ldots, K(v_r)\} \) = \( \{ \sum_{i=1}^{r} \lambda_i K_i ; K_i = K(v_i) \} \)

where: \( \sum_{i=1}^{r} \lambda_i(\theta) = 1 \), \( \lambda_i(\theta) > 0 \).

This technique is known as a convex decomposition technique, and \( \text{Co} \) is the function that generates the convex hull of the polytope vertices. The polytopic coordinates are calculated by fast algorithms in such a way that each vertex \( v_i, i=1,\ldots,r \) has coordinates:

\[ \lambda_j = \prod_{j=1}^{r} \theta_j^i \text{ with } \theta_j^i \]

where \( \theta_j^i \) is a coordinate of \( v_i \) and \( \theta_j^i \) is a coordinate of \( v_j \).

\[ \overline{\theta}_j^i = \left( \overline{\theta}_j^i - \theta_j^i \right) \text{ if } \theta_j^i \text{ is a coordinate of } v_i \]

and \( (\overline{\theta}_j^i, \overline{\theta}_j^i) \) represent the upper and lower bounds of \( \theta_j^i \), and \( l \) fulfills that \( r^2 \).

Finally, the closed-loop system is \( \dot{x} = A_\theta(\theta)x + B_\theta w \), with matrices \( A_\theta(\theta) \) and \( C_\theta(\theta) \) that depend on the parameter vector \( \theta \) described as follows:

\[ A_{j_i} = \left[ \sum_{i=1}^{r} \lambda_i(\theta)(A + B_\theta K_i) \right], B_{j_i} = \left[ \sum_{i=1}^{r} \lambda_i(\theta)(C + D_{\theta} K_i) \right] . \]

4. APPLICATION TO A SINGLE REACH CANAL

4.1 Lunax Gallery

The Lunax gallery is located at Gascogne in southwestern region of France. The dam-gallery is used to supply with water the river Gessse. As depicted in Fig.3, the plant consists in a single reach canal, the dam gate controls the upstream flow. The control objective aims at regulating the downstream flow even if the measurement point is far from the gate, at the output of the gallery. The geometry of the gallery is circular. Geometrical data are: \( d \) (diameter) = 1.8m, \( l \) (length) = 946.65m, \( i \) (slope) = 0.0026rad, Strickler coefficient = 65.

4.2 LPV Model Motivation

The Saint-Venant equations can be simplified by considering the two following assumptions: lateral discharges are null and inertia terms are negligible compared to one representing the energy lost by friction. The approximated equations, known as diffusive wave equation, can be linearized around a reference flow \( Q_{\text{up}} \). According to (Litrico et al., 1999), from the diffusive wave equation and the moment matching method, the following transfer function, known as Hayami model, linking the upstream flow \( Q_{\text{up}} \) and downstream flow \( Q_{\text{down}} \) for a single reach canal of length \( X \) can be derived using a simulator developed by the group of "Modelling and Control of Hydraulic Systems" at the UPC.

\[
G(s) = \frac{Q_{\text{down}}(s)}{Q_{\text{up}}(s)} = \frac{e^{sX(\theta)}}{1 + k_1(\theta)s + k_2(\theta)s^2} \tag{15}
\]

where the scheduling parameter \( \theta = Q_{\text{up}} / Q_{\text{down}} \). In the Lunax Gallery, the time varying parameters of the model (15) can be bounded taking into account that operating range of the scheduling variable is \( \theta = [0, 5] \); \( k_1(\theta) \in [199, 245] \), \( k_2(\theta) \in [8396, 17355] \) and the time delay \( \tau(\theta) \in [73, 252] \) (s).

4.3 Controller Design and Simulation Results

Comparing (15) with (2), it can be obtained \( h_\theta(\theta) = \frac{1}{k_2(\theta)} \), \( a_\theta(\theta) = \frac{1}{k_3(\theta)} \) and \( a_\theta(\theta) = \frac{k_1(\theta)}{k_2(\theta)} \). Then, the control methodology presented in Section 3 can be applied. Additionally, the delay estimation error is considered to be bounded \( \Delta \dot{\tau}(\theta) \in [0, 5] \). The error in the time delay is taken into account in the control design as an LTI unstructured multiplicative uncertainty \( W_\Delta(s, \Delta \dot{\tau}) = 10.25 \). Once the main time varying delay has been compensated by the Smith Predictor (Fig.1) and the remaining delay error considered as the weight \( W_\Delta(s, \Delta \dot{\tau}) \) of a multiplicative dynamic uncertainty (see Section 3.1), a PID controller is designed as a state feedback. This controller should guarantee closed loop stability and the following (step response) performance specifications: 1) tracking error of 0.1, 2) control signal within \([0, 5]\) and 3) closed-loop damping of \( \xi \geq 0.5 \) and settling time in \( t_{\text{ss}} \approx 230(s) \), for any arbitrarily fast parameter variation. The tracking error and the bounded control signal are represented by performance weights \( W_e = 1 \) and \( W_u = 0.4 \), respectively. Furthermore, to achieve this desired transient behaviour and prevent controller fast dynamics, a pole clustering constraint is added. To this end, a LMI region \( \Sigma(h_\theta, h_\theta, \alpha) \) is defined as a combination of three subregions: 1) A conic sector with apex at \( x = 0 \) and angle \( \alpha = 3\pi/4 \), which captures the closed-loop damping constraint \( \xi \geq 0.5 \). 2) Left half plane that guarantees the maximum settling time \( h_\theta = -0.0018 \). The obtained closed loop responses using the LPV PID+SP design are shown in Fig.4.

\[2\] The real canal behaviour is accurately reproduced by Saint-Venant’s equations using a simulator developed by the group of “Modelling and Control of Hydraulic Systems” at the UPC.
Fig. 5 shows the flow released upstream by the gate while Fig. 6 presents the evolution of the model parameters. It can be observed that the performance specifications are achieved for the whole admissible operating range. On the other hand, if a robust LTI $H_\infty$ PID controller with a standard LTI Smith Predictor designed for the worst case set of parameters is used, the time response is slower than the one obtained by its LPV counterpart and do not satisfy the performance specifications for the whole admissible operating range.

5. CONCLUSIONS

The main contribution of this paper is the development of a new approach to design a gain-scheduled Smith PID controller for LPV second order systems plus delay solving a MSP problem with closed loop pole placement constraints. The time varying delay is handled by a “delay-scheduling” Smith predictor and the estimated delay error is treated as an unstructured dynamic uncertainty. Thanks to the second order system structure, the PID controller design can be viewed as an state-feedback controller whose design can be transformed to a convex optimisation problem involving LMI’s. This approach has successfully been applied real case study based on the control of a single reach canal (Lunax Gallery).

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