Humanoid Gait Synthesis Using Trajectory Plot and Relative-ZMP (R-ZMP) Concept

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Abstract: The human body exploits the redundancy of the degree of freedom (DOF) to execute various motions and maintain the body stability. In Humanoid Robots (HRs), the Zero Moment Point (ZMP) and Center of Gravity (COG) are commonly used to evaluate stability. One of the basic motions of a HR is walking. There are two types of walking scheme, namely the Static and Dynamic Gait. The Static Gait uses the COG as the stability criterion. Static Gait usually yields a very slow walking gait. Hence, static walk is now not very common in HRs. On the contrary, Dynamic Gait employs the ZMP as the criterion of stability. Robots like ASIMO and QRIO use Dynamic Gait scheme to achieve impressive walking speed and stability. In this paper, a new trajectory design algorithm by using the trajectory plot and posture plot is proposed. This method is based on the new Relative-ZMP (R-ZMP) concept. The resulting walking gaits were then tested on a humanoid with 18 DOF, LUCY. It is evident that it is able to walk much faster in comparison with other humanoid robots which participated in the Humanoid Robocup Competition 2006.

1. INTRODUCTION
The idea of HRs can be traced back to Leonardo da Vinci’s robot [1] in 1945. At that time, human’s dream was kept in a Pandora’s Box due to the limitations of technology. In 1986, human’s dream takes a first step forward when Honda decided to take up the challenging project, the P-0 Humanoid project (Now known as ASIMO). The aims are to develop robots that are able to coexist and collaborate with human beings, to replace human beings in some of their roles in the interactions between human beings and robots, and to extend human capabilities for interaction with the environments [2]. They started with static gait synthesis on the E1 with a speed of 0.069 m/s and then to a speed of 0.333 m/s, 4.8 times faster using zero moment point (ZMP) concept by Vukobratovic et al [3,4]. From there onwards most successful HRs have commonly adopted the ZMP criterion for stability. However, sometimes, Dynamic Gait can be achieved simply without explicitly considering any stability criterion [5]. Specific trajectories and precise trajectory tracking are not indispensable for HR walking gait. For example, passive biped robots can walk stably down a shallow slope with no sensing or control capabilities. If one would apply ZMP as a criterion in the passive biped walking robot, it would result in an unstable system. Furthermore, by applying the ZMP concept without careful consideration in the HR direct kinematics and structure may result in awkward and impossible posture. Hence, the design of ZMP trajectory is regarded as a complicated and delicate problem. The authors of [6] proposed the sway compensation trajectory to simplify the ZMP or COG trajectory design. This method uses a single mass system to evaluate the single-mass ZMP trajectory first and then apply convergence calculations to refine the single-mass ZMP into a multi-mass ZMP system. In a practical system, the robot capability of tracking a ZMP especially in fast walking is limited by the minimum sampling time of the command to motor sampling time. Hence, convergence calculation in the sway compensation trajectory may not be applicable.

This paper presents a practical way of implementing the humanoid fast dynamic trajectory gait by designing trajectory using the Relativity-ZMP and Trajectory Plot. The resulting walking gaits are applied to an 18 DOF humanoid, LUCY and the performance is compared with HRs which participated in the Humanoid Robocup Competition 2006. This paper is organized as follows: Section 2 describes the development and the application of trajectory and posture plot and the Relative ZMP (RZMP) concept. In Section 3, the approach is demonstrated via several experiments and adjustments. The experimental results are analyzed in details. Finally, we compare the performance with other HR performance that participated in Humanoid Robocup Competition 2006. Section 4 discusses the salient points of our work and Section 5 concludes the paper.

2. TRAJECTORY DESIGN USING THE TRAJECTORY PLOT AND R-ZMP AS STABILITY CRITERIA

2.1 Humanoid Modeling
To solve the HR redundant system, we regard the HRs as made up of 3 simple robots (Fig.1). The Side Model is a simple two-link robot with a fixed frame at the hip of a lift-up leg. And the Front Gamma Model is just a simple trigonometry model. (Note: 2 Side and 1 Front models are...
used to represent the HR. Accordingly, we indirectly apply constraints to it inverse kinematics)

Figure. 1. Parameters and coordinates assignment of a typical humanoid robot

Fig. 1 shows the parameters and the coordinates assignment used as the inputs to the inverse kinematics. The terms \( l_1 \) and \( l_2 \) are the physical length measurements from the HR. \( El \), the effective leg length determines the maximum stretch the leg can achieve. Hence, it is related to how much the HR need to knee down before it walks. We use a safety limit of 0.1 to avoid inverse kinematics singularity.

2.2 Trajectory Design Using Trajectory Plot

The Trajectory Plot (TP) is a multi plot of each \( x \), \( y \) and \( z \) against time in a parallel fashion. This plot is very important to help analysing and realising a trajectory design. We will illustrate a forward walking trajectory design using the TP.

2.2.1 Design of the \( y \) Sway Trajectory

First, let us measure the distance, \( d \), from the origin to the most stable point \( d \) (Fig.2). Then, we determine what kind of sway profile should be used. Usually, sine profile is chosen. This is because the first order derivative continuity guarantees smoothness of joint velocity, while second order derivative guarantees smoothness of acceleration or torque of joints.

Figure. 2. Most stable point

In the LUCY HR, \( d \) is measured to be around 20mm and the total period of a walking cycle is also determined. Here, we would like to design the gait for fastest walking achievable by the LUCY HR, so we used the minimum 24 points design so that we could maintain the sine shape and symmetry of left and right legs. Each point represents a 0.0029s sampling time that is needed for the servo motor response. The aim of the \( y \) sway is to shift the ZMP from one stable point to other stable points and \( a \) is defined as the point where the lifting leg is stable enough to lift off. We can treat the system as the single-mass system first [6]. Hence, we can determine \( a \) using the COG method. The COG must be in the support polygon to lift up the lifting leg. So, \( a \), is the time the \( y \) sway must be at least 70% of \( d \) to touch the support polygon (70% is obtained by measurement), and 0.7 of the sine function is 45 deg. Then, the maximum angle is 90 deg. So, \( 45/90 = 1/2 \). There are a total of 6 points from 0 to \( b \), hence, \( 6*1/2 = 3 \). We add in the safety factor of one sampling point, \( a = 4 \) and \( \Delta b = 2 \). Similarly, \( \Delta d = 4 \) and \( \Delta e = 2 \). Next, the HR is actually a symmetrical robot between right and left side of the robot. Hence, we can use \( \Delta f = a \), \( \Delta g = \Delta b \), \( \Delta h = \Delta c \), \( \Delta i = \Delta d \) and \( \Delta j = \Delta e \).

2.2.2 Design of the \( z \) Component for Both Legs Based on the \( y \) Sway.

Again, we need to determine the profile to use. In this case, we choose the sine function to ensure smooth transitions. In step one, we already know that lifting the right leg can lift up in \( a \) and put down before \( d \) and for left lifting leg is from \( f \) to \( i \). As for the height of the lifting, it depends on the obstacle height. Here, we arbitrary assign the height to 20mm.

2.2.3 Design of \( x \) Component Based on the \( y \) Sway and Both Leg \( z \) Components.

\( x \) component is the most difficult component in the trajectory design. Here, we will design the \( x \) component base on the MBIK coordinate requirement and we will use only the \( a \) model to simplify the design. The step size is determined to be 40mm. Based on the \( y \) sway and the \( z \) components the robot is in the position and transition as shown below.

Figure. 3. X transition

Following the MBIK coordinate system, the transition as seen as the right leg translates from a positive \( x \) to negative \( x \) with the width of the step size. It is the same for the left leg. With this understanding, we are able to design the \( x \) component for both legs.

2.3 Relative-Zero Moment Point(R-ZMP) as Stability Criterion

The Relative-ZMP concept is different from the current ZMP concept. The usual ZMP concept is only concerned with its position and gives no relationships between the links to a control point. The control point is the point that is used to control the robot. This is due to fact that we cannot control all the robot links as they are interconnected. Hence, we usually choose a point as our control point to command a robot to
move in a 3-D space. In our robot, the control point is the Center of Mass (COM) of the robot as shown below.

As can be seen if we control the y sway of the robot, we lose control to the rest of the links to achieve the posture we want. Hence, by finding its ZMP will not help us to fulfill the ZMP criterion.

The R-ZMP concept is about finding the relationship between the COM and the ZMP generated by the dynamic system of the HR. In particular, when we design a trajectory, it is much simpler if we choose COM as the control point and design the trajectory based on the COG method. Next, we apply the supporting polygon criterion using the R-ZMP, and adjust the COG trajectory design accordingly. This method reduces the complexity of traditional ZMP trajectory design problem. This is because the original ZMP trajectory design does not consider the connectivity of the robots links, and hence result in non-unique solutions. Furthermore, it may also cause awkward or impossible posture [11]. By using the R-ZMP concept, we only need to adjust the COG trajectory design to fit the R-ZMP trajectory as close as possible to the desired ZMP one.

The R-ZMP equation can be written as follows:

$$x_{RZMP} = \frac{\sum_{i=1}^{n} m_i (\dot{x}_i + g)(x_i - x_c) - \sum_{i=1}^{n} m_i \dot{x}_i (z_i - z_c) - \sum_{i=1}^{n} I_{iy} \dot{\Omega}_y}{\sum_{i=1}^{n} m_i (\dot{z}_i + g)}$$

where $x_c$ and $z_c$ are the control point’s x component and z component. Similarly,

$$y_{RZMP} = \frac{\sum_{i=1}^{n} m_i (\dot{z}_i + g)(y_i - y_c) - \sum_{i=1}^{n} m_i \dot{y}_i (z_i - z_c) - \sum_{i=1}^{n} I_{ix} \dot{\Omega}_x}{\sum_{i=1}^{n} m_i (\dot{z}_i + g)}$$

Then, $RZMP = (x_{RZMP}, y_{RZMP}, 0)$

2.4 Tuning Parameters to Meet the R-ZMP Stability Criterion

There are 2 tuning parameters that need to be tuned to meet the support polygon criteria, namely the Y sway amplitude and the step coefficient, $\lambda$.

2.4.1 Y Sway Amplitude Parameter

Y sway amplitude adjustment is needed because our original design is based on the COM without considering the acceleration effects of each link involved. This will result in the y component of ZMP swayed more than required and hence result in the stability problem. The R-ZMP is a non zero value because it is impossible to archive 100% matching of the COM trajectory due to the robot link connectivity. (Note that RZMP is a measure of the differences of our desired trajectory and the actual ZMP.)

2.4.2 Step Coefficient, $\lambda$ Parameter

For the x axis of the R-ZMP, $RZMP_x$, the value of $\lambda$ decides how close R-ZMP$_x$ is to the control point. $\lambda = 0$ means that the R-ZMP$_x$ is closest to the control point while $\lambda = 1$ is the farthest the R-ZMP$_x$ to the control point. We can measure how close the R-ZMP$_x$ is by using the Root Mean Square (RMS) of R-ZMP$_x$. Small value of the RMS of R-ZMP$_x$ is desirable if the stable trajectory is known. If the stable trajectory is unknown, the desired trajectory may not be stable. So, R-ZMP$_x$ should be adjusted until it meets the Support Polygon Criterion. In this paper, we treat the stable ZMP trajectory as unknown and the trajectory given in the previous section is not a stable trajectory design. The purpose of assuming that we do not know the stable ZMP trajectory is to allow us to study the adjustments that are needed to make an unstable trajectory stable.

The definition of the Step Coefficient is a fraction of the step size that the front and back legs need to achieve as shown in the figure. Usually, we design our trajectory by assuming $\lambda=0.5$ first and further adjustments will improve the gait stability.

Note : that adjusting the y sway amplitude and the $\lambda$ does not change the walking requirement of a step size and the height of the leg needed to be lifted (Fig. 6). These parameters allow us to change the trajectory design without affecting the walking requirements. Hence, the robot can perform the required task and yet maintain its stabilization.

3. EXPERIMENTS

3.1 The Trajectory Design and Tuning

The initial trajectory design in Fig. 7 was first fed into our 3D simulator and the walking parameters are lifting leg up by 20mm, step size of 40mm and the default $\lambda$ is 0.5. The ZMP positions were then calculated and are shown in Fig. 8.
Figure 6. The effect of $\lambda$ on x axis trajectory legs transition
(a) Show that how $\lambda = 0$ (b) Show that how $\lambda = 1$
task and yet maintain its stability.

Figure 7. Designed trajectory plot

Figure 8. Trajectory and it’s ZMP

In Fig. 8, we can see that the ZMP is different from the
designed trajectory. This difference is the RZMP, which will
allow us to adjust the y sway amplitude and the step
coefficient, and improve the stability without changing the
walking parameters. We can access the robot’s stability by
inspecting the simulator result.

Figure 9. A half cycle walking trajectory before tuning
The purple points in Fig. 9 indicate the ZMP positions. As
can be seen in the sequence number 6,7,9,10, when the lift-
off point of the right leg is too high, the ZMP falls out of the
single phase support polygon. These will result in the robot
falling down. Its corresponding RZMP are given in Fig. 10.

Figure 10. RZMP before tuning

Fig. 9 clearly shows that the original trajectory gait design
fails to meet the support polygon criterion. Hence, the
trajectory is unstable. The y axis of RZMP, RZMP, is
relatively easy to adjust since we are not able to control
RZMP. The only parameter that can be controlled is the
trajectory y sway amplitude, so we simply adjust the y sway
amplitude until the RZMP is inside the support polygon. The
RZMP needs to be adjusted away from the original
trajectory design and hence, the RMS RZMP is bigger in
value for the adjusted trajectory. This adjustment of RZMP
is done by adjusting the Step Coefficient, $\lambda$ and the adjusted
values are shown as follows:

<table>
<thead>
<tr>
<th>Tuning</th>
<th>Before</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.5</td>
<td>0.75</td>
</tr>
<tr>
<td>$y_s$</td>
<td>20</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 1 Tuning Parameters

Figure 11. Trajectory and it’s ZMP after tuning
The tuning parameters affect the stability but did not change the walking parameters requirement. From Fig. 11 we can see that the trajectory is now nearer to the desired trajectory. Fig. 12 on the other hand shows that the ZMP now fulfills the Support Polygon Criterion.

![Figure. 12. A half cycle of the after tuning trajectory](image)

We now inspect the RZMP changes. The RZMP\(_x\) is now bigger in value as planned to improve the trajectory gait stability and the RZMP\(_y\) is now small to ensure the y component follow closely the y sway designed (Fig. 13).

![Figure. 13. RZMP after tuning](image)

The tuning parameters cause the change in trajectory design but maintain its walking parameters. Fig.14 shows that the trajectory change does not affect the walking parameters. On point P, the reading of x component of the 2 legs, \(|30| + |10| = 40\) and the height of the leg lifting is still 20mm, the same as the trajectory before tuning.

3.2 The Trajectory Implementation

The Stable Trajectory in Section 3.1 was implemented on a HR called LUCY. The weight of LUCY is 4.0 kg including batteries. Each leg has 6 DOF, each arm has 2 DOF and the head has 2 DOF, which gives a total of 18 DOF. The salient feature is that it has an onboard CPU using Intel 80386EX. LUCY is able to walk without falling even during the first experiment. However, the joint links are not firmly connected in this HR. Hence, the link tends to drop of the required position especially the lifting leg link. So, a five degree step compensation was included into the hip servo motors to offset the dropping effect. After the correction degree was added, the robot shows a much faster walking speed compared with the performance of the HR who participated in the Humanoid Robocup Competition 2006.

From Fig. 14, the average speed would be \(v = 0.098\text{m/s}\) . Note that 35ms was taken instead of 29ms in view of the limited processing time of LUCY. The performance comparisons of our walking performance with other performers in Humanoid RoboCup Competition 2006 are given in Appendix. A.

4. DISCUSSIONS

Currently the available methods of tuning are to directly change the walking parameters and the ZMP location are all reference to the origin, hence, the relationship between the ZMP and the desired gait cannot be studied. The salient feature of the RZMP is that we can investigate how the tuning parameters affect the stability without altering the walking parameters. This is very important especially in avoiding obstacles by walking over them. The RZMP tuning parameters also reduce the complexity of gait tuning by introducing only 2 tuning parameters, and they are treated separately. The trajectory design using the Trajectory Plot gives a systematic and graphical way of designing a walking trajectory. The proposed separate concern of topology approach in this design algorithm helps to reduce the dimension design problem.

5. CONCLUSIONS

In this paper, a new trajectory design algorithm using the Trajectory Plot, which helps to simplify the trajectory design by separation of concern topology, is proposed. To compliment the stability aspect of the trajectory design, the Relative ZMP (RZMP) was introduced to enable the study of effects and relationships of the gait and the ZMP. This leads us to the tuning parameters of y sway amplitude and the Step coefficient, \(\lambda\). Through the two parameters, we can change the trajectory to improve the stability without changing the walking parameters. Finally, better understanding and tuning of our algorithm and concepts results in faster walking compared with other performers in the RoboCup 2006.

REFERENCES


Appendix A. Comparison of walking speeds with participants in Humanoid Robocup 2006

<table>
<thead>
<tr>
<th>No</th>
<th>University/Institute</th>
<th>Team Name</th>
<th>Weight</th>
<th>Walking Speed</th>
<th>Walking Speed m/s</th>
<th>Relative to 0.098 m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>University of Padua, Italy</td>
<td>Aritisi</td>
<td>2.2kg</td>
<td>1.5m/min</td>
<td>0.0250</td>
<td>3.93</td>
</tr>
<tr>
<td>2</td>
<td>Humboldt Univ. Berlin, Germany</td>
<td>Humanoid</td>
<td>2.1kg</td>
<td>0.05m/s</td>
<td>0.0500</td>
<td>1.96</td>
</tr>
<tr>
<td></td>
<td>Team Numboldt</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>King Mongkut’s University of Technology Thonburi, Thailand</td>
<td>Team KMMUT</td>
<td>3.3kg</td>
<td>1.5m/min</td>
<td>0.0250</td>
<td>3.93</td>
</tr>
<tr>
<td>4</td>
<td>University of Freiburg, Germany</td>
<td>NimbRo</td>
<td>2.3kg</td>
<td>25cm/s</td>
<td>0.0250</td>
<td>3.93</td>
</tr>
<tr>
<td>5</td>
<td>Panamerica University Campus Guadalajara,Mexico</td>
<td>PionerosMexico</td>
<td>2.1kg</td>
<td>0.5m/min</td>
<td>0.0083</td>
<td>11.79</td>
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<td>Singapore Polytechnic</td>
<td>RoboErectus</td>
<td>3kg</td>
<td>2m/min</td>
<td>0.0333</td>
<td>2.96</td>
</tr>
<tr>
<td>7</td>
<td>National University of Singapore</td>
<td>RO PE</td>
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<td>3m/min</td>
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</tr>
<tr>
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<td>12m/min</td>
<td>0.2000</td>
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<tr>
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