Fault Detection based on Orthotopic Set Membership Identification for Robot Manipulators

Vasso Reppa ∗ Anthony Tzes ∗

Abstract: In this article a fault detection algorithm for capturing structural and/or sensor failures in robot manipulators is presented. The robot dynamics is linearizable with respect to a certain parameter. Using this linearization representation, common faults in robot arms, such as failures of actuators or faulty sensor measurements, can be identified as variations encountered in the parameter vector. The proposed algorithm uses an Orthotopic Set Membership Identifier that defines the feasible parameter set and the parameters’ bounds, within which the Weighted Recursive Least Square parameter estimate resides. An Uncertainty Output Predictor that generates the future region of faultless system operation. A fault is detected, when one of the following criteria below is validated: a) the WRLS parameter estimate resides out of the parameters’ bounds, b) there is a sudden increase in the volume of the feasible set and c) the system’s output is not within the predicted interval. Simulation studies are offered to test this fault detection methodology, customized to a two-link robot arm.

1. INTRODUCTION

From a general point of view, the fault diagnosis problem is concerned with the detection of time instants where there is a significant difference in the nominal system’s behavior. The next step is the detection of the reason of the fault occurrence. In case of robot manipulators, De Luca et al. [2005] determine a fault as the unexpected behavior observed in its torques, when a technical failure occurs. Dixon et al. [2000], Shin et al. [1999] report a Fault Detection Scheme targeting failures of actuators or active bias to a sensor measurement etc. Similarly, a false operation in its workspace, because of accidental collision with unknown obstacles or manipulating an unknown load has been reported in De Luca et al. [2005] and Spong [2001], respectively.

The classical statistical methodology for fault detection is based on a fault indicator, or residual, which is computed via a specific model and observation, and defines a fault symptom De Luca et al. [2003], Yen et al. [2000], Greenwood [2005], Zhang et al. [2004]. This method is applied mostly in case of sensor failures in robot manipulators. On the other hand, the deterministic methodology for fault detection concerns set-membership approach, which takes into account a priori knowledge of model uncertainties and measurement errors Adrot et al. [2002], Milanese et al. [2003], Fagarasan et al. [2004], Ploix et al. [2001]. The goal of the set-membership approach is the characterization of a set of all parameter vectors that are consistent with the data, model structure and bounded noise errors, called feasible parameter set. In most techniques, the system output must be linearizable with respect to parameter vector. The benefit of the second methodology is the utilization of the parameter’s intervals that arise from polytopes Chisci et al. [1998], Ingimundarson et al. [2005], bounding the feasible parameter set.

In this paper, a Fault Detection (FD) algorithm based on the interactive relation of an Orthotopic Set Membership Identifier (OSMI) and an Uncertainty Output Predictor is presented. The OSMI uses two geometric approaches: the ellipsoid Cheung et al. [1993], Fogel et al. [1982], Milanese et al. [1982], for the characterization of the feasible parameter set and the orthotope Le et al. [1997], Tzes et al. [1999], bounding the ellipsoid, for the computation of parameters’ bounds. The center of both the ellipsoid and the orthotope is the Weighted Recursive Least Square (WRLS) parameter estimate, and its volume reflects the parameter uncertainties, being induced from the bounded noise error. The vertices of the orthotope represent the parameter interval Walter et al. [1990], Fagarasan et al. [2001]. The bounds of the parameter interval are the inputs to the Uncertainty Input/Output Predictor that generates the limited region of proper system operation. Finally, the fault detection is accomplished, when: a) the WRLS estimate of the parameter vector does not reside within the computed bounds, or b) the volume of the ellipsoid is suddenly increased Reppa et al. [2007] and c) the systems output is not within the predicted limited region Reppa et al. [2006].

This paper is structured in the following manner. The non linear system dynamics of a robot arm and the inherent assumptions that must be satisfied for the proper application of the FD-methodology are presented in the next section. The mathematical preliminaries of the OSMI is detailed in section 3, followed by the simulation studies and the conclusive remarks.

2. PROBLEM STATEMENT

The dynamic equation of an m-link robot manipulator is given from the Euler-Lagrange theory as:

\[ M(q) \ddot{q} + C(q, \dot{q}) + G(q) = \tau \]  

(1)
where $M(q) \in \mathbb{R}^{m \times m}$ is the symmetric positive definite inertia matrix, $C(q, \dot{q}, \ddot{q}) \in \mathbb{R}^{m \times m}$ is the Coriolis and centripetal matrix, $G(q) \in \mathbb{R}^{m}$ is the gravitational vector and $\tau \in \mathbb{R}^{m}$ are the applied torques.

Equation (1) can be formed as:

$$\tau = \theta^T Y(q, \dot{q}, \ddot{q})$$

where $\theta$ is a constant $(r + p)$-parameter vector and $Y$ is an $(r + p) \times n$ matrix of known functions of the generalized coordinates and their derivatives $(q, \dot{q}, \ddot{q})$. The above equation means that the Lagrangian dynamic equations are linearizable, with its discrete dynamics expressed in the most general form as

$$\begin{align*}
y(n) &= \theta(n)^T \phi(n) + e(n), \quad y(n), e(n) \in \mathbb{R}^m \\
\phi(n) &= \begin{bmatrix} \phi_1(u(n-1), \ldots, u(n-r), y(n-1), \ldots, y(n-p)) \\
\phi_2(u(n-1), \ldots, u(n-r), y(n-1), \ldots, y(n-p)) \\
\vdots \\
\phi_m(u(n-1), \ldots, u(n-r), y(n-1), \ldots, y(n-p)) \end{bmatrix}^T \\
\theta(n) &= \begin{bmatrix} \theta_1(n), \theta_{r+1}(n), \ldots, \theta_{r+p}(n) \end{bmatrix}^T
\end{align*}$$

3. FAULT DETECTION BASED ON ORTHOTOPIC SET MEMBERSHIP IDENTIFICATION

3.1 Orthotopic Set Membership Identifier

Ellipsoid Definition: The intricacies behind the OSMI and its utilization in a fault detection scheme are presented in this section. The identified orthotope bounds an ellipsoid which contains the true parameter vector. The ellipsoid is

$$\Omega(n) = \left\{ \theta : (\theta - \hat{\theta}(n))^T C(n) \frac{\theta - \hat{\theta}(n)}{\xi(n)} \leq 1, \theta \in \mathbb{R}^{r+p} \right\}$$

and

$$\xi(n) = \hat{\theta}^T(n) C(n) \hat{\theta}(n) + \sum_{i=1}^{n} \lambda_i \gamma_i (1 - \gamma_i y^2(i))$$

where $\hat{\theta}(n)$ is the WRLS estimate of the parameter vector, $C(n)$ is the covariance matrix, and the symmetric positive definite matrix $W(n) = C(n)/\xi(n)$ determines how far the ellipsoid extends in each direction from $\hat{\theta}(n)$, and $\lambda$ leads to the minimization of the ratio of the ellipsoid's volume at $n$ and $n - 1$, namely

$$\lambda(n) \in \left\{ \lambda_0 : \min_{\lambda} \frac{\det(B(n))}{\lambda \det(B(n-1))} \right\},$$

$$B(n) = \left( W(n) \right)^{-1}$$

Orhtotope Definition: In the sequel, the orthotope oriented parallel to the parameter coordinate axes and centered on the centroid of the ellipsoid, is computed via the equation [28]

$$\Omega^p(n) = \left\{ \theta : \frac{1}{\sigma_i(n)}(\theta_i - \hat{\theta}_i(n)) \leq 1, i = 1, \ldots, r + p \right\}.$$
and the orthotope’s volume

\[ V_{\text{Orthotope}} = 2^{q+p} \sigma_1 \times \cdots \times \sigma_{r+p} \]  

(11)

3.2 Uncertainty Output Predictor

The Uncertainty Output Predictor uses the vertices of the orthotope provided by the OSMI, as the inputs to the uncertainty-predictor for the computation of the \( \lambda \)-step ahead predicted output interval, which is given as:

\[ \hat{y}(n + \lambda | n) = \left[ \hat{y}^-(n + \lambda | n), \hat{y}^+(n + \lambda | n) \right] = \theta(n) \hat{\theta}(n + \lambda) + \epsilon(n), \lambda \geq 1, \]  

(12)

where \( \epsilon(n) \in [-e^M(n), e^M(n)] \) and \( \gamma_n = 1/(e^M)^2 \) in (6). A recursive relationship is derived for generating the \( \lambda \)-step ahead predicted output interval, under the assumption that at time \( n \), all past output quantities are known. The generic expression for \( \hat{y}(n + \lambda | n), (1 \leq \lambda \leq p) \) is given as:

\[ \hat{y}(n + \lambda | n) = \sum_{i=0}^{\lambda-1} B_{A,n}^{i+1} \hat{u}(n + i) + A^\lambda \hat{\epsilon}(n + \lambda) + \sum_{i=1}^{\lambda} \theta_{\lambda+i}(n) \epsilon(n + i), \]  

(13)

where \( A^\lambda, B_{A,n}^{i}, i = 0, \ldots, \lambda - 1 \) are computed using the following recursive scheme:

\[ B_{A,n}^{0} = \sum_{i=0}^{\lambda-1} \theta_{\lambda+i}(n) B_{A,n}^{\lambda-i}, \theta_{\lambda-1}(n), i = 0, \ldots, \lambda - 1 \]

\[ B_{A,n}^{0} = \theta_{\lambda}(n), B_{A,n}^{1} = 0, \quad \lambda - 1 \]

\[ A^\lambda = \sum_{i=1}^{\lambda} \theta_{\lambda+i}(n) A^i + \sum_{j=1}^{\lambda} \theta_{\lambda-j}(n) A^{\lambda-j} \quad \sum_{j=1}^{\lambda} \theta_{\lambda-j}(n) A^{\lambda-j} \quad \sum_{j=1}^{\lambda} \theta_{\lambda-j}(n) A^{\lambda-j} \quad \sum_{j=1}^{\lambda} \theta_{\lambda-j}(n) A^{\lambda-j} \]

(14)

\[ A^\lambda = \sum_{j=1}^{\lambda} \theta_{\lambda-j}(n) A^{\lambda-j} + \sum_{j=1}^{\lambda} \theta_{\lambda-j}(n) A^{\lambda-j} \]

3.3 Fault Detection Criteria

Using the Orthotopic Set Membership identifier and the Uncertainty Predictor Error, the FD-scheme recognizes a fault when:

(1) the centroid \( \hat{\theta} \notin \Omega^p(n) \), or

(2) \( \det(B(n)) > \det(B(n - 1)) \), or

(3) the actual \( y(n) \) is not within the predicted intervals \( y(n) \neq \hat{y}(n + i | n), i = 1, \ldots, \lambda \)

4. SIMULATION STUDIES

The grasping of an unknown mass from a two-link robot arm, which is modelled as a sudden change in the mass of the second link is considered as a FD-Instant. The two-link robot arm manipulating an unknown load is shown in Fig. 2, with \( m_1, m_2 \) being the masses of the links with \( l_1, l_2 \) the corresponding moments of inertia, and \( l_1, l_2 \), their lengths. The dynamic equations of the two-link robot are:

\[ \begin{bmatrix} T_n \end{bmatrix} = \begin{bmatrix} \Phi_1 \Phi_2 \end{bmatrix} \begin{bmatrix} \theta_1 \theta_2 \end{bmatrix} + \begin{bmatrix} e_1 \neq 1 \neq e_2 \end{bmatrix}, \]

(18)

where

\[ \Phi_1 = \begin{bmatrix} \phi_{11} \phi_{12} \phi_{13} \phi_{14} \phi_{15} \end{bmatrix}, \]

(19)

or, written in a compact form,

\[ \left[ \tau_1 \right]_{1 \times 5} = \left[ \theta^T \right]_{1 \times 5} \left[ \Phi_1 \right]_{4 \times 5} + \left[ e_1 \right]_{1 \times 5} \]

(20)

The elements of the regression vector are
\[ \phi_{11}(n) = l_1^2 \ddot{q}_1(n) + l_1 g \cos q_1(n), \]
\[ \phi_{12}(n) = l_1 \left( \cos q_2(n) \left( 2\ddot{q}_1(n) + \ddot{q}_2(n) \right) - l_1 \sin q_2(n) \right), \]
\[ \phi_{13}(n) = \left( \frac{1}{12} l_2^2 + l_1 l_c l \right) \ddot{q}_1(n) + l_1 g \cos q_1(n), \]
\[ \phi_{14}(n) = \dot{q}_1(n) + \dot{q}_2(n), \]
\[ \phi_{21}(n) = 0, \]
\[ \phi_{22}(n) = l_1 \cos q_2(n) \dot{q}_2(n) - l_1 \left( \sin q_2(n) \dot{q}_1(n)^2 \right) + l_1 g \cos q_1(n) + q_2(n), \]
\[ \phi_{23}(n) = 0, \]
\[ \phi_{24}(n) = \dot{q}_1(n) + \dot{q}_2(n) \]

the elements of the parameter vector are
\[ \theta_1 = m_2, \]
\[ \theta_2 = m_2 l c_2, \]
\[ \theta_3 = m_1, \]
\[ \theta_2 = I_2 + m_2 c_2^2 \]

and finally the noise sequence \(|e_i(n)| \leq e_i^M|.

4.1 Case study assumptions

The simulation studies are based on the above system representation for the first link \([\tau_1]_{1 \times n} = ([\theta^T]_{1 \times n} + [e_1]_{1 \times n})\), which satisfies the first assumption for the application of the fault detection methodology. The reason of using only the system representation of the first link is that \(\phi_{21}, \phi_{23}\) are zeros and the corresponding \(\theta_1, \theta_3\) cannot be identified and their bounds cannot be computed. The nominal values of the parameter vector are determined from the entries of Table 1. The set membership identification is realized taking into account that the noise error bound \(\gamma_n\) is constant and equals to \(1/|\epsilon_1|^2\). In this example, \(\epsilon_1^M = 0.02\) is the maximum noise bound for the first input and \(|\tau_1| = 175.2\) is the magnitude of this input. In Fig. 3, the convergence of the four parameters to the aforementioned nominal values and the variation of the ellipsoid's volume are presented.

![Fig. 3. WRLS estimate of the four parameters (\(\theta_i, i = 1, ..., 4\)) and the variation of ellipsoid's volume](image1)

For simulation purposes, it is assumed that the two-link robot arm suddenly grasps a load of mass, \(m = 9 \times m_2\) at the 2501st time instant. The grasping of the unknown mass results in the increment of \(m_2 = 10 \times m_2\). In addition, the applied torque does not equal to the model-based torque, expected in the absence of the collision. An issue of concern is the OSMI behavior after the mass increment.

4.2 Validation of the 1st Fault Detection Criterion

The sudden variation in the mass of the second link of the robot arm is identified, after the “trigger” of the first fault detection criterion. Computing at every time instant the WRLS estimate of the parameter vector and its bounds, it is proven that before and after the 2501st sample the estimate always resides within the parameter interval, except from this particular sample. The recognition of the fault occurrence and the time that this happens are shown in Figures 4-7. In each figure, the performance of the parameter \(\theta_i, i = 1, ..., 4\), the region of the fault occurrence and the logical fault indicator, which equals to one only if the parameter estimate is out of the computed bounds \((\hat{\theta}_i(n) > \theta_i^\ast(n) \lor \hat{\theta}_i(n) < \theta_i^\ast(n))\) are presented.

![Fig. 4. Upper (red line) and lower (green line) bounds and the WRLS estimate (blue line) of the parameter \(\theta_1\) and the fault indicator](image2)

![Fig. 5. Upper (red line) and lower (green line) bounds and the WRLS estimate (blue line) of the parameter \(\theta_2\) and the fault indicator](image3)
4.3 Validation of the 2nd Fault Detection Criterion

The second criterion which leads to the detection of a faulty behavior of the system is the fact that the volume of the ellipsoid grows at the 2501st time instant, as observed in Fig. 8. This fact objects to the basic presupposition of the deterministic algorithms that characterize the feasible parameter set via an ellipsoid, which is the reduction of the ellipsoid’s volume at every time instant, as shown in Fig. 3. In Fig. 9, the ellipsoid and the orthotope, bounding the ellipsoid at the time instant \( n = 2500 \), are presented. On the other hand, at the time instant \( n = 2501 \), not only the orthotope does not bound the ellipsoid, but also the volume of the ellipsoid has been increased excessively in comparison with the volume of the orthotope, presented in Fig. 10.

4.4 Validation of the 3rd Fault Detection Criterion

The detection of a “fault” in the behavior of the robot manipulator is also verified via the Uncertainty Output Predictor, defined in equation 13. It is already referred that the dynamic effect of the unexpected grasping is the augmentation of joint torques to the commanded ones.

5. CONCLUSION

A Fault Detection scheme is utilized for capturing sudden variations of the parameter vector encountered in the linearizable model of a robot arm. The primary goal is the parameter estimation of the system, when the data are corrupted by unknown but bounded error, consistent with the measurements, the model and the error description (feasible parameter set). A “false” performance is detected.
Fig. 11. The predicted upper (red line) and lower bound (green line) and the actual (input) torque of the first link (blue line) and the fault indicator either when the WRLS estimate resides out of the bounds that are defined via the orthotopic set membership, or a sudden increase in the ellipsoid's volume of the feasible region, or when the predicted values of the robot's output vector are not within a certain interval. Simulation studies are used to investigate the efficiency of the presented algorithm.

REFERENCES


