Robust Fault Diagnosis of Energetic System with Parameter Uncertainties

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Abstract: In this paper, a bond graph model based approach for robust FDI (Fault Detection and Isolation) in presence of parameter uncertainties is presented. Due to the energetic and multi physical properties of the Bond Graph, the whole of nonlinear model, structural analysis, residual with adaptive thresholds generations, and residual sensitivity analysis, can be synthesized using only one tool. This method is applied online for industrial steam generator. Experimental results are given to support the theoretical development.

Keywords: Steam Generator, Bond Graph, Parameter Uncertainties, FDI, Linear Fractional Transformations, Sensitivity Analysis.

1. INTRODUCTION

Recently, robust fault diagnosis has been the subject of several researches, due to the increase of system complexity, and the industrial requirement around the safety and the yield. FDI (Fault Detection and Isolation) procedures consist of comparison between the actual process behavior and the theoretical reference process behavior, represented by its mathematical model. In literature, two fault diagnosis approaches exist: quantitative and qualitative. Among works published these last years on the robust diagnosis using these approaches, one can find: (M. Basseville (1998) [4]), (O. Adrot et al. (1999) [1]), (J. Armengol et al. (2000) [2]), (Z. Han et al. (2002) [8]), (K. Hising-Chia et al. (2004) [10]), (D. Henry et al. (2005) [9]).

Due to the bond graph’s behavioral, structural and causal properties, this tool is more and more used for modelling and fault diagnosis. From FDI point of view, the causal properties of the bond graph tool were initially used for the determination of the faults’ origin. For an electromechanical system application, FDI scheme is proposed in (M. A. Djeziri et al. (2006) [5]), in order to detect the presence of a perturbed backlash phenomenon, in presence of parameter uncertainties. These latter are assumed as normally distributed’ Gaussian signals with zero mean and a known variance. In (M. A. Djeziri et al. (2007) [6]), the normalized gradient method is applied in real time on an electromechanical test bench for parameter and uncertainties identifications.

In this paper, a bond graph methodology is used to synthesis a robust FDI method for nonlinear system in presence of parameter uncertainties, and tested on an industrial steam generator. This FDI method is summarized by the following steps:

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2. LFT AND ROBUST DIAGNOSIS

The main advantages of the bond graph model in LFT form for robust diagnosis, are given as follows:

- Modeling of studied system using bond graph tool with standard LFT form;
- Generation of Analytical Redundancy Relations (ARRs) from the uncertain model by decoupling the nominal and the uncertain parts. Residuals correspond to the ARR nominal part, while their adaptive thresholds represent the ARR uncertain parts;
- Residual’ sensitivity analysis is done by using the ARR uncertain part, in order to calculate the fault detectability indices.

This paper is organized as follows: Section 2 presents briefly the LFT form modelling and its advantages for robust diagnosis. In Section 3, the bond graph LFT modelling of the steam generator in presence of parameter uncertainties is given. Section 4 describes the robust ARRs generation algorithm and the residual sensitivity analysis. The experimental results are shown in Section 5. Finally, conclusion is given in Section 6.

3. UNCERTAIN MODELING OF THE STEAM GENERATOR SYSTEM

The steam generator proposed for this application is a complex and non stationary energetic system. This process
can be decomposed in three principal subsystems: The tank, the pump with a pipe and the boiler.

3.1 Modeling hypothesis

The steam generator is modeled by considering the following hypothesis:

- Water and steam are supposed in thermodynamic equilibrium state, due to their good emulsion homogenization;
- Boiler mixture is considered under uniform pressure;
- Steam generator has a heat capacity and sudden heat losses by conduction towards external environment;
- Fluid in the feeding circuit is incompressible.

3.2 The tank

Determinist and uncertain bond graph models of the tank are respectively given in Fig.1-(a) and (b). The storage of hydraulic and thermal energies is modelled respectively by the bond graph elements \( C : C_h \) and \( C : C_t \). The input mass flow \( S_f : \dot{m}_{in} \) is assumed equal to zero and the tank is initially full filled-in. Then the following equation is deduced from the junction \( 0_h \) of the bond graph determinist model in derivative causality:

\[
\dot{m}_{out} = -C_h \frac{dP_1}{dt} \quad (1)
\]

where \( \dot{m}_{out} \) is the tank output mass flow, \( C_h \) represents the hydraulic capacity of the tank, \( P_1 \) is the fluid pressure measurement inside the tank. Knowing that the studied tank is cylindrical, \( C_h \) can be expressed as follows:

\[
C_h = A_T \cdot (\rho_T \cdot g)^{-1} \quad (2)
\]

where \( A_T \) is the tank section, \( \rho_T \) is the fluid density which is function of the fluid pressure and \( g \) is the gravity acceleration.

![Fig. 1. (a): Tank determinist model. (b): Tank uncertain model.](image)

The uncertainty on \( C_h \) noted \( \delta C_h \) corresponds to the whole of uncertainty on the tank section \( A_T \), and uncertainty on the fluid density \( \rho_T \). \( \delta A_T \) is due to the extra thicknesses of corrosion, and its value with nominal value \( A_{Tn} \) are given by the manufacturer. The fluid density function \( \rho_T \) is calculated using polynomial interpolation method, and the uncertainty \( \delta \rho_T \) is equal to the estimation error given by this polynomial interpolation.

The relation between \( C_h \), and \( \delta C_h \) is given by the following expression:

\[
C_h = C_{hn} + \delta C_h, C_{hn} \quad (3)
\]

where \( C_{hn} \) is the nominal value of \( C_h \).

The modulated input \( w_{C_h} \) in Fig.1 corresponds to an effort variable deduced from \( \delta C_h \) and expressed by the following equation:

\[
w_{C_h} = - \left( \frac{\delta C_{hn}}{\rho_C} + \delta \frac{A_{Tn}}{\rho_C} \right) \frac{A_{Tn}}{\rho_C} \cdot \frac{dP_1}{dt} \quad (4)
\]

\( w_{C_h} \) is taken with a negative sign, because it is considered as a fictive flow input source (Fig.1).

Since the input mass' flow is assumed equal to zero, the energetic assessment calculated from the junction \( 0_t \), on the determinist model of Fig.1 is given as follows:

\[
\dot{H}_5 = -\dot{m}_{out} \cdot c_p \cdot T_2 \quad (5)
\]

where \( \dot{H}_5 \) is the enthalpy flow at the output of the \( C : C_t \) element, \( c_p \) is the fluid specific heat at constant pressure, and \( T_2 \) is the sensor measurement of the fluid temperature inside the tank.

The uncertainty on the enthalpy flow \( \dot{H}_5 \) is due to the uncertainty \( \delta C_t \) on the \( C_t \) parameter. This uncertainty is issued from the variation of the fluid specific heat at constant pressure \( c_p \), which is also in function of the fluid temperature. The nominal value \( c_{pn} \) and its uncertainty \( \delta c_p \) are calculated using the polynomial interpolation algorithm, then the uncertainty on \( C_t \) element is calculated as follows:

\[
w_{C_t} = -\delta c_p, (\dot{m}_{out} \cdot c_{pn}, T_2) \quad (6)
\]

3.3 Pump with pipe

![Fig. 2. (a): Pump and pipe determinist model. (b): Pump and pipe uncertain model.](image)

In bond graph models of Fig. 2, the pump and pipe are represented separately by two resistances. The pump is modeled by a resistance \( R_p \) and modulated by the pump characteristic of equation (7). This latter describes the relation between the pressure \( \Delta P = P_{14} - P_3 \) and the mass flow generated by the pump \( \dot{m}_{14} \). The pump characteristic of equation:

\[
\dot{m}_{14} = b \cdot (k_1 \cdot (P_{14} - P_3) + k_2) \quad (7)
\]
$P_3$ and $P_{14}$ are respectively the input and output pressures of the pump. $k_1$ and $k_2$ are the pump characteristic parameters. $b$ is the boolean control parameter.

The modulated source $MSf : w_{R1p}$ on the bond graph model of Fig. 2-(b) represents the uncertainty on the mass flow at the pump output and is given by the following expression:

$$w_{R1p} = -\delta_k (k_1 \cdot (P_{14} - P_3) + \delta_k \cdot k_2)$$

(8)

The flow parameter $R_{Z1}$ depends on the tubing details and is a function of the valve opening. The nominal value of $R_{Z1}$ and its uncertainty can be calculated using equation (9), where the Poiseuille flow case is considered.

$$R_{Z1} = \frac{8 \rho_l L_p}{\pi r_{1p}}$$

$$\delta R_{Z1} = \delta p_i \cdot (\delta L_p + \delta p_i + \delta r_{1p} + \delta p_i \cdot \delta L_p + \delta p_i \cdot \delta r_{1p})$$

(9)

with $L_p$ is the pipe length and $r_{1p} = r_p^4$, where $r_p$ is the pipe radius.

The mass flow $\dot{m}_{17}$ is calculated using the Bernoulli law:

$$\dot{m}_{17} = \frac{1}{R_{Z1}} \sqrt{P_{14} - P_{17}}$$

Then, the uncertainty on the effort at the output of the pipe is determined as follows:

$$w_{R_{Z1}} = -(\delta R_{Z1} + 2 \cdot \delta R_{Z1}) \cdot (R_{Z1} \cdot \dot{m}_{17})^2$$

(10)

with $R_{Z1}$ and $\delta R_{Z1}$ are respectively the nominal value and multiplicative uncertainty value of the flow parameter $R_{Z1}$. $F_3$ is the mass flow measurement given by the detector $DF : F_3 = \dot{m}_{17}$

The enthalpy flow through the pipe is convected by the fluid as follows:

$$\dot{H}_{13} = T_6 \cdot c_p \cdot \dot{m}_3$$

(11)

The coupling of thermal and hydraulic energies is modelled by a multiport element $R : R_{P1}$. The variables $\dot{m}_3$ and $T_6$ are measured respectively by the $F_3$ and $T_2$ sensors

The uncertainty on the thermal energy transmitted by the pump to the boiler is due to the variation of the specific heat at constant pressure $c_p$ according to the fluid temperature. This last temperature is assumed as a variation between the temperature of the water in the tank and the ambient temperature $T_a$. It is expressed by the following equation:

$$w_{R_{P1}} = -\delta_c \cdot (T_2 \cdot c_p \cdot F_3)$$

(12)

### 3.4 The boiler

Determinist and uncertain bond graph models of the boiler are given in Fig. 3-(a) and (b)

The storage of hydraulic and thermal energies is modelled by the two ports element $C : C_{ht}$. The thermal energy stored by the boiler wall is modeled by a simple one port $C$ element, and the heat transfer from the boiler to the environment is modeled by $R : R_a$ element. The boiler is instrumented with two redundant sensors of temperature($De : T_5$ and $De : T_6$), two redundant volume sensors ($De : L_8$ and $De : L_9$), a pressure sensor ($De : P_7$), a mass flow sensor at the output of the boiler ($DF : F_{10}$), and a sensor of the power provided to the thermal resistor ($DF : Q_4$).

Dissipation of the heat flow $\dot{H}_{28}$ via the boiler wall can be determined using the thermal conductivity $\lambda$, thickness $e_B$, the difference between the these sides’ temperature $T_b - T_a$ and the section $A_B$ of the boiler wall, according to the following relation:

$$\dot{H}_{28} = \frac{\lambda A_B}{e_B} (T_b - T_a)$$

(13)
Where $T_a$ and $T_b$ are respectively the ambient and boiler temperatures. The heat transfer coefficient via the boiler wall is $Ra = \frac{\Delta T}{\sqrt{T_a T_b}}$, where its uncertainty $\delta Ra$ is the combination of $\delta a_H$ and $\delta T_m$.

Then the flow source ($MSf : w_{Ra}$) of Fig. 3-(b) which represents the uncertainty on the heat flow dissipated via the boiler wall is expressed as follows

$$w_{Ra} = -\delta Ra.Ra_n.(T_5 - T_a) \quad (14)$$

The flow at the output of the $C : CH_t$ element represents the interaction of the mass flow $\dot{m}_{Cn}$ and the heat flow $\dot{H}_{Cn}$, in the boiler, given in the system of equations (15).

$$\begin{align*}
\dot{m}_{Cn} &= \frac{d}{dt}(p_l.V_l + p_v.V_v) \\
\dot{H}_{Cn} &= \frac{d}{dt}(p_l.h_l.V_l + p_v.h_v.V_v - P_B.V_B)
\end{align*} \quad (15)$$

where $p_l, h_l, V_l$ and $p_v, h_v, V_v$ are respectively the density, the specific enthalpy and the volume of water and steam inside the boiler. $P_B$ is the boiler pressure given by the pressure measurement $P_B$. $V_B$ is the known volume of the boiler.

All variables $p_l, h_l, p_v$ and $h_v$ are function of the pressure $P_l$ and calculated using a polynomial interpolation algorithm. $\delta p_l, \delta h_l, \delta h_v$ and $\delta p_v$ represent the estimation errors of the polynomial interpolation algorithm.

Taken into account the uncertainties on the variables $p_l, h_l, V_l, p_v, h_v, V_v$, the uncertainties on the $C : CH_t$ element are given as follows

$$\begin{align*}
\delta_{1CH_t} &= \delta p_l.\delta V_l + \delta p_l.\delta V_l + \delta p_v.\delta V_v + \delta p_v.\delta V_v + \delta h_l + \delta h_v \\
\delta_{2CH_t} &= \delta_{1CH_t} + \delta h_l.\delta V_l + \delta h_l.\delta V_l + \delta h_v.\delta V_v + \delta h_v.\delta V_v + \delta h_l + \delta h_v
\end{align*} \quad (16, 17)$$

where $\delta_{1CH_t}$ represents the hydraulic uncertainty on $CH_t$, and $\delta_{2CH_t}$ represents the thermal uncertainty on $CH_t$.

Then, the flow sources $MSf : w_{1Cn}$ and $MSf : w_{2Cn}$ are expressed by the following expressions:

$$\begin{align*}
w_{1Cn} &= -\delta_{1CH_t}.\frac{d}{dt}(p_n.V_n + p_v.V_v) \\
w_{2Cn} &= -\delta_{2CH_t}.\frac{d}{dt}(p_n.h_n.V_n + p_v.h_v.V_v)
\end{align*} \quad (19)$$

The bond graph element $RS$ of Figs. 3-(a) and (b) represents the thermo-resistance, its nominal value $RS_n$ and its uncertainty $\delta RS$ are calculated using the electrical power given by the sensor $Q_4$. Then the flow source $MSf : w_{RS}$ which represents the uncertainty on the heat flow provided to the boiler is given in equation (20).

$$w_{RS} = -\delta_{RS}.(RS_n.Q_4) \quad (20)$$

According to model of Fig. 3-(a), the heat flow driven by the steam at the boiler output is given in equation (21).

$$\dot{H}_{43} = T_6.c_v.F_{10} \quad (21)$$

with $c_v$ is the steam heat capacity at constant volume.

The uncertainty on the heat flow at the boiler output is due to the uncertainty $\delta c_v$, then the flow source $MSf : w_{RT2}$ is modulated as follows:

$$w_{RT2} = -\delta c_v.(T_6.c_v.F_{10})$$

4. ROBUST ARRS GENERATION

The generation of the robust ARRs from a proper and observable bond graph model is summarized in the following steps:

**1st step:** The bond graph model is made in derivative causality with LFT form;

**2nd step:** The unknown variables are eliminated by covering the causal paths from the bond graph elements to the detectors;

**3rd step:** The ARRs are generated by expressing energetic assessments on the junctions 1 and 0, where all the unknown connected variables are determined from the 2nd step.

The energetic assessments is expressed by two fundamental laws for each junction:

**Junction 1:**

$$f_1 = f_2 = f_3 = \ldots$$

$$ARR : \sum e_{i_n} + \sum w_i = 0 \quad (22)$$

for 0 junction, $e_{i_n}$ is replaced by $f_{i_n}$

$$i_n \in \{R_n, I_n, C_n, TF_n, GY_n, RS_n, MSf_n, MSf_{n}\}$$

**4th step:** The obtained ARRs at the 3rd step are composed of two perfectly separate parts, a nominal part called $r$ which describes the residual, and an uncertain part called $a$, represents the sum of fictive input values $\sum w_i$. This uncertain part is used to calculate the normal operating thresholds.

The residual $r$ and the uncertain part $a$ are expressed as follows:

**Junction 1:**

$$r = \sum e_{i_n} \quad (23)$$

$$a = \sum |w_i| \quad (24)$$

for 0 junction, $e_{i_n}$ is replaced by $f_{i_n}$

Uncertain $ARR$ part cannot be quantified perfectly, it is evaluated to generate a normal operation’ threshold which satisfies the following inequality:

$$-a \leq r \leq a \quad (25)$$

The first $ARR$ is generated from junction 1, connected to detector $F_3$. This $ARR$ is sensitive to the stopper at the level of the pump output (pipe radius variation). Knowing that the studied system is extremely perturbed, the expression of $ARR_1$ given in equation (26) takes into account the Bernoulli’s law.

$$\begin{align*}
\begin{cases}
\dot{r}_1 &= -(R_{z_1}.F_3)^2 - \frac{A_T}{b.k_1.\rho.T.G} \frac{dP_1}{dt} \\
\dot{a}_1 &= \frac{w_{Cz}}{b.k_1} + \frac{w_{RP}}{b.k_1} + |w_{Rz_1}|
\end{cases}
\end{align*} \quad (26)$$
The second ARR is generated from the junction 0h of the boiler model, for the detection of the mass leak:

\[ ARR_2 : \begin{cases} r_2 = \frac{d}{dt} \left( \rho_l V_i + \rho_v V_v \right) - F_{10} \\ a_2 = |w_{1C_{hi}}| \end{cases} \quad (27) \]

Finally, the third ARR is generated from the junction 0i of the boiler model, in order to detect the thermo-resistance fault:

\[ ARR_3 : \begin{cases} r_3 = F_{3} + c_{p_a} T_2 + R S_n Q_4 \\ \frac{-d}{dt} (\rho_l V_i + \rho_v V_v) - \frac{\rho_i V_i}{T_m} (T_3 - T_a) - F_{10} + c_{p_a} T_5 \\ a_3 = \left[ w_{2C_{hi}} \right] + \left| w_{R_S} \right| + \left| w_{R_a} \right| + \left| w_{RT_1} \right| + \left| w_{RT_2} \right| \end{cases} \quad (28) \]

### 4.1 Residuals sensitivity analysis

The residuals sensitivity analysis is made using the normalized partial derivative of the residual uncertain part compared to the parameter uncertainty, as shown in equation (29) (M. A. Djeziri et al. (2007) [6]).

\[ SI_k = \left[ \frac{\delta_j a}{a} \right] = \left[ \frac{|w_i|}{a} \right] \quad (29) \]

where \( \delta_j \) is the multiplicative uncertainty on the parameter \( i \), \( a \) is the ARR uncertain part. \( SI_k \) is the sensitivity index according to \( \delta_j, w_i \) is the fictive input according to the uncertainty on the parameter \( i \).

A fault detectability indexes \( DI \) expressed in the equation (30) is defined as the ability of the residual to detect a physical fault. Two types of faults are considered: parameter fault (noted \( Y_i \)), and structural fault (noted \( Y_s \))

- ARR generated from junction 1

\[ DI = Y_i \left( Y_i \right) + Y_s - \sum w_i \]

\[ \begin{cases} Y_s > \sum |w_i| & \text{Fault is detectable} \\ Y_s \leq 0 & \text{Fault is not detectable} \end{cases} \quad (30) \]

or 0 junction, \( e_{i_n} \) is replaced by \( f_{i_n} \)

with \( DI \) the fault detectability index. \( Y_i \) is the fault detectable rate according to the parameter \( i \). \( Y_s \) is the structural fault detectable value. \( \sum w_i \) represents the sum of fictive inputs. \( e_{i_n} \) and \( f_{i_n} \) are respectively the nominal values of effort and flow given by the nominal parameter \( i \).

Fault rate \( (Y_i) \) corresponds to an abnormal deviation of the parameter from its nominal value, which causes a failure of the system.

Structural fault \( (Y_s) \) causes a modification in the system structure, and consequently in its model. It creates an imbalance in the energetic assessments calculated in normal operation.

From equations (30) with considering \( Y_s = 0 \), the fault detectable rate \( Y_i \) on parameter \( i \) can be defined by the following equations (31)

- ARR generated from junction 1

\[ Y_i > \sum \left| \frac{|w_i|}{e_{i_n}} \right| \quad (31) \]

for 0 junction, \( e_{i_n} \) is replaced by \( f_{i_n} \)

From equations (30), and by considering \( Y_i = 0 \), the structural fault detectable value can be defined by equation (32)

\[ Y_s > \sum |w_i| \quad (32) \]

### 5. EXPERIMENTAL RESULTS

Experimental senary consists on the generation of the residuals and the normal operation thresholds using the acquired data from the real system. Four situations are considered:

1. Residuals generation without fault using the real sensors data; 2. Introduction of the first fault, by considering a stopper at the level of the pump output; 3. Introduction of the second fault as a fluid leak at the boiler level; 4. Introduction of the thermo-resistance breakdown.

![Fig. 4. The residuals \( r_1, r_2 \) and \( r_3 \) without faults.](image)

![Fig. 5. (a): \( Y_{R_z} \), (b): \( R_z \) variation. (c): Residual \( r_1 \)](image)
Fig. 6. (a): The fluid leak detectable value. (b): The introduced fault variation. (c): Residual $r_2$

Fig. 7. (a): $Y_{RS}$. (b): $RS$ variation. (c): Residual $r_3$

Figures 4-(a), (b) and (c) show respectively the residuals $r_1$, $r_2$ and $r_3$ without faults. The thresholds of normal operation are given with dot lines.

Fig. 5-(a) represents the fault detectable rate of $R_z$ noted $Y_{R_z}$. Fig. 5-(b) shows the $R_z$ nominal value, where a fault is introduced gradually between times $t = 4s$ and $t = 16s$. Fig. 5-(c) shows the reaction of the residual $r_1$ to this fault. The fault is detected when the rate of $R_z$ deviation exceeds the detectable rate $Y_{R_z}$.

Fig. 6-(a) represents the fluid leak detectable value. Fig. 6-(b) shows the introduced fault, where the fluid leak is introduced gradually between times $t = 4s$ and $t = 16s$. Fig. 6-(c) shows the reaction of the residual $r_2$. The fault is detected when its energy being higher than that introduced by the whole uncertainties.

Fig. 7-(a) represents the fault detectable rate of $RS$ noted $Y_{RS}$. Fig. 7-(b) shows the $RS$ nominal value where a fault is introduced gradually between times $t = 4s$ and $t = 16s$. Fig. 7-(c) shows the reaction of the residual $r_3$ to this fault. The fault is detected when the rate of $RS$ deviation exceeds the detectable rate $Y_{RS}$.

6. CONCLUSION

Modeling and robust FDI of a steam generator are presented in this paper. Interactions of different phenomenon are taken into account by using the energetic properties of the bond graph tool. The nominal part of ARRs is perfectly separated from uncertain one. This latter is used to generate the adaptive thresholds of normal operation, then the parameter fault detectable rate and the structural fault detectable value are calculated. The use of bond graph as an integrated design tool for modeling and robust monitoring of energetic systems is well justified by the obtained results performances on the steam generator system.

REFERENCES