Linear Quadratic Energy-Motion Regulators of Electric Motorcycles

Boe-Shong Hong* Chun-Chia Huang**
Ching-Huang Chiu***

*Department of Mechanical Engineering, National C.C. University, Chia-Yi 62102, Taiwan, R.O.C. (email: imehbs@ccu.edu.tw)
**Department of Mechanical Engineering, National C.C. University, Chia-Yi 62102, Taiwan, R.O.C.
***Department of Vehicle Engineering, National Formosa University, Yun-Lin 632, Taiwan, R.O.C.

Abstract: With the application of $H_\infty$-control theory to dynamics of electric motorcycles, we obtain a feedback regulator served for keeping the balance between energy consumption and motion responses. This regulator parameterises an ad hoc vehicular personality: the trade-off between economy in energy and quickness in motion. Computer simulations and road tests are provided to verify the practicability of such a parameterisation. In performing experiments, we develop the technology of making industrial controllers, inclusively of robust DSP programs for implementing feedback dynamics into PIC microcontrollers.

Keywords: Motor drives, differential game, energetic engineering, $H_\infty$ control.

1. INTRODUCTION

For energy efficiency in (hybrid) electric vehicles, there are four fields of engineering currently under investigation: (a) Management of active power sources; (b) Assistance with passive energy storage; (c) Restructuring of power converters; and (d) Implementation of control rules for economy in energy.

Most of power sources available in the industry of electric motorcycles (EMs) suffer high ratio of energy either to cost or to weight. For examples, lithium-ion battery (Kennedy et al., 2000), fuel cell (Rajesh et al., 2004, Andreas et al., 2005, Corbo et al., 2006, Chau et al., 2002), high-capacity ultracapacitor, and ultrahigh-speed flywheel are too costly, and lead-acid battery is too heavy. Therefore, it becomes important for EM industry to manage batteries to reduce consumption of electric energy. For examples, Meissnerm and Richter (2003), Gregory (2004), Rand (1997) and Saakes et. al. (2005) on-line measured the state-of-charge and/or state-of-health of considered batteries, and then regulated current outputs for conserving energy and increasing battery’s lifespan.

Besides, it is also necessary to increase storage of mechanical energy. For examples, Eckhard et. al. (2007) and Eugenio et. al. (1999) installed ultracapacitors between batteries and power converters to increase the energy transformation in braking phases from mechanical kinetics to electricity. Since capacitors are passive elements, they can efficiently absorb the impulsive currents due to boost choppers during the braking stages, while highly switching currents input to lead-acid batteries merely become heat being dissipated instead of electricity being stored. Moreover, ultracapacitors deliver initial overshoots of currents to start engines, and thus are indispensable to fuel cell -powered hybrid electric vehicles (Vahidi et al., 2006, Timmermans et al., 2005). However, most EMs are lightweight so that the amount of energy conserved in braking phases is quite limited, and, for lead-acid batteries -equipped EMs, the initial torques are large enough to start engines.

Restructuring power converters can lead to vital reduction of electricity-to-heat, which has been well investigated in the field of power electronics. For examples, Martinez and Ray (1994) designed bi-directional DC/DC power converters to “soften” switching of currents, which was later improved with four power MOSFETs (Caricchi et al., 1998); Hofsajer et. al. (1995) used RCD snubber to reduce energy consumption from high-frequency on-offs of power transistors; and He and Nelms (2003) embedded microprocessors into power converters for optimal timing of switching to suppress electricity loss. In the fields of (a), (b) and (c), additional components are needed, as has been discussed, for the fulfilment of saving energy.

On the other hand, additional hardware in the field of engineering (d) is unnecessary for improving economy in energy. Specifically, we can program a regulator of energy consumption into the microcontroller of the EM controller, an already existent component, to boost energetic economy. In literature, fuzzy logic (Niels et al., 2003), genetic algorithm (Morteza et al., 2006), rule-based strategy (Jalil et al., 1997), instantaneous optimisation strategy (Guezenne et al., 2003) and global optimisation strategy (Lin et al., 2003) are found as methods to make energy-saving programs. In this paper, we turn to consider the interaction between energy utilization and motion responses of EM dynamics, since, as we have experienced, an irrational emphasis of energetic economy will dull the motions, and vice versa. It is hoped that manufacturers can prepare a set of feedback-dynamics implemented in the microcontroller of the EM controller for a...
diversity of traffic conditions. For example, the EM can switch to the best mode of conserving energy for a less jammed traffic.

Roughly speaking, we first examine based on a reduced version of mathematical model the particular personality of EM dynamics: the trade-off between economy-in-energy (EE) and quickness-in-motion (QM). The reduced model is then served for $H_\infty$-control synthesis that treats the EE and QM as two players entering into negotiations with a negotiation parameter being assigned for quantification of the EE-QM trade-off of the controlled dynamics. The applicability of this quantification will pass examinations of computer simulations and road tests in this paper. In advance of road tests, we design and manufacture an industrial controller for the experiments that installs the EE-QM regulation.

Including this introduction section, achievements of the work are presented with five sections. Section 2 performs modelling parameterisation for EM dynamics with specific simplifications, and based on which develops the EE-QM regulator through constructing a generalized plant. Section 3 presents the implementation of feedback dynamics produced by the EE-QM regulator into PIC16X microcontroller. Also presented is the procedure to improve controller designs for EM industry and the final discusses findings of computer simulations and road tests. In the section 4, we conclude the work.

2. SYNTHESIS OF EE-QM REGULATION

The electric motorcycles (EMs) under consideration appear as the one in Figure 1. Throughout this section, the EMs dynamics served for control analyses and syntheses takes the following simplifications:

(A1) There is no slippage between the belt and gears;

(A2) The inertia of moment of the DC-motor armature is negligible;

(A3) Counter Electromagnetic Force coefficient of the DC-motor equals the torque coefficient;

(A4) The torque coefficient, equivalent inductance and equivalent resistance of the DC motor are invariant to changes of the load.

(A5) The tires of EM are purely rolling on the road;

(A6) The mass centres of tires are coincident with the centres of bearing;

(A7) The wind resistance is negligible; and

(A8) The motorcycle is moving along a straight line of no inclination.

2.1 Mathematical modelling

With the Assumption (A5), the minimum realization of the EM dynamics is of two state variables, designated as the motor current $i$ and the tire speed $\omega$. The equivalent inertia of moment of the EM plus its rider, $J$, can be calculated by the following equation:

$$\frac{1}{2} J \omega^2 = \frac{1}{2} (M_1 + M_2) i^2 \omega^2 + 2(\frac{1}{2} I \omega^2),$$

which indicates that the equivalent kinetic of rotation is the sum of the kinetics of translation of the EM plus the rider and the kinetics of rotation of two tires. Explicitly,

$$J = (M_1 + M_2) i^2 + 2I.$$

Figure 1 shows the equivalent circuit of the EM dynamics, where besides the equivalent inertia of moment $J$, other parameters such as equivalent inductance $L$, resistance $R$ and damping $B$ are identified in advance of control/observer syntheses. Chou et. al. (2006) devised an inverse dynamics equipment to identify those parameters.

Kirchhoff theorems upon the equivalent circuit in Figure 1 realizes the state space of the EM dynamics by

$$\frac{d}{dt} i = -\rho \omega - R i + u,$$

$$\frac{d}{dt} \omega = \rho i - B \omega. \quad (1)$$

The time $t$, state $x$ and input $u$ in (1) are further made dimensionless by

$$t \rightarrow \frac{t}{\tau_m}, \quad \omega \rightarrow \frac{\omega}{\dot{\omega}}, \quad i \rightarrow \frac{i}{i}, \quad u \rightarrow \frac{u}{u}, \quad (2)$$

where $\tau_m$ stands for the time constant of mechanical parts $J / B$; $\bar{u}$ for the maximum of average voltage across the motor; and $\bar{\omega}$ and $\bar{i}$ for the steady-state tire speed and motor current, respectively, as the average voltage delivered by the power converter is kept at $\bar{u}$. The motor voltage $\bar{u}$ is identical to the voltage across the battery if the power converter of the controller adopts buck choppers.

Fig. 1. Equivalent representation for dynamics of electric motorcycles

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With the dimensionless setting of (2), the steady state of (1) signifies that \( \rho k \dot{\tau} = B \ddot{\dot{\tau}} \) and \( \rho k \dot{\omega} + R \dot{\omega} = \pi \). Therefore, the non-dimensional state-space realization of the EM dynamics can be rewritten as

\[
\begin{align*}
\frac{di}{dt} &= -\beta \dot{i} - \varphi \varphi + (\beta + \varphi)u \\
\frac{d\omega}{dt} &= i - \omega,
\end{align*}
\]

where \( \beta \) indicates the mechatronic ratio of AC characteristics, \( \beta = \tau_e / \tau_c \), where \( \tau_e \) is the time constant of electric parts, \( \tau_c = L / R \), and \( \varphi \) indicates the mechatronic ratio of power ratings, \( \varphi = J\omega^2 / L \). These two dimensionless parameters define the mechatronic property of the considered EM, which should be determined first during the EM design process. It is inferred from (3) that the non-dimensional motor current \( i \) and tire speed \( \omega \) are equal in steady state, valued within \([0,1] \).

2.2 Control synthesis

We employ the following symbols to construct a generalized plant served for the synthesis of \( H_\infty \)-negotiation based EE-QM regulator. There are four state variables:

1) \( x_1 \): the motor current \( i \) related to the rate of energy dissipated from the resistor;

2) \( x_2 \): the tire speed \( \omega \) related to the rate of energy dissipated from the damper;

3) \( x_3 \): the integration of tracking error of motor current \( e_1 = r - i \), which indicates the quickness of acceleration under servocontrol; and

4) \( x_4 \): the integration of tracking error of tire speed \( e_2 = r - \omega \), which indicates the quickness of speed under servocontrol, where \( r \in [0,1] \) is the referenced commands of motor current and also tire speed.

The former two state variables, \( x_1 \) and \( x_2 \), quantify the current rate of energy consumption, and the latter two state variables, \( x_3 \) and \( x_4 \), memorize the past responses of motions.

Also defined are two performance variables and two corresponding performance indexes:

5) \( z_1 = [x_3 \ x_4 \ u]^T \), the performance variable pointed to QM;

6) \( z_2 = [x_1 \ x_2 \ u]^T \), the performance variable pointed to EE;

7) \( \gamma_1 \): the performance index of QM, being the bound of the \( L_2 \)-gain from the reference \( r \) to the QM variable \( z_1 \), that is,

\[
\int_0^T \|y_1(t)\|^2 dt \leq \gamma_1^2 \int_0^T \|r(t)\|^2 dt \ , \ r(t) \in [0,1] \ \forall t \in [0,T] \ , \ \forall T > 0 
\]

8) \( \gamma_2 \): the performance index of EE, being the bound of the \( H_\infty \)-norm from the reference \( r \) to the EE variable \( z_2 \), that is,

\[
\int_0^T \|y_2(t)\|^2 dt \leq \gamma_2^2 \int_0^T \|r(t)\|^2 dt \ , \ r(t) \in [0,1] \ \forall t \in [0,T] \ , \ \forall T > 0
\]

A smaller QM index \( \gamma_1 \) implies that the time for reaching referenced speed or acceleration is shorter, that is, the motion of EM is quicker under servocontrol. A smaller EE index \( \gamma_2 \) means that heat dissipated from mechanical and electric resistances are less before the motions arrive at steady state. It is to be noted that the QM variable \( z_1 \) and the EE variable \( z_2 \) include the motor voltage \( u \) to protect it from becoming unreasonably large for regulating the EE and QM.

With the introduction of new symbols above, the EM dynamics of (3) can be extended to the following generalised plant that incorporates the EE and QM performances:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix} =
\begin{bmatrix}
\beta + \varphi & 0 & 0 & 0 \\
1 & -\varphi & 0 & 0 \\
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & \beta - \varphi
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
+ \begin{bmatrix}
r + \varphi \\
r + \varphi \\
r + \varphi \\
r + \varphi
\end{bmatrix} \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix} u
\]

(6)

\[
z_1 = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix} u
\]

(7)

\[
z_2 = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix} u
\]

(8)

where we choose small values \( \varepsilon_1 \) and \( \varepsilon_2 \) to replace zeros so that \( H_\infty \)-control syntheses escape from numerical singularity.

Let us briefly write (6)-(8) as

\[
\begin{align*}
\dot{x} &= Ax + B_1 r + B_2 u \\
z_1 &= C_1 x + D_{11} r + D_{12} u \\
z_2 &= C_2 x + D_{21} r + D_{22} u
\end{align*}
\]

(9)

and employ the state-feedback

\[
u = -Kx,
\]

(10)
with the feedback gain $K$ being to-be-determined. Figure 2
shows the configuration of the proposed feedback system,
where a current sensor is installed on the board of power
converter with its output being sent to a Luenberger observer
for reconstructing the tire speed and filtering noises out of
the measured signals of motor current. The Luenberger observer
is synthesised based on the EM dynamics in (3). Since the
time constant of electric parts $\tau_e$ is much smaller in practice
than that of mechanical parts $\tau_m$, the observed tire speed is
taken as a really measured one.

Substituting (10) into (9) yields the dynamics of feedback
system:

$$
\begin{align*}
\dot{x} &= (A - B_2K)x + B_1r \\
z_1 &= (C_1 - D_{12}K)x + D_{11}r \\
z_2 &= (C_2 - D_{22}K)x + D_{21}r
\end{align*}
$$

Based on the Bounded Real Lemma (Boyd et al., 1994), (4)
and (5) hold if there exists a positive-definite, symmetric
matrix simultaneously satisfying the following two matrix
inequalities:

$$
\begin{align*}
&\begin{bmatrix}
X(A-B_2K)^T + (A-B_2K)X & X(C_1-D_{12}K)^T B_1 \\
(C_1-D_{12}K)X & -\gamma_1 I + D_{11}
\end{bmatrix} < 0 \quad (12) \\
&\begin{bmatrix}
X(A-B_2K)^T + (A-B_2K)X & X(C_2-D_{22}K)^T B_1 \\
(C_2-D_{22}K)X & -\gamma_2 I + D_{21}
\end{bmatrix} < 0 \quad (13)
\end{align*}
$$

The introduction of a new matrix $Y$ by $Y = KX$ transforms
(12) and (13) into two linear matrix inequalities (LMIs):

$$
\begin{align*}
&\begin{bmatrix}
AX + (AX)^T & (C_1X)^T B_1 \\
C_1X & 0
\end{bmatrix} < \begin{bmatrix}
B_2 Y + (B_2Y)^T & (D_{12}Y)^T \\
(D_{12}Y) & -\gamma_1 I
\end{bmatrix} \quad (14) \\
&\begin{bmatrix}
AX + (AX)^T & (C_2X)^T B_1 \\
C_2X & 0
\end{bmatrix} < \begin{bmatrix}
B_2 Y + (B_2Y)^T & (D_{22}Y)^T \\
(D_{22}Y) & -\gamma_2 I
\end{bmatrix} \quad (15)
\end{align*}
$$

Any quadruple $(X, Y, \gamma_1, \gamma_2)$ satisfying (14)-(15) defines a
feasible feedback $K = YX^{-1}$ guaranteeing the performances
of feedback dynamics to have QM index $\gamma_1$ and EE index
$\gamma_2$. It is easy to check that the set of feasible pairs of indexes
$(\gamma_1, \gamma_2)$ is convex.

Based on the convexity mentioned above, we provides a
numerical procedure in the following to represent the set of
feasible performances $(\gamma_1, \gamma_2)$ in a 2D plane coordinated by
$\gamma_1$ and $\gamma_2$:

[Step 1] Assign a negotiation parameter $\alpha$ and then set
$\gamma_2 = \alpha \gamma_1$ in (14)-(15). Solve the optimal solution
$(X, Y, \gamma_1, \gamma_2)$ with the feedback gain $K$ being to-be-determined.

Figure 2 shows the configuration of the proposed feedback system,
... verified in Section 3. 

Fig. 2. $H_\infty$-negotiation via generalised plant

Fig. 3. Trade-off between EE and QM
3. RESULTS OF COMPUTER SIMULATION AND ROAD TESTS

We purchase an electric motorcycle for experiment, keeping its DC motor and lead-acid battery \( \mu = 24 \text{ V}; \; \bar{i} = 20 \text{ A}; \; \bar{\omega} = 3200 \text{ rpm} \) and taking away its controller. An improved controller is designed and manufactured, which installs an ACS754SCB-200 Hall-effect current sensor/transducer (single bias; 0-100A input; 0-5V output). A toy DC-motor is pivoted on the axial of the EM motor for sensing tire speeds, and a Laptop Dspace is carried on the running EM for the real-timed signal acquisition of motor currents and tire speeds. The signal conditioning inside the PIC16F87X chip is as follows: (1) the command signal 0V-5V corresponds to the motor current 0A-20A; (2) the output of current sensor 0V-5V corresponds to the motor current 0A-100A; (3) the frequency of the PWM is 20kHz with its duty cycle 0%-100% being proportional to motor voltage 0V-24V; and (4) the sampling time in the MIT program is \( T_s = 0.05 \text{ s} \).

Firstly, we choose two types of feedback dynamics to check the ability of the developed EE-QM regulator to quantify the trade-off between QM and EE. One is an economic type with the negotiation parameter being \( \alpha = 1.1 \) and corresponding EE-QM indexes \( \gamma_1, \gamma_2 \) being \( 2.80, 3.08 \); and the other is a quick type with the negotiation parameter being \( \alpha = 2.0 \) and corresponding indexes \( \gamma_1, \gamma_2 \) being \( 1.71, 3.42 \). Figure 4 records the responses of road tests of these two types of controllers. We find that Allegro ACS754SCB-200 Hall-effect current sensor is of wide bandwidth and, as of such, lures high-frequency noises into the loop. Though noises contaminate measurement, we can discern the coincidence between measured responses and simulated responses, which validates the accuracy of computer simulations upon the reduced EM dynamics with identified parameters for studying the personality of the considered EM.

![Fig. 4. Measured responses of economic type and quick type](image)

It is computationally convenient to change the independent variable of the EM dynamics in (3) from the time \( t \) to the angular displacement \( \theta \) \( (d\theta/dt = \omega) \):

\[
\frac{d\theta}{dt} = -\beta \frac{i}{\omega} - \phi + (\beta + \phi) \frac{u}{\omega}.
\]

We simulate based on (16) the responses of energy consumption, motor current and tire speed versus the angular displacement \( \theta \) of the EM driven in a jammed traffic. Figure 5 shows situations in a jammed traffic. Their responses of energy consumption indicate that the economic type \( (\alpha = 1.1) \) saves about 25% energy with respect to the quick type \( (\alpha = 2.0) \). Energy is saved because of reduced electric dissipation resulted from the depression of current overshoots during the acceleration and deceleration. This also protects the lead-acid battery from overcharging and the DC motor from overheating, and thus saves energy. That is out of the scope of this paper.

![Fig. 5. Motion responses and energy consumption in a jammed traffic](image)

![Fig. 6. Energy saved versus negotiation parameter](image)

In Figure 6, we plot the relationship between the amount of energy being saved and the negotiation parameter chosen in the EE-QM regulator. It is found that the percentage of energy saved with respect to the quick type \( (\alpha = 2.0) \), \( \eta \), is almost proportional to the reciprocal of negotiation parameter \( 1/\alpha \). This linear relationship verifies the practicability of taking negotiation parameters to parameterise the trade-off between EE and QM in EM dynamics.
The findings from computer simulations correspond to our experiences: commands of rapid acceleration and deceleration tend to shorten driving ranges of EMs, however, it is usually necessary for rapid accelerations and decelerations to safely drive an EM in a crowded condition. Therefore, every condition of traffic has its own negotiation parameter when the manufacturer involves our $H_{\infty}$-negotiation-based EE-QM regulator in designing EM controllers.

4. CONCLUSIONS
This work contributes an example to the academic realisation of our common experience that quicker motions lead to larger consumption of energy. To the industry of electric motorcycles, this work provides a motion-energy regulator to achieve the trade-off between economy in energy and quickness in motion.

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