Heart Rate Regulation During Exercise with Various Loads: Identification and Nonlinear $H_\infty$ Control

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Abstract: A model for the heart rate response to treadmill walking exercise is proposed in this paper. The parameters of the model were experimentally identified which involved subjects walking at different speeds. A 2-degree-of-freedom controller was then developed for the regulation of the heart rate response during treadmill exercise. The controller consists of a piecewise LQ and an $H_\infty$ sub-controllers. Experimental results demonstrated that the heart rate of the subjects were regulated by the proposed controller.

Keywords: Biomedical systems; Non-linear systems modelling; Robust control design; Application; Piecewise affine system; Experimental study

1. INTRODUCTION

In order to meet the metabolic demand during exercise, the heart rate (HR) of an exerciser increases. Thus, knowing how HR responds to exercise will improve our understanding of exercise physiology. In addition, it may also be useful for predicting cardiovascular disease mortality (Savonen et al. [2006], Cole et al. [1999]). The understanding of HR response may also lead to an improvement in developing training protocols for athletics and more efficient weight loss protocols for the obese, and in facilitating assessment of physical fitness and health of individuals (Achten and Jeukendrup [2003]). Furthermore, knowing the cardiovascular system responses to the stress induced by physical exercise provides us another perspective on how this system functions. For instance, this may give us some measures for the prevention of cardiac failure from dialysis.

HR response during exercise have been widely studied, e.g. Brodan et al. [1971], Hajek et al. [1980], Rowell [1993], Coyle and Alonso [2001], Su et al. [2007]), among them a number of models have been proposed. Broden et al. Brodan et al. [1971] and Hajek et al. Hajek et al. [1980] modelled the HR response from a regulation point of view. Their models are reliable for short duration exercises, but are not sufficient for explaining long duration exercises. As shown in, e.g. Coyle and Alonso [2001], HR will continue to increase during prolonged exercise. In reference (Su et al. [2007]), exercising HR response was modelled by a Hammerstein system1. Besides modelling, they also studied the control of the HR response during exercise.

The ability to control the HR during exercise is of importance in the design of exercise protocols for patients with cardiovascular diseases and in developing rehabilitation exercises to aid patients recovering from cardiothoracic surgery. The control of heart rate response during exercise has been reported in the references (Kawada et al. [1999], Cooper et al. [1998], Su et al. [2007]). Among them, a number of different control strategies or algorithms have been successfully applied, e.g. classical PID control, $H_\infty$ control, and model reference control. Each has its merits or disadvantages and therefore, it is interesting to investigate the usefulness of other control algorithms and techniques that have been developed by the control society.

The objective of this paper is twofold. First, a nonlinear model is proposed to describe the HR response to treadmill walking exercise during both the exercising and the recovery phases. Secondly, using the proposed model, we develop a controller-using the treadmill’s speed as a control variable-that regulates the HR during exercise.

2. THE MODEL

In this paper, we propose the following nonlinear state-space control systems to model the HR response to treadmill walking exercise:

$$\dot{x}(t) = -a_1 x(t) + a_2 x(t) + a_2 u^2(t)$$
$$\dot{x}(t) = -a_3 x(t) + \phi(x(t))$$
$$z(t) = x(t)$$
$$\phi(x(t)) := \frac{a_4 x(t)}{1 + \exp\left(-\left(x(t) - a_5\right)\right)}$$

where $x(0) = [x_1(0) \ x_2(0)]^T = 0$, $z(t)$ describes the change in HR from rest, and $a_1, \ldots, a_5$ are positive scalars. The control input $u(t)$ represents the normalised speed of...
the treadmill and it is normalised by 8 km/h, assuming the maximum walking speed is 8 km/h. The unit of time $t$ is in minutes.

System (1) can be viewed as a feedback interconnected system, i.e. $x_1$ in the forward path and $x_2$ in the feedback path. The component $x_1(t)$ can be viewed as the change of HR due to the neural response to exercise, including both the parasympathetic and the sympathetic neural inputs (see e.g. Rowell [1993]). The component $x_2$ is utilised in describing the complex slow-acting peripheral effects from, e.g. the hormonal systems, the peripheral local metabolism, and/or the increase in body temperature, etc. Generally, these effects cause vasodilatation and hence HR needs to be increased in order to maintain the arterial pressure (see McArdle et al. [2007]). So, the feedback signal $x_2$, which can be thought of as a dynamic disturbance input to the $x_1$ subsystem, is a reaction to the peripheral local effects. By observing system (1), the input $u$ drives the system nonlinearly, describing the nonlinear increase of the HR in response to the increase in walking speed. It has been observed that there is a curvilinear relationship between aerobic demand and walking speed (see, e.g. McArdle et al. [2007]). The quadratic increase in HR in response to an increase in walking speed was observed in Johnson [2007].

2.1 Experimental Setup

The parameters in system (1) were identified from experimental data. The setup of the experiment is described in this section.

Subject: Six healthy male subjects were studied. The physical characteristics of the subjects are given in Table 1.

<table>
<thead>
<tr>
<th>mean</th>
<th>Height (cm)</th>
<th>Weight (kg)</th>
<th>BMI (kg/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>174</td>
<td>68.5</td>
<td>22.5</td>
<td></td>
</tr>
<tr>
<td>std</td>
<td>3.4</td>
<td>12.6</td>
<td>3.4</td>
</tr>
<tr>
<td>range</td>
<td>169–178</td>
<td>53–85</td>
<td>18–27</td>
</tr>
</tbody>
</table>

Table 1. Physical characteristics of the subjects: age, height, weight, and BMI (Body Mass Index)

System (1) is first written in a state-space form as follows:

\[
\begin{align*}
\dot{x}(t) &= f(x(t), a, u(t)) \\
z(t) &= Cx(t)
\end{align*}
\]

where $x(t) = [x_1(t), x_2(t)]^T$, $u(t) = [u_1(t), u_2(t)]^T$, $z(t) = [z_1(t), z_2(t)]^T$, and $a = [a_1, a_2, a_3]^T$. The measurement matrix $C$ was defined as:

\[
C_{ij} = \begin{cases} 1 & \text{if } j = 2i - 1 \\ 0 & \text{otherwise} \end{cases}
\]

for $i = 1, 2, \ldots, 18$ and $j = 1, 2, \ldots, 36$. To make the estimation more robust, the output $z_i(t)$ from the input $u_i(t)$ was defined as $z_i(t) = (HR_i(t) - 74)/4$, where $HR_i(t)$ is the absolute HR at time $t$, 74 bpm is the average resting HR for all the subjects (resting HR was estimated from the 3-minute resting period before exercise), and 4 bpm is a normalising factor.

For estimation, the objective function was chosen as

\[
S(a) = \sum_{i=1}^{N} (z_i(t_i) - \hat{z}_i(t_i, a))^2 (z_i(t_i) - \hat{z}_i(t_i, a))
\]

where, for $i = 1, 2, \ldots, N$, $z_i(t_i)$ is the measurement of the output vector at time $t_i$ and $\hat{z}_i(t_i, a)$ is the output of system (2) with the parameter vector $a$. With the objective function (4), the Levenberg-Marquardt method was used to determine an estimate of $a$ which was denoted as $\hat{a} := [\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_5]^T$ (see, e.g. Bard [1974], Stortelder [1996], Englezos and Kalogerakis [2001]). Based on a linear approximate method (see e.g. Stortelder [1996]), an approximate $100(1-\alpha)%$ independent confidence interval for each estimate is given by $(\hat{a}_i - \delta a_i, \hat{a}_i + \delta a_i)$, for $i = 1, 2, \ldots, 5$, with

\[
\delta a_i = \sqrt{\frac{\alpha}{N-\alpha}} F_{\alpha}(p, Nm - p)[A^{-1}]_{ii}
\]

where $F_{\alpha}(p, Nm - p)$ denotes the upper $\alpha$ quantile for Fisher’s $F$-distribution with $p$ and $Nm - p$ degrees of freedom and the scalar $[A^{-1}]_{ii}$ is the $(i, i)$ diagonal element of the inverse matrix $A$ that is defined as

\[
A := \sum_{i=1}^{N} G'(t_i)C'CG(t_i), \quad G(t_i) := \frac{\partial f}{\partial a}|_{a=a_i, t=t_i}
\]

In this study, $N = 180$, $m = 18$ and $p = 5$. An $\alpha$ level of 0.05 was used for obtaining the confidence intervals of parameter estimates. Table 2 summaries the estimated parameters of the model (1).

3. CONTROLLER DESIGN

In the second part of this paper, a controller design is proposed for the regulation of HR. The controller essentially controls the speed of the treadmill and in turn controls the HR during treadmill exercise. System (1) is first written in a state-space form as follows:
\[ \dot{y}(t) = A\eta(t) + B_1\Phi(\eta(t)) + B_2g(u(t)) \]
\[ g(t) = C\eta(t) \]
where
\[ A = \begin{bmatrix} -3.07 \times 10^{-2} & 2.7 \times 10^{-2} \\ 0 & -1.06 \times 10^{-3} \end{bmatrix}, \quad \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}, \]
\[ B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 2.7 \times 10^{-2} \\ 0 \end{bmatrix}, \quad C' = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \]
\[ \Phi(\eta_1(t)) := \frac{8.03 \times 10^{-4}\eta_1(t)}{1 + \exp(-15(\eta_1(t) - 5.55 \times 10^{-1}))}, \]
\[ g(u(t)) := u^2(t). \]
To obtain (7), we have defined the normalised output
\[ y(t) = \eta(t) := \frac{x_2(t)}{15} \text{ (i.e., } y(t) := \frac{(HR(t) - HR_{\text{rest}})}{60}). \]
The time unit of \( t \) in (7) is in seconds, instead of minutes as in (1).

System (7) is a nonlinear system with nonlinearity \( \Phi(\eta_1) \) and nonlinear control input \( g(u) \). To overcome the control input nonlinearity, a transformed input
\[ v(t) = g(u(t)) \]
is defined. As for the nonlinear function \( \Phi(\eta_1) \), it can be approximated by a piecewise linear function
\[ \gamma(\eta_1) = \begin{cases} 0 & \text{if } \eta_1 \leq 0.419 \\ 1.52 \times 10^{-3}\eta_1 - 6.34 \times 10^{-4} & \text{if } \eta_1 > 0.419. \end{cases} \]
In fact, \( \gamma(\eta_1) \) is obtained by linearising the function \( \Phi(\eta_1) \) at \( \eta_1 = 0 \) and 0.5. As a result, system (7) can be approximated by a piecewise affine system (see e.g. Rantzer and Johansson [2000]).

In this paper, we adopt a two-degree-of-freedom (2-DOF) controller consisting of a piecewise LQ feedforward and a \( H_\infty \) feedback controllers, as shown in Figure 2, for the control and regulation of heart rate responses.

**3.1 LQ Feedforward Controller Design**

First, we design a feedforward controller using the piecewise LQ optimal control technique of Rantzer and Johansson [2000]. In doing so, we also incorporate an integral action in the controller, see e.g., Burl [1999].

Define two partitions of the state space as shown in Figure 3:
\[ X_1 := \{[\eta_1 \eta_2] \in \mathbb{R}^2 | \eta_1 < 0.419\} \]
\[ X_2 := \{[\eta_1 \eta_2] \in \mathbb{R}^2 | \eta_1 \geq 0.419\}. \]
### 3.2 $H_\infty$ Controller Design

Next, we design a feedback controller based on the $H_\infty$ control technique (see e.g. Petersen et al. [2000], Petersen and Savkin [1999], Moheimani et al. [1998]). We first linearise the system (7) and then formulate the control problem as a mixed sensitivity problem (see e.g. Skogestad and Postltheawite [1996], Zhou and Doyle [1998] for details). In a mixed sensitivity problem, the idea is to choose some weighting functions, namely $W_1(s)$, $W_2(s)$ and $W_3(s)$ to satisfy the control objectives.

The system (7) was first linearised at $x_0 = [0.5 \ 0.115]^T$, $v_0 = 0.452$, and the transfer function of the linearised model is given by

$$G(s) = \frac{0.027s + 2.86 \times 10^{-5}}{s^2 + 0.0317s - 8.65 \times 10^{-6}}.$$  

(20)

Then, the weighting functions were chosen as

$$W_1(s) = \frac{0.1(s + 0.083)}{(s + 8.33 \times 10^{-5})}, \quad W_2(s) = \frac{70(s + 6.25 \times 10^{-4})}{(s + 4.38)}.$$  

(21)

$$W_3(s) = \frac{100(s + 0.966)}{(s + 8.33)}.$$  

The weighting function $W_1(s)$ is chosen as a high-gain lowpass filter approximating an integral action in order to ensure good tracking accuracy. A first-order high-pass filter is chosen for $W_2(s)$ to limit the controller bandwidth and magnitude. The weighting function $W_3(s)$ is chosen to accommodate any multiplicative modelling error. The inverse of the weighting functions $W_i(s)$, $i = 1, 2, 3$, are shown in Figure 4.

By using MATLAB Robust Control Toolbox, we then obtain a robust controller

$$K(s) = \frac{8.91 \times 10^{-3}s^4 + 0.11s^3 + 0.33s^2 + 0.01s + 2.6 \times 10^{-6}}{s^4 + 9.1s^3 + 6.43s^2 + 4.2s^2 + 4.61 \times 10^{-4}s + 3.56 \times 10^{-8}}$$

(22)

and $\gamma = 0.9$ so that $|S(j\omega)| \leq \gamma|W_1(j\omega)|$, $|T(j\omega)| \leq \gamma|W_3(j\omega)|$ and $|K(j\omega)S(j\omega)| \leq \gamma|W_2(j\omega)|$ for all $\omega$. The functions $S$ and $T$ are the sensitivity and the complementary sensitivity functions, respectively (see Figure 4).

### 4. CONTROLLER VERIFICATION

By using the control design presented in Section 3, a controlled treadmill system was implemented for the heart rate regulation (see Figure 5).

The Powerjog fully motorised treadmill was connected to the controller via an RS232 serial port. The heart rate was collected in a similar way as in the stage of identification of the model (see Section 2.1). Except that here the computer (LabVIEW) collected heart rate signal from the wireless Polar system every 6 seconds. Also, an exponential smoothing with filter coefficient $\alpha = 0.75$ was employed on-line (see Diggle [1990]).

As for the controller, it was implemented in LabVIEW. The feedforward controller (19) was pre-computed offline, whereas the robust feedback controller (22) was discretized using the zero-order-hold method with a sampling period $T = 6$ seconds. The control signal was then sent to the treadmill via the serial port.

To validate the controller, the 6 subjects participated in system identification were requested to exercise on the treadmill and prescribed with 2 sets of pre-defined exercise heart rate profiles. The goal was to regulate subjects’ heart rate according to the profile. The pre-defined heart rate profiles may be viewed as prescribed training or exercise protocols. The first profile had 3 10-minute stages involving 2 heart rate levels, namely 100bpm and 115bpm. The first 10-minute stage with heart rate 100bpm was considered as a warm-up period, the second 10-minute stage with heart rate 115bpm was the exercise period, and the last stage with heart rate 100bpm was the cool-down period.

The second exercise heart rate profile also includes the warm-up, exercise and cool-down stages. The differences between the first and the second profiles were: 1) the warm-up period was 3-minute long with gradually increase of heart rate to 110bpm from rest; 2) the heart rate level during the exercise phase was 110bpm; 3) the cool-down period was 7 minutes long with heart rate gradually decreasing to the subjects’ recovery heart rate from the exercise phase. As shown in Figure 6, the heart rate of each subject closely followed the first exercise heart rate profile and regulated at the pre-defined levels, i.e. 100 and
115 bpm, by using the proposed controller. Similar results were obtained for the second heart rate profile that are shown in Figure 7.

![Figure 6](image1.png)

Fig. 6. Regulation of heart rate at 100bpm and 115bpm for all 6 subjects.

![Figure 7](image2.png)

Fig. 7. Heart rate regulation at 110bpm for all 6 subjects with gradually warm-up and cool-down periods.

5. CONCLUDING REMARKS

A nonlinear model describing the heart rate response to the treadmill walking exercise was proposed. The proposed model is a feedback interconnected system. The subsystem in the forward path may be used to describe the neural or the central response, whereas the feedback subsystem may be utilised to describe the peripheral local response. Moreover, the model would be useful in describing the interactions between these two responses. Using the nonlinear model, a controller was developed for the regulation of heart rate during treadmill exercise. The controller consists of a piecewise LQ feedforward and an $H_\infty$ feedback controller. Experimental results demonstrated that the proposed controller had the ability to regulate heart rate for all the experimental subjects. By applying the controller, the heart rate of the subjects could follow two pre-defined heart rate profiles that may represent two kinds of exercise protocols.

REFERENCES


