Robust Mixed $H_2/H_\infty$ Control of Uncertain Neutral Systems with Time-Varying Delays

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Abstract: This paper considers the problem of robust mixed $H_2/H_\infty$ delayed state feedback control for a class of uncertain neutral systems with time-varying discrete and distributed delays. Based on the Lyapunov-Krasovskii functional theory, new required sufficient conditions are established in terms of delay-range-dependent linear matrix inequalities (LMIs) for the stability and stabilization of the considered system using some free matrices. The desired robust mixed $H_2/H_\infty$ delayed control is derived based on a convex optimization method such that the resulting closed-loop system is asymptotically stable and satisfies $H_2$ performance with a guaranteed cost and a prescribed level of $H_\infty$ performance, simultaneously. Finally, a numerical example is given to illustrate the effectiveness of our approach.

1. INTRODUCTION

Delay systems represent a class of infinite-dimensional systems largely used to describe propagation and transport phenomena or population dynamics. Neutral delay systems constitute a more general class than those of the retarded type. Stability of these systems proves to be a more complex issue because the system involves the derivative of the delayed state. Especially, in the past few decades increased attention has been devoted to the problem of robust delay-dependent stability or delay-dependent stabilization and stabilization via different approaches for linear neutral systems with delayed state and/or input and parameter uncertainties (see, Han, 2004; He et. al., 2007; Han and Yu, 2004; Lam et. al., 2005). Among the past results on neutral delay systems, the LMI approach is an efficient method to solve many control problems such as stability analysis and stabilization (Fridman, 2001; Chen and Zheng, 2007) and $H_\infty$ control problems (Chen, 2005; Fridman and Shaked, 2003; Gao and Wang, 2003; Xu et. al., 2001; Chen, 2006; Xu et. al., 2002). It is also worth citing that some appreciable works have been performed to design a guaranteed-cost (observer-based) control for the neutral system performance representation (Karimi, 2008; Chen et. al., 2006; Lien, 2005; Park, 2003; Xu et. al., 2003). To the best of our knowledge, a robust mixed $H_2/H_\infty$ delayed state feedback control for uncertain neutral systems with time-varying discrete and distributed delays has not been fully investigated in the past and remains to be important and challenging.

This paper develops an efficient approach for robust mixed $H_2/H_\infty$ delayed state feedback control problem of uncertain neutral systems with discrete and distributed time-varying delays. The main merit of the proposed method is the fact that it provides a convex problem such the control gain can be found from the LMI formulations. New required sufficient conditions are established in terms of delay-range-dependent LMIs combined with the Lyapunov-Krasovskii method for the existence of the desired robust mixed $H_2/H_\infty$ control such that the resulting closed-loop system is asymptotically stable and satisfies both, $H_2$ performance with a guaranteed cost and a prescribed level of $H_\infty$ performance. A numerical example is given to illustrate the use of our results.

2. PROBLEM DESCRIPTION

Consider a class of neutral systems with discrete and distributed delays and norm-bounded time-varying uncertainties represented by

$$\begin{align}
&x(t)-A_1x(t-d(t))=(A+A(t))x(t)+(A_1+A_1(t))x(t-h(t)) \\
&+(A_2+A_2(t)) \int_{t-h(t)}^{t} x(s)ds +(B+B(t))u(t) + (B_1+B_1(t))w(t),
\end{align}
$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^n$, $w(t) \in L_2^2[0,\infty]$ and $z(t) \in \mathbb{R}^r$ are state, input, disturbance and controlled output, respectively. The time-varying function $\phi(t)$ is continuous vector valued initial function and the time-varying delays $h(t), d(t)$ and $\tau(t)$ are functions satisfying, respectively,

$$\begin{align}
&h_1 \leq h(t) \leq h_2, \quad h(t) \leq h_3, \quad (2a) \\
&0 \leq d(t) \leq d_1, \quad d(t) \leq d_2 < 1, \quad (2b) \\
&0 \leq \tau(t) \leq \tau_1, \quad \tau(t) \leq \tau_2 < 1, \quad (2c)
\end{align}
$$

Moreover, $\Delta A(t), \Delta A_1(t), \Delta A_2(t), \Delta B(t), \Delta B_1(t), \Delta C(t),$ and $\Delta D(t)$ are bounded uncertainties and defined as follows:

$$\begin{align}
&[\Delta A(t) \quad \Delta A_1(t) \quad \Delta A_2(t) \quad \Delta B(t) \quad \Delta B_1(t)] \\
&= H_1 \Delta(t)[E \quad E_1 \quad E_2 \quad E_3 \quad E_4],
\end{align}
$$

$$\begin{align}
&[\Delta C(t) \quad \Delta D(t)] = H_2 \Delta(t)[E_5 \quad E_6 \quad E_7 \quad E_8 \quad E_9].
\end{align}
$$

$$\begin{align}
&\Delta A(t), \Delta A_1(t), \Delta A_2(t), \Delta B(t), \Delta B_1(t), \Delta C(t), \text{ and } \\
&\Delta D(t) \text{ are bounded uncertainties and defined as follows:}
\end{align}
$$
where the uncertain matrix $\Delta(t)$ satisfies $\Delta^T(t)\Delta(t) \leq I$.

**Definition 1:** The $H_2$ and $H_\infty$ performance measures of the system (1) are defined, respectively, as

\[
J_2 = \mathbb{E} \left[ \mathcal{X}(x(t)) S, \mathcal{X}(x(t)) + u(t) S, u(t) \right] dt,
\]

\[
J_\infty = \mathbb{E} \left[ \mathcal{X}(x(t)) z(t) - \gamma^2 w^T(t) z(t) \right] dt,
\]

where the operator $\mathcal{X}(x(t))$ in (4a) is defined by

\[
\mathcal{X}(x(t)) = x(t) - \frac{1}{\gamma^2} A_2 x(t - d(t))
\]

and $S_1 > 0$, $S_2 > 0$ and the positive scalar $\gamma$ are given.

**Assumption 1:** The full state variable $x(t)$ is available for measurement.

In this paper, the authors’ attention will be focused on the design of the following robust mixed $H_2/H_\infty$ delayed state feedback control law,

\[
u(t) = K \mathcal{X}(x(t))
\]

where the matrix $K$ of the appropriate dimension is to be determined such that for any delays satisfying (2) the resulting closed-loop system is asymptotically stable and $J_2 \leq J_0$, where the constant scalar $J_0$ is an upper bound of the $H_2$ performance measure which satisfies an $H_\infty$ norm bound $\gamma$. It can be easily seen that the resulting closed-loop system (1) and (6) is of the following form,

\[
\dot{x}(t) = (A + \Delta A(t) + B + \Delta B(t)) x(t) + \frac{1}{\gamma^2} \sum_{i=1}^n (A + \Delta A_i(t)) x(t - h_i(t)) + (A_2 + \Delta A_2(t)) x(t - d(t))
\]

\[
\dot{x}(t) = \mathcal{X}(x(t)) z(t) - \gamma^2 w^T(t) z(t) + \dot{V}(t)
\]

**Lemma 1:** (Wang et al., 1992) Given matrices $Y = Y^T$, $D$, $E$ and $F$ of appropriate dimensions with $F^T F \leq I$, then the matrix inequality $Y + \text{sym}(DFE) < 0$, the operator $\text{sym}(A)$ represents $A + A^T$, holds for all $F$ if and only if there exists a scalar $\varepsilon > 0$ such that

\[
Y + \varepsilon D D^T + \varepsilon^{-1} E E^T < 0
\]

3. ROBUST CONTROL SYNTHESIS

In this section, both the asymptotic stability and mixed $H_2/H_\infty$ performance of the interconnection of plant and the control are investigated such sufficient stability conditions are derived for the existence of the control (6) combined with the Lyapunov method in terms of LMIs. In the literature, extensions of the quadratic Lyapunov functions to the quadratic Lyapunov–Krasovskii functionals have been proposed for time-delayed systems (Park, 1999). Now, we choose a Lyapunov functional candidate for the uncertain neutral system (1) as

\[
V(t) = \sum_{i=1}^n V_i(t), \quad \text{where}
\]

\[
V_i(t) = \chi(x(t))^T P \chi(x(t)), \quad V_2(t) = \sum_{i=1}^n \int_{t-i}^{t} \dot{x}(s)^T R_i \dot{x}(s) ds
\]

\[
V_1(t) = \int_{t-d(t)}^{t} \dot{x}(s)^T R_1 \dot{x}(s) ds, \quad V_3(t) = \int_{t-th_i(t)}^{t} \dot{x}(s)^T R_3 \dot{x}(s) ds d\theta
\]

Differentiating $V_i(t)$ in $t$ we obtain

\[
\dot{V}_1(t) = 2 \chi(x(t))^T P [(A + \Delta A(t)) + (B + \Delta B(t))] \chi(x(t)) + \int_{t-th_i(t)}^{t} \dot{x}(s)^T R_3 \dot{x}(s) ds d\theta
\]

\[
\dot{V}_2(t) = \int_{t-th_i(t)}^{t} \dot{x}(s)^T R_i \dot{x}(s) ds - \int_{t-d(t)}^{t} \dot{x}(s)^T R_1 \dot{x}(s) ds - \int_{t-th_i(t)}^{t} \dot{x}(s)^T R_3 \dot{x}(s) ds
\]

\[
\dot{V}_3(t) = \int_{t-th_i(t)}^{t} \dot{x}(s)^T R_i \dot{x}(s) ds - \int_{t-d(t)}^{t} \dot{x}(s)^T R_1 \dot{x}(s) ds - \int_{t-th_i(t)}^{t} \dot{x}(s)^T R_3 \dot{x}(s) ds
\]

Moreover, from the Leibniz-Newton formula, the following equations hold for $\{N_i\}_{i=1}^n$ with appropriate dimensions:

\[
2 \chi^T(t) N_i(x(t) - x(t-h_i(t))) \dot{x}(t) - x(t-h_i(t)) - \int_{t-th_i(t)}^{t} \dot{x}(s) ds = 0
\]

\[
2 \chi^T(t) N_i(x(t) - x(t-h_i(t))) \dot{x}(t) - x(t-h_i(t)) - \int_{t-th_i(t)}^{t} \dot{x}(s) ds = 0
\]

Now, to establish the $H_\infty$ performance measure for the system (1), assume zero initial condition, then we have $V(t)|_{t=0} = 0$. Consider the index $J_\infty$ in (4b), then along the solution of (1) for any nonzero $w(t)$ there holds

\[
J_\infty \leq \int_{0}^{T} \dot{V}(t) z(t) - \gamma^2 w^T(t) z(t) + \dot{V}(t) dt
\]

Substituting (1c), (7) and (8)-(16) and adding the left sides of equations (17) and (18) into (19), we obtain

\[
J_\infty \leq \int_{0}^{T} \ddot{\theta}^T(t) \Sigma \dot{\theta}(t) dt
\]

where $\dot{\theta}(t) := \text{col} \{\chi(x(t)), x(t-h_i(t)), x(t-h_i(t)), x(t-d_i(t))\}$, $\dot{x}(t-d_i(t)), \int_{t-d_i(t)}^{t} \dot{x}(s) ds$, $w(t)$, is an augmented state vector and the matrix $\Sigma$ is given by

\[
\Sigma = \Pi + \Pi^T(h_1 R_1 + h_2 R_2 + R_3) \Pi^T + h_1 M_1 R_1^T M_1^T + h_2 M_2 R_2^T M_2^T
\]

where

\[
\Pi =
\begin{bmatrix}
\Pi_{11} & \Pi_{12} & 0 & 0 & 0 & 0 & P(A_1 + \Delta A_1(t)) & P(B_1 + \Delta B_1(t)) \\
\Pi_{12} & \Pi_{22} & 0 & 0 & 0 & 0 & P(A_2 + \Delta A_2(t)) & P(B_2 + \Delta B_2(t)) \\
\Pi_{22} & \Pi_{22} & 0 & 0 & 0 & 0 & P(A_3 + \Delta A_3(t)) & P(B_3 + \Delta B_3(t)) \\
0 & 0 & 0 & 0 & 0 & 0 & P(A_4 + \Delta A_4(t)) & P(B_4 + \Delta B_4(t)) \\
0 & 0 & 0 & 0 & 0 & 0 & P(A_5 + \Delta A_5(t)) & P(B_5 + \Delta B_5(t)) \\
0 & 0 & 0 & 0 & 0 & 0 & P(A_6 + \Delta A_6(t)) & P(B_6 + \Delta B_6(t)) \\
\end{bmatrix}
\]
with \( M_1 = \text{col} \{ N_1, N_2, 0 \} \), \( M_2 = \text{col} \{ N_2, N_3, 0 \} \) and
\[
\tilde{J} = [A + \Delta A(t) + (B + \Delta B(t))K, A + \Delta A(t), 0, 0, \frac{1}{\tau_d^2} (A + \Delta A(t))A_2, A_2A + \Delta A(t), B_1 + \Delta B(t)]
\]
\[
\Pi_{11} = \text{sym} \{ P ((A + \Delta A(t)) + (B + \Delta B(t))K) + N_1 \}
\]
\[
+ K^T (B + \Delta B(t))^T (R_1 + h_2 R_2) (B + \Delta B(t)) K
\]
\[
+ (C + \Delta C(t)) + (D + \Delta D(t))K) ^T (C + \Delta C(t)) A
\]
\[
+ (D + \Delta D(t))K) ^T \sum_{i=1}^3 R_i + R_4 + \tau_i R_4,
\]
\[
\Pi_{12} = P (A + \Delta A(t)) - N_1 - N_2 - \tau_i R_4,
\]
\[
\Pi_{13} = \frac{1}{\tau_d^2} (\sum_{i=1}^3 R_i + R_4 + \tau_i R_4) A_2 + \frac{1}{\tau_d^2} (P + N_1) A_2
\]
\[
+ \frac{1}{\tau_d^2} (C + \Delta C(t)) + (D + \Delta D(t))K) ^T (C + \Delta C(t)) A
\]
\[
\Pi_{22} = -(1-\tau_1) R_4 - \text{sym}(N_4 + N_2),
\]
\[
\Pi_{14} = -(1-\tau_d) R_4 + (1 - \tau_d)^2 A_2 ^T (C + \Delta C(t)) ^T (C + \Delta C(t)) A
\]
where the symbol * denotes the elements below the main diagonal of a symmetric block matrix. Thus, if the inequality \( \Sigma < 0 \) holds, the inequality \( J_\Sigma < 0 \) is satisfied. The inequality \( \Sigma < 0 \) yields (by Schur complement)
\[
\begin{bmatrix}
\tilde{H} & \tilde{H}_1 & \tilde{H}_2 & \tilde{H}_3 & \tilde{H}_4 & \tilde{H}_5 \\
\star & -h_2 R_4 & 0 & 0 & 0 & 0 \\
\star & \star & -h_2 R_4 & 0 & 0 & 0 \\
\star & \star & \star & -R_4 & 0 & 0 \\
\star & \star & \star & \star & -h_2 R_4 & 0 \\
\star & \star & \star & \star & \star & -h_2 R_4
\end{bmatrix} < 0. \tag{22}
\]

Let \( \xi = \text{diag} \{ X, R_1, R_1, R_2, R_2, R_2, R_3, R_3, R_3, R_3, R_4, R_4, R_4, R_4 \} \) where \( X = P^{-1} \) and \( R_i = R_i^{-1} \) for \( i = 1, \ldots, 8 \). Pre-multiplying \( \xi \) and post-multiplying \( \xi^T \) to the matrix inequality (22) yield
\[
\begin{bmatrix}
\tilde{H}_1 & \tilde{H}_2 & \tilde{H}_3 & \tilde{H}_4 & \tilde{H}_5 \\
\star & \tilde{H}_1 & 0 & 0 & 0 \\
\star & \star & -h_2 R_4 & 0 & 0 \\
\star & \star & \star & -h_2 R_4 & 0 \\
\star & \star & \star & \star & -h_2 R_4 \\
\star & \star & \star & \star & \star & -h_2 R_4
\end{bmatrix}
+ \text{sym}(\Psi, \text{diag}(\Pi, \tilde{A}))(\Psi) < 0. \tag{23}
\]

and by Lemma 1 and applying Schur complement, the following inequality is obtained for any scalar \( \epsilon_i > 0 \),
\[
\begin{bmatrix}
\tilde{H}_1 & \epsilon \Psi & \epsilon \Psi^T \\
\star & -\epsilon \Psi & 0
\end{bmatrix} < 0.
\tag{24}
\]

where
\[
\begin{bmatrix}
\tilde{H}_1 & \tilde{H}_2 & \tilde{H}_3 & \tilde{H}_4 & \tilde{H}_5 \\
\star & \tilde{H}_1 & 0 & 0 & 0 \\
\star & \star & -h_2 R_4 & 0 & 0 \\
\star & \star & \star & -h_2 R_4 & 0 \\
\star & \star & \star & \star & -h_2 R_4 \\
\star & \star & \star & \star & \star & -h_2 R_4
\end{bmatrix}
\]

On the other hand, by applying the same Lyapunov-Krasovskii functional candidate (8) for the uncertain neutral system (1), under \( w(t) = 0 \), for the index \( J_2 \) in (4a) we get
\[
J_2 \leq \int_0^t \chi_t(x(t))S \chi_t(x(t)) + \chi_t(x(t))K^T S_2 K \chi_t(x(t)) + \hat{V}(t) \ dt \leq \int_0^t \hat{\sigma}(t) S \hat{\sigma}(t) \ dt,
\tag{25}
\]
where \( \hat{\sigma}(t) := \chi_t(x(t)), x(t) = h(t), x(t) = h(t), x(t) = h(t), x(t) = h(t) \), and the matrix \( \hat{\Sigma} \) is given by
\[
\hat{\Sigma} = \tilde{\Pi} + \tilde{K} \tilde{K} (h_2 R_4 + h_2 R_4 + R_3) \tilde{A} + h_2 M_4 R_1^{-1} M_3^T + h_2 M_2 R_1^{-1} M_2^T
\tag{26}
\]
\[
\begin{align*}
\dot{\Pi}_1 &= [\Pi_{i1}, \Pi_{i2}, N_i, 0] [\Pi_{j2}, 0] P(A_i + \Delta A_i(t))
\end{align*}
\]

with
\[
\tilde{A} = [A + \Delta A(t) + (B + \Delta B(t)) K, A_i + \Delta A_i(t), 0, 0, \frac{1}{1 - \tau_1}, -\Delta A(t) A_i, A_i, A_i + \Delta A_i(t)]
\]

\[\Pi_{i1} = \text{sym} \{ \Pi ((A + \Delta A(t)) + (B + \Delta B(t)) K) + N_i + K^T (B + \Delta B(t))^T \}
\]

\[
\Pi_{i1} = \frac{1}{2} \sum_{j=1}^{T} (R_i + \tau_i R_i) A_i + \frac{1}{\tau_i} (P + N_i) A_i,
\]

\[
\Pi_{i2} = -(1 - d_i) R_i + \sum_{k=1}^{T} R_i + \tau_i R_i) A_i.
\]

Therefore, the condition \( \hat{\Sigma} < 0 \) in (25) implies
\[
\dot{\tilde{V}}(t) dt = \lim_{\tilde{V}(0) \to V(0)} \frac{1}{2} \sum_{j=1}^{T} (R_i + \tau_i R_i) A_i + \frac{1}{\tau_i} (P + N_i) A_i,
\]

By Theorem 1.6 of the reference Kolmanovskii and Myshkis (1992), we conclude that the system (1) with \( w(t) = 0 \) is asymptotically stablizable by (6). Now, by considering the asymptotically stability of the system (1) by (6) the \( H_2 \) performance measure for the system is established as
\[
\int_0^\infty \chi^T (x(t)) S_i \chi (x(t)) \dot{x}^T (x(t)) + \chi^T (x(t)) K^T S_i K \chi (x(t)) dt \leq V(0) = J_0,
\]

where
\[
J_0 = (\phi(0) - \frac{1}{\tau_i} A_i \phi(-d_i(0)))^T P (\phi(0) - \frac{1}{\tau_i} A_i \phi(-d(0))) + \frac{1}{2} \sum_{j=1}^{T} (\phi(s))^T R_i \phi(s) ds + \frac{1}{2} \int_{0}^{\infty} (\phi(s))^T R_i \phi(s) d\theta + \frac{1}{2} \int_{0}^{\infty} (\phi(s))^T R_i \phi(s) ds + \frac{1}{2} \int_{0}^{\infty} (\phi(s))^T R_i \phi(s) ds + \frac{1}{2} \int_{0}^{\infty} (\phi(s))^T R_i \phi(s) d\beta
\]

Similar to the case of \( H_\infty \) performance measure, after applying some matrix manipulations to the inequality \( \hat{\Sigma} < 0 \) we obtain for any scalar \( \epsilon_1 > 0 \),
\[
\begin{bmatrix}
\hat{\Pi} & \epsilon_1 \Psi_1 & \Psi_1^T
\end{bmatrix}
\begin{bmatrix}
0 & \epsilon_1 I & 0
\end{bmatrix}
< 0
\]

where
\[
\hat{\Pi} = \begin{bmatrix}
\Pi_{i1} & h_i A_i & h_i A_i & h_i \Delta A_i & h_i M_i & h_i \Delta M_i
\end{bmatrix}
\]

\[
\Pi_i = \begin{bmatrix}
\Pi_{i2} & N_i & \Pi_{i2} & \Pi_{i2} & \Pi_{i2} & \Pi_{i2}
\end{bmatrix}
\]

Theorem 1: Consider the system (1)-(3) and let \( \gamma > 0 \) be a given scalar. If there exists scalars \( \{\epsilon_i\}_{i=1}^{T} \) and a matrix \( Y \) and positive definite matrices \( X \) and \( \{\hat{R}_i\}_{i=1}^{T} \), satisfying the following LMIs,
\[
\begin{bmatrix}
\hat{\Pi} & \epsilon_1 \Psi_1 & \Psi_1^T
\end{bmatrix}
\begin{bmatrix}
0 & \epsilon_1 I & 0
\end{bmatrix}
< 0
\]

where
\[
\hat{\Pi} = \begin{bmatrix}
\Pi_{i1} & h_i A_i & h_i A_i & h_i \Delta A_i & h_i M_i & h_i \Delta M_i
\end{bmatrix}
\]

\[
\Pi_i = \begin{bmatrix}
\Pi_{i2} & N_i & \Pi_{i2} & \Pi_{i2} & \Pi_{i2} & \Pi_{i2}
\end{bmatrix}
\]

\[
\Pi_i = \begin{bmatrix}
\Pi_{i1} & X & N_i & \Pi_{i2} & \Pi_{i2} & \Pi_{i2}
\end{bmatrix}
\]

\[
\epsilon_1 \Psi_1 = \begin{bmatrix}
0 & \epsilon_1 I & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
\epsilon_1 \Psi_1 = \begin{bmatrix}
0 & \epsilon_1 I & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
\hat{\Pi} = \begin{bmatrix}
\Pi_{i1} & X & N_i & \Pi_{i2} & \Pi_{i2} & \Pi_{i2}
\end{bmatrix}
\]

\[
\epsilon_1 \Psi_1 = \begin{bmatrix}
0 & \epsilon_1 I & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
\epsilon_1 \Psi_1 = \begin{bmatrix}
0 & \epsilon_1 I & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
\hat{\Pi} = \begin{bmatrix}
\Pi_{i1} & X & N_i & \Pi_{i2} & \Pi_{i2} & \Pi_{i2}
\end{bmatrix}
\]

\[
\epsilon_1 \Psi_1 = \begin{bmatrix}
0 & \epsilon_1 I & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
\epsilon_1 \Psi_1 = \begin{bmatrix}
0 & \epsilon_1 I & 0 & 0 & 0 & 0
\end{bmatrix}
\]
Consider the system (1) with the following matrices

\[
A = \begin{bmatrix}
-1 & 0 \\
0.2 & -1.2
\end{bmatrix}; \quad A_1 = \begin{bmatrix}
0.01 & -0.04 \\
0.02 & 0.01
\end{bmatrix}; \quad A_2 = \begin{bmatrix}
0 & 0.1 \\
0 & 0.1
\end{bmatrix}; \quad A_3 = \begin{bmatrix}
0.1 & 0 \\
0 & 0.1
\end{bmatrix}; \quad B_1 = \begin{bmatrix}
2 \\
1
\end{bmatrix}; \quad B_2 = \begin{bmatrix}
0 \\
1
\end{bmatrix}; \quad C = \begin{bmatrix}
0 & 0
\end{bmatrix}; \quad D = \begin{bmatrix}
0 \\
1
\end{bmatrix}; \quad H_1 = \begin{bmatrix}
1 \\
0
\end{bmatrix}; \quad H_2 = \begin{bmatrix}
0 \\
1
\end{bmatrix}; \quad \phi(t) = \begin{bmatrix}
0.5 \\
-0.3
\end{bmatrix}; \quad \theta(t) = \begin{bmatrix}
0.5 \\
0.2
\end{bmatrix}; \quad E_1 = \begin{bmatrix}
0.1 \\
0
\end{bmatrix}; \quad \xi = \begin{bmatrix}
0.1 \\
0
\end{bmatrix}; \quad E = \begin{bmatrix}
0.1 \\
0.1
\end{bmatrix}; \quad \tau \leq 0.3.
\]

Using Theorem 1 and solving LMIs (30a, b) with parameters \( h_1=0.1, h_2=0.8, h_3=0.7, d_1=d_2=0.3, \quad \tau = 0.3, \quad \tau \leq 0 \), the corresponding suboptimal \( H_2 \) performance measure of the resulting closed-loop system is given by \( J_y=1.5539 \) and the minimum value of the parameter \( \gamma \) in optimal \( H_\infty \) performance measure is obtained as \( 0.578 \). Hence, according to Theorem 1, a robust mixed \( H_2/H_\infty \) delayed state feedback control law is given by

\[
u(t) = [-0.5835 - 0.0008] x(t) + [0 0.0835] x(t - 0.3 \sin(t)^2)
\]

For simulation purpose, we simply choose a unit step in the time interval \([1, 2]\) as the disturbance, \( \Delta(t) = \sin(t) \) as the norm-bounded uncertainty and select time delays as \( h(t) = 0.1 + 0.7 \sin(t)^2, \quad d(t) = 0.3 \sin(t)^2 \) and \( \tau(t) = 0.3 \). The simulation results are shown in Figures 1 and 2. Responses of two states, i.e., \( x_1(t), x_2(t) \), of the closed-loop system are depicted in Figure 1 and compared with the corresponding state trajectories in the open-loop system under the initial condition \( x(0) = [0.5 - 0.3]^T \). It is seen from Figure 1 that the closed-loop system is asymptotically stable. The corresponding control signal (32) is also shown in Figure 2.

5. CONCLUSION

The problem of robust mixed \( H_2/H_\infty \) delayed state feedback control was proposed for a class of uncertain neutral systems with time-varying discrete and distributed delays. Based on the Lyapunov-Krasovskii functional theory, new required sufficient conditions were established in terms of delay-range-dependent LMIs for the stability and stabilization of the considered system with considering a mixed \( H_2/H_\infty \) performance measure.

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REFERENCES


Fig. 1. Response of the states $x_1(t)$ and $x_2(t)$: closed-loop system (solid line) and b) open-loop system (dashed line).

Fig. 2. Control law for system.