Abstract: This paper presents a bilateral control scheme designed to achieve the two main goals of a teleoperation system: stability and transparency. The control scheme allows that the slave follows the master, and that the force displayed to the operator was exactly the reaction force from the environment. In addition, the interaction force of the slave with the environment is adapted to the master/slave ratio when it is reflected to the operator, improving the transparency of the system. The bilateral control scheme can be used in contact situations or non-contact situations (free motion) of the slave with the environment. Together with the control scheme, the paper describes an analytical design method that allows to obtain the control gains.

Keywords: Telerobotics; teleoperation; control system; state-space models; control laws.

1. INTRODUCTION

In a telerobotics system the slave is controlled to follow the motion of the master that is manipulated by the human operator. Habitually, the interaction force of the slave with the environment is reflected to the operator to improve the task performance. In this case, the teleoperator is bilaterally controlled, Hanaford (1989).

From a control point of view, the main goals of the bilateral control schemes are maintain the stability of the closed-loop system, and to achieve the transparency of the system between the environment and the operator, Hokayem and Spong (2006). The teleoperation system is transparent if, ideally, the human feels as if directly performing the task in the remote environment, Raju et al. (1989). Or alternatively, the system is transparent if the master and slave positions are equal, and the force displayed to the human is exactly the react of force from the environment, Yokokohji and Yoshikawa (1994). Often, achieving the stability and transparency of a teleoperation system is an incompatible task.

In Azorin et al. (April 2004) a design and bilateral control method of teleoperation systems based in the state convergence was presented. The design method allows to achieve the stability, and that the slave follows the master. In addition, it allows to establish the dynamics of the teleoperation system. However, the control scheme does not achieve the transparency. This paper describes a new bilateral control scheme that achieves the transparency of the teleoperation system, and considerably improves the previous control scheme by state convergence.

The paper is organized as follows. Section 2 explains the limitations of the previous bilateral control scheme by state convergence that have motivated the development of the new control scheme. In Section 3, the new transparent bilateral control method of telerobotics is described. Section 4 shows some simulation results to verify the performance of the new control scheme. Finally, Section 5 summarizes the key features of this control scheme.

2. MOTIVATION

In Azorin et al. (April 2004) a bilateral control scheme of teleoperation systems by state convergence was presented. Fig. 1 shows the modeling on the state space of this control scheme, where:

- \( F_m \) is the operator force,
- \( x_m \) and \( x_s \) are the master and slave state,
- \( u_m \) and \( u_s \) are the master and the slave control signals, and
- \( y_m \) and \( y_s \) are the master and slave position.

A teleoperation system of one dof was considered to explain the design and control method. The simplified linear model of an element with one dof is:

\[
J\ddot{\theta}(t) + b\dot{\theta}(t) = u(t)
\]

where \( J \) is the inertia of the element, \( \theta(t) \) is the rotate angle, \( b \) is the viscous friction coefficient, and \( u(t) \) is the control torque applied. The representation on the state space of the master and the slave is obtained considering as state variables the position \( (x_1(t) = \theta(t)) \) and the velocity \( (x_2(t) = \dot{\theta}(t)) \).
The new control gain $G_1 = g_1$ has been inserted in the control scheme. This gain defines the influence in the reaction force from the environment $f_s(t)$, but it is affected by the master state feedback $K_m x_m(t)$. So, the transparency is not achieved in the bilateral control scheme by state convergence.

On the other hand, in the control scheme by state convergence, considering that the operator exerts the same force, the final position of the slave does not depend on the environment, but it depends on the desired dynamics of the slave, Azorin et al. (2003).

In addition, the control scheme by state convergence can be only applied to contact situations of the slave with the environment. However, it will be suitable that the control scheme can be also applied to non-contact situations (free motion).

Finally, it would be suitable that the reaction force of the slave displayed to the human was adapted to the master/slave ratio. For example, if the master is bigger than the slave, the reaction force displayed to the human must be amplified in order to improve the transparency of the system.

### 3. TRANSPARENT BILATERAL CONTROL SCHEME

This section presents the new bilateral control scheme for telerobotics. This control scheme solves the limitations of the previous scheme described in the last section. The characteristics of this bilateral control scheme are the following:

- The teleoperation system is stable.
- The transparency of the teleoperation system is improved, because the force displayed to the human is exactly the reaction force from the environment.
- The slave position follows the master position, and the desired dynamics of the master-slave error is established.
- The final position of the slave, considering the same operator force, depends on the environment where the slave interacts.
- The force displayed to the human is adapted to the master/slave ratio.
- The control scheme can be applied to contact or non-contact situations of the slave with the environment.

#### 3.1 Modeling of the Teleoperation System

The next changes have been made in the modeling of the teleoperation system shown in Fig. 1 to obtain the new bilateral control scheme:

- The control by state feedback has been removed in the master, i.e. the matrix $K_m$ has been eliminated.
- The structure of the matrix $R_m$ to consider force feedback from the slave to the master does not include the force feedback gain ($k_f$), i.e.:

$$ R_m = [r_{m1} \ r_{m2}] = [k_e \ b_e] $$

- The new control gain $G_1 = g_1$ has been inserted in the control scheme. This gain defines the influence in

$$ u_m(t) = K_m x_m(t) + R_m x_s(t) + F_m(t) $$

$$ = K_m x_m(t) + k_f f_s(t) + F_m(t) $$

Therefore, the force displayed to the human is not exactly the reaction force from the environment $f_s(t)$, but it is affected by the master state feedback $K_m x_m(t)$. So, the transparency is not achieved in the bilateral control scheme by state convergence.
the master of the interaction force of the slave with the environment.

Fig. 2 shows the modeling of the master side in the new control scheme. The modeling of the slave side in the new control scheme has not changed. The elimination of the control by state feedback in the master improves the transparency of the system, because the force displayed to the human is exactly the interaction force of the slave with the environment. In addition, the final position of the master (and the slave) will not be independent of the environment, but it depends on the environment. If the slave does not contact with the environment, the master (and the slave) will have free motion, and the final position of the master and slave will not reach a established constant value. On the other hand, the control gain $G_1$ allows adjusting the force displayed to the human according to the master/slave ratio. This way, if the master is bigger than the slave, this gain will be greater than 1 in order to amplify the interaction force with the environment, and to improve the transparency of the system.

The master and the slave system are represented on the state space like:

$$\dot{x}_m(t) = A_m x_m(t) + B_m u_m(t)$$
$$y_m(t) = C_m x_m(t)$$
$$\dot{x}_s(t) = A_s x_s(t) + B_s u_s(t)$$
$$y_s(t) = C_s x_s(t)$$

Considering one dof, the representation on the state space of the master is:

$$\begin{bmatrix} \dot{x}_{m1}(t) \\ \dot{x}_{m2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{b_m}{J_m} & 0 \end{bmatrix} \begin{bmatrix} x_{m1}(t) \\ x_{m2}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J_m} \end{bmatrix} u_m(t)$$

$$y_m(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_{m1}(t) \\ x_{m2}(t) \end{bmatrix}$$

and the slave is represented in a similar way.

In the new bilateral control scheme, the master control signal, $u_m(t)$, and the slave control signal, $u_s(t)$, are respectively:

$$u_m(t) = F_m(t) - g_1 R_m x_s(t)$$
$$u_s(t) = K_s x_s(t) + R_s x_m(t) + g_2 F_m(t)$$

If the master and slave control signals in the master state equation (7) and in the slave state equation (8) are replaced respectively by the expressions (11) and (12), the next state equations are obtained:

$$\dot{x}_m(t) = A_m x_m(t) - g_1 B_m R_m x_s(t) + B_m F_m(t)$$
$$\dot{x}_s(t) = (A_s + B_s K_s) x_s(t) + B_s R_s x_m(t) + g_2 B_s F_m(t)$$

The equations (13) and (14) can be represented in a matrix way as:

$$\begin{bmatrix} \dot{x}_s(t) \\ \dot{x}_m(t) \end{bmatrix} = \begin{bmatrix} A_s + B_s K_s & B_s R_s \\ -g_1 B_m R_m & A_m \end{bmatrix} \begin{bmatrix} x_s(t) \\ x_m(t) \end{bmatrix} + \begin{bmatrix} g_2 B_s \\ B_m \end{bmatrix} F_m(t)$$

3.2 Design Methodology by State Convergence

There are six control gains in the new bilateral control scheme: $K_s = [k_{s1} k_{s2}], R_s = [r_{s1} r_{s2}], G_1 = g_1$ and $G_2 = g_2$. To calculate these control gains, six design equations must be obtained. In order to get these design equations, the state convergence methodology is going to be applied, Azorin et al. (April 2004).

If the next linear transformation is applied to the system (15):

$$\begin{bmatrix} x_s(t) \\ x_s(t) - x_m(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ I & -I \end{bmatrix} \begin{bmatrix} x_s(t) \\ x_m(t) \end{bmatrix}$$

the next state equation is obtained:

$$\ddot{\tilde{x}}(t) = \tilde{A} \tilde{x}(t) + \tilde{B} F_m(t)$$

where

$$\tilde{x}(t) = \begin{bmatrix} x_s(t) \\ x_s(t) - x_m(t) \end{bmatrix}$$

$$\tilde{A} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix}$$

$$\tilde{A}_{11} = A_s + B_s K_s + B_s R_s$$
$$\tilde{A}_{12} = -B_s R_s$$
$$\tilde{A}_{21} = A_s + B_s K_s + g_1 B_m R_m + B_s R_s - A_m$$
$$\tilde{A}_{22} = A_m - B_s R_s$$

$$\tilde{B} = \begin{bmatrix} g_2 B_s \\ g_2 B_s - B_m \end{bmatrix}$$

Let $x_c(t)$ be the error between the slave and the master, $x_c(t) = x_s(t) - x_m(t)$. From (17) the error state equation between the slave and the master will be:

$$\ddot{x}_c(t) = (A_s + B_s K_s + g_1 B_m R_m + B_s R_s - A_m) x_s(t) + (A_m - B_s R_s) x_c(t) + (g_2 B_s - B_m) F_m(t)$$

If the error evolves as an autonomous system, the slave-master error can be eliminated, and the slave will follow the master. To achieve that the error evolves as an autonomous system, the next equations must be verified:
\[ g_2B_s - B_m = 0 \] (26)
\[ A_s + B_sK_s + g_1 B_mR_m + B_sR_s - A_m = 0 \] (27)

From equation (26) the next design equation is obtained:
\[ g_2 = \frac{J_s}{J_m} \] (28)

Therefore, \( g_2 \) depends on the master/slave ratio, and the operator force sent to the slave will be amplified or reduced depending on this control gain.

Operating in equation (27) the next design equations are obtained:
\[ k_{s1}J_m + g_1 r_{m1}J_s + r_{s1}J_m = 0 \] (29)
\[ b_sJ_m - k_{s2}J_m - g_1 r_{m2}J_s - r_{s2}J_m - b_mJ_s = 0 \] (30)

Therefore satisfying the equations (28) – (30) the error will evolve as an autonomous system. In this case, the dynamics of the slave and the error can not be fixed. It has been verified that the control gains can be only calculated fixing the error dynamics. In this case the design equations are (29), (30), (34) and (35). From these equations, the control gains are obtained:
\[ r_{s1} = q_0J_s \] (37)
\[ r_{s2} = q_1J_s - \frac{J_s}{J_m}b_m \] (38)
\[ k_{s1} = -k_e - q_0J_s \] (39)
\[ k_{s2} = b_s - b_e - J_s q_1 \] (40)

Since the slave dynamics can not be established, the stability of the slave dynamics must be analyzed. The dynamics of the slave is given by the next characteristic polynomial, see (32) and (33):
\[ p(s) = s^2 + \left( b_e - \frac{J_s}{J_m} b_m \right)s + \frac{k_e}{J_s} \] (41)

The slave dynamics depend on the slave and master parameters \((J_s, J_m, \text{ and } b_m)\), and the environment parameters \((k_e \text{ and } b_e)\). Therefore the slave dynamics will be always stable, because \(J_s, J_m, b_m, k_e\) and \(b_e\) are positive for any master, slave and environment. The slave dynamics will be stable even if the environment is modeled only by the stiffness \(k_e\), i.e. \(b_e = 0\).

4. SIMULATION RESULTS

This section shows the simulation results obtained using the new bilateral control scheme. First, a teleoperation system where the slave is bigger than the master is considered. The next parameters have been considered:
\[ J_m = 1 \text{kgm}^2 \quad b_m = 2 \frac{\text{Nm}}{\text{rad/s}} \] (42)
\[ J_s = 10 \text{kgm}^2 \quad b_s = 60 \frac{\text{Nm}}{\text{rad/s}} \] (43)

To design the control system, the error poles have been placed in the position \(-11\) of the \(s\) plane. In all the cases, the force exerted by the operator over the master has been simulated as a constant step of 1 Nm.

Fig. 3 shows the master and slave evolution considering that the slave interacts with a hard environment (\(k_e = 100 \text{Nm/rad} \text{ and } b_e = 0 \frac{\text{Nm}}{\text{rad/s}}\)). It can be verified that the slave position and velocity follow without error the master position and velocity, respectively. Fig. 4 shows the master and slave control signals (top part), and the operator force and the reaction force displayed to the human (bottom part). The reaction force displayed to the operator is adapted to the master/slave ratio. This reaction force opposes to the operator force. When the reaction force displayed to the human is equal to the operator force, the master and the slave stop in the same final position. The control system is stable, and the slave poles are placed in \(s_{1,2} = -1 \pm 3i\).

In top part of Fig. 5 the master and slave position considering that the slave interacts with a soft environment (\(k_e = 10 \text{Nm/rad} \text{ and } b_e = 0 \frac{\text{Nm}}{\text{rad/s}}\)) is shown. As in the...
previous case the slave position follow without error the
master position. However, comparing with Fig. 3, as the
environment stiffness decreases, the final position of the
slave (and the master) increases because the opposition to
the slave advance is lesser. The control system designed is
stable, and the slave poles are placed in $s_{1,2} = -1$. The
bottom part of Fig. 5 shows the master and slave position
when the slave does not interact with any environment
(free motion). In this case the slave follows the master and
they do not stop in a constant position because there is not
any opposition to the slave motion. Therefore the control
scheme can be used in contact and non-contact situations
of the slave with the environment.

The new control scheme has been compared with the
classical force-position bilateral control scheme, Flatau
(1977). A force feedback gain $k_f = 0.1$, and a PD controller
($P = 100$, $D = 32$) have been considered in the force-
position scheme. Fig. 6 shows the master and slave position
considering the hard environment (top part), the soft
environment (central part), and free motion (bottom part).
In all the cases there is a position error between the master
and slave. In addition, there is not a design procedure to
obtain the controller parameters. The controller must be
experimentally tuned by trial-and-error.

The new control scheme has been also compared with the
previous control scheme by state convergence shown in
Fig. 1. It has been assumed that $k_f = 0.1$, and the slave
and error poles are placed in the location $-11$ of the $s$
plane. Fig. 7 shows the simulation results when the slave
interacts with the hard environment. The slave follows
the master. However, the final position of the slave and
the master does not depend on the environment, but it
depends on the desired dynamics of the slave. If a different environment or free motion of the slave is considered, similar results are obtained. On the other hand, the force displayed to the operator is not only the weighted reaction force, but it is the weighted reaction force plus the master state feedback. Therefore the teleoperation system is not transparent. Both problems have been solved with the new bilateral control scheme.

Finally, some simulation results have been obtained considering that the master is bigger than the slave, and a very soft environment \((k_e = 0.5N\text{m}/\text{rad})\), e.g. in a telesurgical system. The next parameters have been considered:

\[
J_m = 1kgm^2 \quad b_m = \frac{2Nm}{\text{rad/s}}
\]

\[
J_s = 0.1kgm^2 \quad b_s = 0.06\frac{Nm}{\text{rad/s}}
\]

To design the control system, the error poles have been placed in the position \(-11\) of the s plane. Fig. 8 shows the master and slave evolution. In spite of the fact that the reaction force is amplified by \(g_1 = 10\) when it is displayed to the human, the slave follows the master without any error, and the system is stable. The slave poles are placed in \(s_{1,2} = -1 \pm 2t\).

5. CONCLUSION

A new transparent bilateral control scheme by state convergence has been presented. The characteristics of this control scheme are the next:

- The system is stable, and the slave follows the master. In addition, the error dynamics is established.
- The system is transparent because the force displayed to the human is the reaction force of the slave with the environment.
- The reaction force displayed to the operator, and the operator force provided to the slave are adapted according to the master/slave ratio. Therefore, the transparency of the system is improved.
- A simple analytical design procedure is provided to calculate the control gains.

REFERENCES


