ROBUST OPTIMAL CONTROL OF FLEXIBLE SPACECRAFT DURING SLEWING MANEUVERS

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Abstract: In this paper, slewing maneuver of a flexible spacecraft with large angle of rotation is considered and assuming structural frequency uncertainties a robust minimum-time optimal control law is developed. Considering typical bang-bang control commands with multiple symmetrical switches, a parameter optimization procedure is introduced to find the control forces/torques. The constrained minimization problem is augmented with the robustness constraints, which in turn increases the number of switches in the bang-bang control input to match the total number of the constraint equations. The steps of the solution algorithm to obtain the time optimal control input are discussed next. The developed control law is applied on a given satellite during a slewing maneuver, and the simulation results show that the robust control input with just few switching times can significantly lessen the vibrating motion of the flexible appendage in the presence of structural frequency uncertainties, which reveals the merits of the developed control law. Copyright © 2005 IFAC

Keywords: Optimal control, Robust control, Space vehicles, Flexible arms, Simulation.

1. INTRODUCTION

Space robotic systems are expected to play an important role in future, e. g. in the servicing, construction, and maintenance of space structures in orbit. Before long, coordinated teams of robots might deploy, transport, and assemble structural modules for a large space structure (Jacobsen, et al., 2002). In order to control such systems, it is essential to develop proper kinematics/dynamics model for the system. This has been studied under the assumption of rigid elements, (Vafa and Dubowsky, 1987; Moosavian and Papadopoulos, 1998; Moosavian and Papadopoulos, 2004), and also elastic elements (Baillicul and Levi 1987; Mah, et al., 1990; Cyril, et al., 1991; Kuang, et al., 1998). There have been also various studies on the control problem of such systems with both rigid and flexible elements (Joshi, et al., 1990; Carusone, et al., 1993; Papadopoulos and Moosavian, 1994-95; Moosavian and Rastegari, 2000).

Due to maneuver time limitations in space, the optimal control with a time minimization constraint is of main concern. It should be noted that high

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speeds, in turn, may stimulate the system flexible modes which may drastically affect the control system performance. Space projects involving large structures and satellites with antennas or solar panels, in general, and robotic manipulators, are examples where one should consider achieving rapid maneuvers without stimulating flexible modes (Scriverener and Thompson, 1994). Therefore, the minimum-time optimal control for the rigid mode and \( n \) flexible modes has become the focus of several articles (Meirovitch and Sharony, 1991; Bobrow, et al., 1985; Singh, et al., 1989; Barbieri and Ozgunar, 1993; Bassam, 2002). Robust time-optimal control problems for slewing spacecraft have been recently received attention (Fontes and Magni, 2003; Boskovic et. al., 2001; Baldelli et. al., 2001; Singh, 1988; Liu and Wie, 1992). In all of these published works, the time-optimal controller is obtained by solving the state and co-state equations, considering the Pontryagins minimum principle. Spacecraft and satellites in orbit usually operate in the presence of various disturbances, including gravitational torque, aerodynamic torque, radiation torque, and other environmental and nonenvironmental torques. The problem of disturbance rejection is of main concern particularly in the case of Low-Earth-Orbiting satellites that operate in the altitude ranges where their dynamics is substantially affected by most of the preceding disturbances. In addition, as some dynamic parameters of spacecraft is not known exactly, the controller design should take these parametric uncertainties into account. In this paper, slewing maneuver of a flexible spacecraft with large angle of rotation is considered and assuming structural frequency uncertainties a robust minimum-time optimal control law is developed. Considering typical bang-bang control commands with multiple symmetrical switches, a parameter optimization procedure is introduced to find the control forces/torques. The constrained minimization problem is augmented with the robustness constraints, which in turn increases the number of switches in the bang-bang control input to match the total number of the constraint equations. The steps of the solution algorithm to obtain the time optimal control input are discussed next. The developed control law is applied on a given satellite during a slewing maneuver, and the simulation results show that the robust control input with just few switching times can significantly lessen the vibrating motion of the flexible appendage in the presence of structural frequency uncertainties, which reveals the merits of the developed control law.

2. PROBLEM FORMULATION

Considering a linear model of a flexible spacecraft with one rigid-body mode and \( n \) flexible modes during a slewing maneuver, the system can be represented as

\[
\mathbf{M} \ddot{\mathbf{q}} + \mathbf{K} \mathbf{q} = \mathbf{G} \mathbf{u}
\]

where \( \mathbf{M} \) and \( \mathbf{K} \), the so-called mass and stiffness matrices, respectively, and \( \mathbf{G} \) the control input distribution. The system described by Eqs. (1) can be transformed into the decoupled modal equations using the eigenvalue and eigenvector information of the system (for more details see Ebrahimi et. al. 2004):

\[
\ddot{q}_i + \omega_i^2 q_i = \Phi_i \mathbf{u} \quad i = 1, \cdots, n
\]

where \( q_i(t) \) is the i-th modal coordinate, \( \omega_i \) is the i-th modal frequency (i-th diagonal element of eigenvalue matrix), and scalars \( \Phi_i \) are defined by:

\[
[\Phi_1 \Phi_2 \cdots \Phi_n]^T = \mathbf{A} \mathbf{G}
\]

where \( \mathbf{A} \) is an \( n \times n \) matrix which its columns are the corresponding eigenvectors, and \( n \) is the number of modes considered in control design. The control input \( u(t) \) is a single bounded one

\[
-u_{\text{max}} \leq u(t) \leq u_{\text{max}}
\]

where \( u_{\text{max}} \) is the maximum value of control input. It is desired to convey the system described by Eqs. (2) from the initial conditions \( \mathbf{q}(0) = [0 0 0 \cdots 0]^T \), to final conditions \( \mathbf{q}(t_f) = [\theta_f 0 0 \cdots 0]^T \) subjected to the control constraints (4) in minimum time. Therefore the performance index can be defined as

\[
J = \int_0^{t_f} \mathbf{u} \, dt = t_f
\]

where the initial time \( t_0 \) is set equal to zero and \( t_f \) is the final time of the maneuver.

3. TIME OPTIMAL CONTROL DESIGN

From the optimal control theory and Pontryagins minimum principle, (see Kirk, 1970), it is known that the solution of the above time optimal control problem is in the form of bang-bang control input with \((2n-1)\) switches which is symmetric about \( t = t_f / 2 \). A bang-bang input with \((2n-1)\) switches can be represented as

\[
u(t) = u_{\text{max}} \sum_{j=0}^{2n-1} b_j \hat{t}(t - t_j)
\]

where \( b_j \) defines the magnitude coefficient at \( t_j \) \( \hat{t}(t) \) defines unit step function, and \( t_{2n} = t_f \).

To obtain the switching times \( t_j \), one should obtain the constrains of the problem. Considering the rigid body mode equation with \( \omega_1 = 0 \), yields

\[
\ddot{q}_1 = \Phi_1 \mathbf{u}
\]

with the following initial conditions

\[
q_1(0) = 0, \quad q_1(t_f) = \theta_f
\]

\[
\dot{q}_1(0) = 0, \quad \dot{q}_1(t_f) = 0
\]

Substituting Eq. (6) into Eq. (7-a) and integrating with respect to time twice, using initial conditions, it is obtained

\[
\theta_f = \Phi_1 u_{\text{max}} \sum_{j=0}^{2n-1} b_j (t_f - t_j)^2
\]

which describes the constraint for the rigid body
motion mode. Next the flexible modes should be considered

$$q_i(t) = \sum_{j=0}^{\Phi_i - L_i} \frac{\Phi_i}{\omega_i} \cos \omega_i (t - t_j) + \sin \omega_i (t - t_j) \right]$$

where all related initial conditions are set equal to zero. Substituting Eq. (6) into Eq. (9), and following a similar procedure it is obtained

$$q_i(t) = -\frac{\Phi_i \mu_{\text{max}}}{\omega_i} \sum_{j=0}^{\Phi_i - L_i} b_j \cos \omega_i (t - t_j) \right]$$

Substituting $t - t_j = (t - t_n) - (t_n - t_j), Eq.(10-a)$ can be rewritten as

$$q_i(t) = -\frac{\Phi_i \mu_{\text{max}}}{\omega_i} \sum_{j=0}^{\Phi_i - L_i} b_j \cos \omega_i (t - t_j) \right]$$

Note that the sine function is an odd one and $t_j$ is symmetric about $t_n$, which is equal to $t/2$, hence the second term vanishes, and the following holds for any bang-bang input

$$\sum_{j=0}^{\Phi_i - L_i} b_j \cos \omega_i (t - t_j) = 0$$

Therefore, to have $q_i(t) = 0$ for $t \geq t_j$, i.e. no residual structural vibration, the following flexible mode constraints is obtained as

$$\sum_{j=0}^{\Phi_i - L_i} b_j \cos \omega_i (t - t_j) = 0 \quad i \geq 2$$

To solve the minimum-time optimal control problem, we have to determine $(2n-1)$ unknown switching times such that the final time $t_f$ be minimized. This can be formulated as a constrained parameter optimization problem, i.e. minimization of the performance index of Eq. (5) subjected to the following constraints

$$f_i(t_1, t_2, \ldots, t_j, \ldots, t_{2n}) =$$

$$\theta_j - \frac{\Phi_i \mu_{\text{max}}}{2} \sum_{j=0}^{\Phi_i - L_i} b_j (t_j - t_n)^2 = 0$$

$$f_j(t_1, t_2, \ldots, t_j, \ldots, t_{2n}) =$$

$$\sum_{j=0}^{\Phi_i - L_i} b_j \cos \omega_i (t - t_j) = 0 \quad j = 1, 2, \ldots, 2n$$

To satisfy the necessary and sufficient condition for optimality, the Hamiltonian can be introduced as

$$H = t_f + \lambda_i f_i$$

where $\lambda_i$ are defined as Lagrange multipliers. Setting up the following equations, a set of $3n$ equations, can be solved to determine $3n$ unknowns, i.e. $(2n-1)$ switching times, one final time $t_f$, and $n$ Lagrange multipliers

$$g_j = \frac{\partial H}{\partial \lambda_j} = 0 \quad j = 1, 2, \ldots, 2n$$

$$g_k = \frac{\partial H}{\partial \lambda_k} = 0 \quad k = 2n + 1, \ldots, 3n$$

This set of equations are often coupled and nonlinear, which can be solved employing numerical methods, see (Gerald and Wheatley, 1999).

4. ROBUST TIME OPTIMAL CONTROL DESIGN

Constraint equations (13) can be represented as

$$f(T, P) = 0$$

where $T$ represents a set of switching times and $P$ consists of flexible mode frequencies that are considered uncertain. Expanding $f(T,P)$ about its normal value $P^0$ it can be obtained

$$f(T, P) = f(T, P^0) + \frac{\partial f}{\partial P} \sum_{i=1}^{n} (P_i - P_i^0)^2$$

Then, the set of switching times $T$ can then be redesigned to satisfy the following two sets of constraints:

$$f(T, P^0) = 0$$

$$\frac{\partial f}{\partial P} \sum_{i=1}^{n} (P_i - P_i^0)^2 = 0$$

where the second constraint is called the first-order robustness constraint, because it limits the amplitude of residual structural vibrations caused by uncertainties of $P$. By taking the derivative of Eq. (10-b) with respect to $\omega_i$, for each flexible mode it can be obtained

$$\sum_{j=0}^{\Phi_i - L_i} b_j \cos \omega_i (t - t_j) = 0$$

Letting $dq_{i,m} / d\omega_i = 0$ for all $m \neq t_f$, it is obtained

$$\sum_{j=0}^{\Phi_i - L_i} b_j \cos \omega_i (t - t_j) = 0$$

which describes the first-order robustness constraint. If these robustness constraints are in the constrained minimization problem formulation described by Eq.(14), the number of switches in the bang-bang control input, must be increased to match the number of the constraint equations. Adding one robustness constraint will require, two more switches. Then robust time optimal control problem can be formulated as a constrained parameter optimization problem, i.e. minimization of the performance index of Eq. (5) subjected to the constraints set (14) and the new additional set of constraints (21), i.e. previous ones plus robustness constraints.

5. SOLUTION ALGORITHM

Collecting $g_j$ functions defined by Eq. (16) in a $(3n \times 1)$ vector as

$$g = [g_1, g_2, \ldots, g_{2n}, \ldots, g_{3n}]^T$$

the steps of the solution procedure and numerical algorithm to obtain the time optimal control and robust time optimal control inputs are given below.

1- Determine the numbers of flexible modes $(n)$.
2- Define the bang-bang input with $(2n-1)$ switches.
3- Apply this control input function for rigid body mode and flexible modes to obtain constrains of problem with performance index, $J=t_f$. At this step, the time optimal control problem is converted into parameter optimization problem. Parameters of this
The problem can be reduced by consideration of time symmetry property of bang-bang input function.

4- Form vector \( \mathbf{g} \), using Eqs. (14)-(22).

5- Form a \((3n \times 1) \) vector \( \mathbf{h} \) for unknowns

\[
\mathbf{h} = [t_1, \ldots, t_n, \lambda_1, \ldots, \lambda_n]^T
\]  

(23)

6- Assume some starting values for \( \mathbf{h} \) as \( \mathbf{h}_0 \).

7- Calculate the \( 3n \times 3n \) Jacobian matrix

\[
J = \frac{\partial \mathbf{y}}{\partial \mathbf{h}} = 
\begin{bmatrix}
\frac{\partial y_1}{\partial h_1} & \ldots & \frac{\partial y_1}{\partial h_n} \\
\frac{\partial y_2}{\partial h_1} & \ldots & \frac{\partial y_2}{\partial h_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial y_n}{\partial h_1} & \ldots & \frac{\partial y_n}{\partial h_n}
\end{bmatrix}
\]  

(24)

where \( J_{ij} = \frac{\partial y_i}{\partial h_j} \) are calculated using the current values of \( \mathbf{h} \).

8- Calculate the step direction as

\[
\Delta = J^{-1} \mathbf{g}
\]  

(25)

9- Update the unknown variables

\[
\mathbf{h} = \mathbf{h}_c - \Delta
\]  

(26)

where \( \mathbf{h}_c \) denotes the current value of \( \mathbf{h} \).

10- Repeat Steps 7-9 until:

\[
\| \mathbf{g} \| \leq \varepsilon
\]  

(27)

where \( \| \cdot \| \) refers to Euclidean norm, and \( \varepsilon \) is a chosen threshold.

11- The unknown variables are obtained as

\[
\mathbf{h} = \mathbf{h}_c
\]  

(28)

Next, to illustrate the developed optimal control law and described numerical procedure, the slewing maneuver of a given satellite is simulated.

6. SIMULATIONS

The system parameters and maneuver specifications are listed in Table 1. To see the inherent behavior of the system, the first five modes are retained in the developed model in the simulation routine prepared in MATLAB environment, in which a single torque actuator is located on the rigid central body to control the maneuver. The task is to control the satellite orientation during a rest-to-rest maneuver in minimum time. Table 2 shows the natural frequencies \( \omega_i \), in radian per second, and the components of \( \varphi_i \) in Eq. (2), for the first five modes.

For the first trial, just the rigid body mode is considered, and so there exists just one switching time and the control torque will be defined as

\[
u_1(t) = \frac{1}{2} \left[ \Theta(t) - 2[\Theta(t - t_1) + \Theta(t - t_f)] \right]
\]  

(29)

By applying the presented algorithm, the middle and final time, \( t_1 \) and \( t_f \), are obtained as shown in Table 3. The input amplitude is given \( u_{\text{max}} = 20 \) N.m as shown in Fig. (1a), and the attitude of the central rigid body due to this input torque varies according to curve (1) in Fig. (2a). To find the vibration of the end point of appendages (solar panels), due to this input torque, if we solve the equation:

\[
\ddot{y}_2 + \omega_2^2 y_2 = \Phi_2 u_1
\]  

(30)

and transform the solution back to the physical coordinate, the end point vibration will be obtained as Fig. (2b). As seen, the amplitude is considerably large and may cause significant damage of the spacecraft. Therefore, at least the first flexible mode, i.e. \( n = 2 \), should be considered.

To consider the first flexible mode \( (n = 2) \), in order to apply the control input \( u_2(t) \), three switching times will be introduced as

\[
u_2(t) = \frac{1}{2} \left[ \Theta(t) - 2[\Theta(t - t_2) + \Theta(t - t_f)] \right] + 2[\Theta(t - t_2) - 2[\Theta(t - t_3) + \Theta(t - t_f)]
\]  

(31)

According to the presented algorithm, the switching and final times are obtained as shown in Table 3, and the control input is illustrated in Fig. (1b).

Table 1. System Parameters and Maneuver Specifications

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central body inertia</td>
<td>1.1</td>
</tr>
<tr>
<td>Solar panels Length</td>
<td>4.0</td>
</tr>
<tr>
<td>Solar panels Thickness</td>
<td>0.02</td>
</tr>
<tr>
<td>Solar panels Width</td>
<td>0.50</td>
</tr>
<tr>
<td>Solar panels material stiffness</td>
<td>20.10 Nm²</td>
</tr>
<tr>
<td>Solar panels material density</td>
<td>0.81 Kg/m²</td>
</tr>
<tr>
<td>Maximum torque available</td>
<td>20 N.m</td>
</tr>
<tr>
<td>Total mass of spacecraft</td>
<td>800 Kg</td>
</tr>
<tr>
<td>Total slewing angle</td>
<td>20 deg</td>
</tr>
</tbody>
</table>

Table 2. Flexible Modes Specifications

<table>
<thead>
<tr>
<th>( i )</th>
<th>( \omega_i )</th>
<th>( \Phi_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.0628</td>
</tr>
<tr>
<td>2</td>
<td>1.2355</td>
<td>-0.0328</td>
</tr>
<tr>
<td>3</td>
<td>6.9311</td>
<td>0.0092</td>
</tr>
<tr>
<td>4</td>
<td>19.3320</td>
<td>0.0043</td>
</tr>
<tr>
<td>5</td>
<td>38.2100</td>
<td>-0.0026</td>
</tr>
</tbody>
</table>

Table 3. Switching and final maneuver times (sec.)

<table>
<thead>
<tr>
<th>Times for ( u_1(t) )</th>
<th>Times for ( u_2(t) )</th>
<th>Times for ( u_3(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_1 = 2.104 )</td>
<td>( t_2 = 1.498 )</td>
<td>( t_3 = 1.012 )</td>
</tr>
<tr>
<td>( t_1 = 4.208 )</td>
<td>( t_2 = 2.2755 )</td>
<td>( t_3 = 2.235 )</td>
</tr>
<tr>
<td>( t_1 = 4.012 )</td>
<td>( t_2 = 3.851 )</td>
<td>( t_3 = 3.851 )</td>
</tr>
<tr>
<td>( t_1 = 5.509 )</td>
<td>( t_2 = 5.467 )</td>
<td>( t_3 = 5.467 )</td>
</tr>
<tr>
<td>( t_1 = 6.690 )</td>
<td>( t_2 = 7.702 )</td>
<td>( t_3 = 7.702 )</td>
</tr>
</tbody>
</table>
The response of the central rigid body and the flexible appendage are shown as Curve (2) in Fig. (2a), and Fig. (2c), respectively. As shown in Fig. (2c), by applying $u_2(t)$, the vibration of the appendage in its first flexible mode does completely vanish, however it seems so that the second flexible($n=3$) mode is excited. Therefore, to investigate this, the amplitude of vibrations for the second flexible mode is shown in Fig. (2d). As seen in the figure, the amplitude is about 0.5 mm which is reasonably small. This will be substantially reduced by considering the robust control input $u_3(t)$.

The robust control input $u_3(t)$ can be defined by adding first-order robustness criterion described in Eq. (21), as follows

$$u_3(t) = u_{\text{opt}} + \{t(t) - 2[t(t-t_s)] + 2[t(t-t_s)] - 2[t(t-t_s)] + [t(t-t_s)]\}$$

Following presented algorithms, switching and final times for this case will be obtained as shown in table 3. The control profile $u_3(t)$, and obtained responses are illustrated in Fig. (1)-(2).

Finally, the end point motion of the appendage is compared for the three cases in Fig. (3). It should be noted that the vibration amplitude has been decreased by 99% due to $u_3(t)$, compared to that of $u_1(t)$, whereas the maneuver time has increased by 83%.

![Fig. 1. Optimal input (a) $u_1(t)$, (b) $u_2(t)$, and (c) $u_3(t)$.](image1)

![Fig. 2. (a) Central rigid body responses to inputs $u_1(t)$, $u_2(t)$ and input $u_3(t)$; (b) Flexible appendage response due to $u_1(t)$; (c) Flexible appendage response due to $u_2(t)$; (d) The second flexible mode response due to $u_2(t)$; (e) The second flexible mode response due to robust input $u_3(t)$.](image2)

![Fig. 3- Attitude of end point of solar array for input $u_1(t)$ $u_2(t)$ and robustified $u_3(t)$.](image3)

7. CONCLUSIONS

Focusing on slewing maneuver of a flexible spacecraft with large angle of rotation, and assuming structural frequency uncertainties a robust minimum-time optimal control law was developed in this paper. Employing the assumed modes method for the flexible appendage, and the Euler-Bernoulli beam assumption, the system dynamics was modeled. Considering typical bang-bang control commands with multiple symmetrical switches, a parameter optimization procedure was introduced to find the control forces/torques. Augmenting the constrained minimization problem with the robustness constraints, the number of switches in the bang-bang control input was increased to match the total number of the constraint equations. The steps of the solution algorithm to obtain the time optimal control input were discussed next. The developed control law was applied on a given satellite during a slewing maneuver, where the first five modes were considered in the simulated model of the system. The simulation results accentuated that the robust control input, exerted by a single torque actuator located on the rigid central body, with just few switching times significantly diminishes the vibrating motion of the flexible appendage.

REFERENCES


