Abstract: Some problems on robust gyromoment motion control of agile spacecraft (SC) for remote sensing the Earth surface are considered. Elaborated methods for dynamic research of the SC programmed angular motion at principle modes under external, structural and parametric disturbances, partial discrete measurement of the state and digital control of the gyro moment cluster (GMC) by the minimum-excessive gyrodine scheme, are presented. Copyright © 2005 IFAC

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1. INTRODUCTION

The dynamic requirements to the attitude control system (ACS) for the remote sensing SC are:

- guidance the telescope’s line-of-sight to a predetermined part of the Earth surface with the scan in designated direction;
- stabilization of a image motion at the onboard optical telescope focal plane.

Moreover, for the remote sensing spacecraft these requirements are expressed by rapid angular manoeuvring and spatial compensative motion with a variable vector of angular rate, see Fig. 1. Increased requirements to such information satellites (lifetime up to 10 years, exactness of spatial rotation manoeuvres with effective damping the SC flexible construction oscillations, robustness, fault-tolerance as well as to reasonable mass, size and energy characteristics has motivated intensive development the gyro moment clusters (GMCs) based on excessive number of gyrodes (GDs) — single-gimbal control moment gyros.

Mathematical aspects of the SC nonlinear gyromoment control were represented in a number of research works (Junkins and Turner, 1986; Singh and Bossart, 1993; Hoelscher and Vadali, 1994; Schaub et al., 1998) including authors’ papers (Anshakov et al., 1995; Somov, 1997; Matrosov et al., 1997; Somov, 1998; Somov et al., 1999a,b,c; Kozlov et al., 1999; Somov, 2000, 2001, 2002; Somov et al., 2002; Matrosov and Somov, 2003; Somov et al., 2003a,b,c, 2004; Anshakov et al., 2004) The paper suggests new results on nonlinear and robust gyromoment control of the agile SC.
AM variation without non-passed singular states for condition $V_k^m \subseteq V_k^n$. In Crenshaw (1973) collinear pair of the stop-less GDs was named Scissored Pair Ensemble (SPE) and scheme based on two collinear pairs — as 2-SPE. The angels $\gamma^s, 0 \leq \gamma^s \leq \pi/2, s = 1,2$ define the axis suspension positions for the GDs pairs. Topological analysis of all natural sets by singular GMC’s states and results were presented in Somov et al. (2003b).

3. MATHEMATICAL MODELS

The spacecraft BRF attitude with respect to the inertial reference frame (IRF) is defined by quaternion $\Lambda = (x_0, x, \lambda)$, $\lambda = (\lambda_1, \lambda_2, \lambda_3)$. Assume that $\Lambda^p(t)$ is a quaternion of the programmed SC body’s motion in the IRF. The error quaternion is $\kappa = (\kappa_0, \kappa, \lambda)$, and the attitude error’s matrix is $C_{\kappa} = C(\kappa)$.

For a fixed position of the SC flexible structures with some simplifying assumptions and $t \in T_{u} = [t_0, +\infty)$ a SC angular motion model appears as:

$$\dot{\Lambda} = \Lambda \omega/2; \quad \Lambda^p = (\omega, \bar{q}, \bar{\beta}) = (F^p, F^q, F^\beta), \quad (1)$$

$$F^p = M^s - \omega \times G + M^p + Q^2; \quad M^s = -\dot{\Lambda} = -A_h \dot{\beta}; \quad F^q = -a_j^{\rho}((\delta/\pi)\Omega_j y_j + (\Omega_j)^2 q_j) + Q^2(\omega, \bar{q}, \bar{\beta}); \quad F^\beta = A_h \omega + M^p + M^q + Q^3; \quad A_h = [\partial(h(\beta)/\partial \beta)]$$

the GMC’s AM vector $h(\beta) = h_g \sum h_p(\beta_p)$; a damping torque vector $M^{\alpha}_p$ is continuous function, and vector $M^{\alpha}_a$ of the friction torques in the GD’s bearings is discontinuous function.

The components of the GMC control vector $M^p$ are described as $M^p = a^p \omega^p(t)$, where the digital control voltages $u^p(t) = Z(x) \text{Sat} (\text{Qnt} u^p_k) = B^p_p, T_u$ and $\omega^p$ are constants. Functions $u^p_k$ are the discrete outputs of nonlinear control law (NCL), and functions $\text{Sat}(x, a)$ and $\text{Qnt}(x, a)$ are general-usage ones, while the holder model with the period $T_u$ is of the type: $y(t) = Z(x_k, T_u) = x_k \quad \forall t \in [t_k, t_{k+1})$.

4. THE GMC ADJUSTMENT

Within precession theory of control moment gyro the the GMC torque vector $M^\beta$ is presented as

$$M^\beta = -\kappa \times A_h \dot{\beta}; \quad \kappa = \bar{\beta} = \bar{u}; \quad u = v, \quad (2)$$

where matrix $A_h = \{[1, 0, 0, 0], [0, s, \gamma, 0], [0, -c_x, c_y, 0], [0, -c_y, -c_x, 0]\}$.

$$A_h(\beta) = [\partial(h(\beta)/\partial \beta)] = \begin{bmatrix} -y_1 & -y_2 & -z_3 & -z_4 \\ x_1 & x_2 & 0 & 0 \\ 0 & 0 & -x_3 & -x_4 \\ \end{bmatrix}; \quad \kappa = \bar{\beta} = \bar{u}; \quad \bar{u} = v, \quad \bar{v} = \kappa$$

$$h(\beta) = h_g h^c; \quad h^c = \{x, y, z\} = A_h c_\alpha; \quad \gamma = \gamma^s_1 = \gamma^s_2; \quad \alpha = [x_{12}, x_{34}]; \quad y = y_1 + y_2; \quad z = -z_3 + z_4; \quad x_p = c_{\beta_p};$$

Fig. 1. The scanning pattern of given targets

2. THE GMC SCHEME

Primary problem consists in an estimating the domain $V_k^m$ of nominal variation of the SC body normed angular momentum (AM), starting from the SC purposes. The SC inertia tensor $J$ usually is given in the BRF with any admissibility, therefore guaranteeing calculation is carried out for the inertia tensor $J^p$, by which corresponds an inertia ellipsoid $J^p = \text{diag} \{J_{x}^p, J_{y}^p, J_{z}^p\} \equiv [J^p]$ with principle central axes on body reference frame (BRF) axes, and enveloping (i.e. including in itself) all possible the SC inertia ellipsoids. The domain of required variation by the SC vector programmed angular rate $\omega^p(t)$ conveniently is defined as ellipsoid $V_k^p = \{\omega^p : |\omega^p| \leq \omega_m\}$, where $\omega_m$ are given constants. Then the domain $V_k^p$ of nominal variation of the SC normed AM $k^p$ for its components $k^p_i(t) = J_{i}^p \omega^p_i(t)/\omega_m$ is obtained also in the form of ellipsoid $V_k^p = \{k^p : |k^p_i| \leq k^p_i\}$. Every $k^p_i = J_{i}^p \omega^p_i/\omega_m$ present its semi-axes.

For information SC it is characteristics that the semi-axes dimensions of an ellipsoid $V_k^p$ have the essentially different values. For the agile remote sensing SC there is possible to stand out two classes of programmed angular rates of the SC movements — the course movements (CMs) and the rotational maneuvers (RMs):

- class $A$, when $\omega^p_m = \omega_m$ and $|\omega^p(t)| \leq \omega_m$;
- class $B$, when the ellipsoid’s $V_k^p$ semi-axes are related among themselves as $a^p_1 : a^p_2 : a^p_3$ at the fast movements with priority on the pitch axis $Oz$. For typical remote sensing SC the values of inertial moments by its body $J_{x_0}^p$ and $J_{y_0}^p$ on the axes roll $Ox$, and pitch $Oz$ are compared among themselves, and no less twice exceeded the inertial moment by the SC body $J_{z_0}^p$ on the axis yaw $Oz$. Therefore the semi-axes $k_{x_0}^p, k_{y_0}^p, k_{z_0}^p$ of ellipsoid $V_k^p$ are related among themselves approximately as 2:1:2 on the SC motions in class $A$ and as $2a^p_1 : a^p_2 : 2a^p_3$ in the basis $B$ on the SC motions in class $B$. Domain $V_k^m$ of the SC body AM required variation must be given in the BRF with something to spare with respect to domain $V_k^p$. It is obviously that in class GD-systems the GMC mass will minimal, if at least number $m \geq 4$ applied GDs it ensures the domain $V_k^m \subseteq V_k^n$ of the normed
there are defined the branch functions $\lambda_{\tau}(t)$ with notation where the values

$$\lambda_{\tau}(t) = \begin{cases} \epsilon_{\tau} \tau_{1}(1 - \tau_{1}), & \sigma_{1}(t) = \omega_{\tau} \tau_{1}(3 - 2 \tau_{1}), \\ \sigma_{2}(t) = \sigma_{2}(1/2 + (1 - \tau_{2}^{3}/2)), & t = t_{\nu}, T_{\nu} + \tau_{1} \\ \sigma_{2}(t) = - \epsilon_{\tau} \tau_{1}(1 - \tau_{2}), & \sigma_{1}(t) = \omega_{\nu} (1 - t_{\nu}^{3}/2), \sigma_{2}(t) = \sigma_{2}(1/2 + (1 - \tau_{2}^{3}/2)), & t = t_{\nu} + T_{\nu} + \tau_{1} \\ \end{cases}$$

with the norm time parameters $\tau_{1}, \tau_{2} \in [0, 1].$ For notation $S_{\mu} = \text{Sign}(p_{\nu} - p_{\nu})$ the function $\rho(t)$ and its time derivatives are presented in explicit form $\rho(t) = p_{\nu} + S_{\mu} \cdot \sigma(t); \dot{\rho}(t) = S_{\mu} \sigma_{1}(t); \ddot{\rho}(t) = S_{\mu} \sigma_{2}(t)$.

5. PROGRAMMED CONTROL

The analytic matching solution have been obtained for problem of the SC programmed angular RM synthesis at given time interval $t \in T_{\nu} = [t_{0}, t_{f}]$, $t_{f}^{n} \equiv t_{f}^{n} + T_{n}$, when a space opto-electronic observation is carried out. This problem consists in determination of quaternion $\mathbf{A}(t)$ by the SC BRF $\mathbf{B}$ with respect to the IRF $\mathbf{I}$, angular rate vector $\omega(t)$, vectors of angular acceleration $\varepsilon(t) = \{\epsilon_{1}(t)\} = \omega(t)$ and its derivative $\dot{\varepsilon}(t) = \dot{\varepsilon}(t) \times \varepsilon(t)$ in the form of explicit functions. Moreover values of vectors $\omega_{\nu} \equiv \omega(t)$ for the discrete time moments $t_{\nu} \in T_{\nu}$ with period $T_{\nu} = t_{\nu+1} - t_{\nu}, s = 0, 1, 2...n_{\nu} = 0: n_{\nu}, n_{\nu} = T_{\nu}/T_{\nu}$ and initial value $\mathbf{A}(t_{0}) = \mathbf{A}_{0}$ are given.

Solution is based on a vector composition of all elemental motions in geodetic reference frame with regard to initial headings and the scan azimuth, the Sun zenith angle and a current observation perspective, proceed from principle requirement: optical image of the Earth given part must to move by desired way at focal plane of the telescope. The solution is obtained by extrapolation of the vector $\omega_{\nu} \equiv \omega(t_{\nu})$ values which are defined in the time moments $t_{\nu} \in T_{\nu}$ with step $T_{\nu} = t_{\nu+1} - t_{\nu}, k = 0; n, n \equiv T_{\nu}/T_{\nu}$, for the period’s ratio $k_{2}^{0} \equiv T_{\nu}/T = k_{2}^{0}$ by degree $k_{2}^{0} \geq 2.$ For such approximation the quaternion $\mathbf{M}(t)$ and vector $\mathbf{p}(t)$ have been appeared as explicit functions of a time $t \in T_{\nu}$ which are near to the quaternion $\mathbf{A}(t)$ and vector $\omega(t) \forall t \in T_{\nu}$, respectively, and in explicit form — the vector functions $\mathbf{q}(t) = \mathbf{p}(t)$ and $\dot{\mathbf{q}}(t)$, corresponding to $\varepsilon(t)$ and $\dot{\varepsilon}(t)$. Extrapolation of the discrete-assigned trajectory $\omega_{\nu}$ by vector $\mathbf{p}(t) \forall t \in T_{\nu}$ at the conditions $\mathbf{p}(k_{2}^{0}) \equiv \mathbf{p}_{k} = \omega_{\nu}, k = 0 : (n - 1), \mathbf{p}(t_{0}) = \mathbf{p}(t_{0}) = \omega_{\nu}$ is obtained by set of $n$-degree vector splines $\mathbf{p}_{k}(\tau)$ at normed time $\tau = (t - t_{k})/T_{\nu} \in [0, 1]$, with analytical obtaining a high-precise approximation both vector of programmed angular acceleration $\mathbf{q}(t) = \mathbf{p}(t)$ with its local derivative $\dot{\mathbf{q}}(t)$ and quaternion $\mathbf{M}(t)$.

These functions are applied at onboard computer for the time moments $t_{\nu} \in T_{\nu}$.

For control torque $M^{*} \equiv 2$ and the SC model as a free rigid body the simplified controlled object is such:

$$\dot{\Lambda} = \Lambda \cdot \omega_{\nu}/2; \dot{\Lambda} \mathbf{w} + [\omega \times \mathbf{G}^{*}] = \mathbf{M}^{*}; \beta = u; \dot{u} = v. \quad (4)$$

The computer-aided algorithm for synthesis of the strict optimal control $\mathbf{v} = \mathbf{v}(t), t \in T_{\nu} = [t_{0}, t_{f}]$ for the GMAC’s DL (3) during the SC rapid RMs with the preassigned boundary conditions on initial and final states $\mathbf{A} = \mathbf{A}(t_{f}), \omega_{f} = \omega(t_{f})$, $s = i, f, \beta = \beta(t_{f})$ with $h(\beta(t)) \subseteq \mathcal{S} \subseteq \mathcal{S}$ and $\beta(t_{f}) = 0$ in the optimization problem of the energy index

$$I = \frac{1}{2} \int_{I} (q Q_{1}(u(\tau)) + p Q_{2}(u^{2}(\tau)) d\tau \iff \text{min} (5)$$

for $\mathbf{A} = \mathbf{A}, \mathbf{w} \iff \mathbf{A}^{f}, \mathbf{w}^{f}$, where $q, p, a_{1} = const$ and $q \in [0, 1], p = 1 - q$. $Q_{1}(x) \equiv x^{2}$ and a preassigned time $T_{f} = t_{f} - t_{0}$ for $h(\beta(t)) \subseteq \mathcal{S} \forall t \in T_{f} \subset T_{f}$, $\mathbf{A}$ has been created (Somov, 2000). Fast onboard algorithms with parametric optimization $\mathbf{v}^{(0)}(t)$ in the problem (5) for model (4) and restrictions to $\omega(t), \dot{\omega}(t), \ddot{\omega}(t)$, corresponding restrictions to $h(\beta(t)), \dot{\beta}(t) = \mathbf{u}(t), \ddot{\beta}(t) = \mathbf{v}(t)$ in a class of the SC angular motions, were elaborated. At analytical synthesis of the SC RM
programme under given interval of a time \( t \in T_p \equiv [t^p_0, t^p_f] \); \( t^p_f \equiv t^p_0 + T_p \) a problem consists in determination the explicit time functions — quaternion \( \Lambda(t) \), vectors \( \omega(t), \varepsilon(t) \) and \( \dot{\varepsilon}(t) = \varepsilon^*(t) + \omega(t) \times \varepsilon(t) \) for the boundary conditions on left (\( t = t^p_0 \)) and right (\( t = t^p_f \)) trajectory ends:

\[
\Lambda(t^p_f) = \Lambda_f; \omega(t^p_f) = \omega_f; \varepsilon(t^p_f) = \varepsilon_f; \dot{\varepsilon}(t^p_f) = \varepsilon^*_f + \omega_f \times \varepsilon_f.
\]

In (7) last condition presents requirements for a smooth conjugation of rotation maneuver with next the SC route motion. Developed approach to the problem is also based on necessary and sufficient condition for solvability of Darboux problem.

At general case the solution is presented as result of composition by six \((k = 1:6)\) simultaneously derived elementary rotations of embedded bases \( E_k \) about units \( e_k \) of Euler axes, which position is defined from the boundary conditions (6) and (7) for initial spatial problem. For all 6 elementary rotations with respect to units \( e_k \) the boundary conditions are analytically assigned, moreover for first five elementary motions there are ensured zero value for local (own) derivative of acceleration on right end of trajectory. Into basis \( I \) the quaternion \( \Lambda(t) \) is defined by the production

\[
\Lambda(t) = \Lambda_0 \odot \Lambda_1(t) \odot \Lambda_2(t) \odot \Lambda_3(t) \odot \Lambda_4(t) \odot \Lambda_5(t) \odot \Lambda_6(t),
\]

where \( \Lambda_{k}(t) = (\cos(\varphi_k(t)/2), \sin(\varphi_k(t)/2) e_k) \); \( e_k \) is unit of Euler axis by \( k \)'s rotation, and functions \( \varphi_k(t) \) present the elementary rotation angels in analytical form. These functions are selected in class of splines by relative degree. At initial denotations for vectors \( \omega^{(1)}(t) = \omega_1(t), \varepsilon^{(1)}(t) = \varepsilon_1(t) \) \( \dot{\varepsilon}^{(1)}(t) = \dot{\varepsilon}_1(t) \) vector of angular rate \( \omega(t) \), vectors of angular acceleration

Fig. 2. Coordinates of the SC and the GMC at the programmed rotation maneuver: a — without a limit of the SC angular rate in a position transfer; b — with such limit.
\[ \omega_k^{(k)}(t) = \dot{\Lambda}_k(t) \omega^{(k-1)}(0) \lambda_k(t), \]
\[ \omega_{\epsilon}^{(k)}(t) = \dot{\Lambda}_k(t) \epsilon^{(k)}(t) \]
\[ \epsilon_k^{(k)}(t) = \dot{\epsilon}_k(t) + \epsilon_k^{(k)}(t) \times \omega_k(t); \quad (9) \]

and in result quested vectors are obtained in the form \( \omega(t) = \omega^{(0)}(t), \epsilon(t) = \epsilon^{(0)}(t), \dot{\epsilon}(t) = \dot{\epsilon}^{(0)}(t). \)

In (9) a module of a motion rate in a position transfer \( k=3 \) may be limited (Somov and Butyrin, 2003). The technique is based on the generalised integral's properties for the AM of the mechanical system "SC+GMC" and allows to evaluate vectors \( \beta(t), \dot{\beta}(t), \ddot{\beta}(t) \) in the analytical form for an arbitrary preassigned SC motion \( \Lambda(t), \omega(t), \dot{\omega}(t), \ddot{\omega}(t) \forall t \in T_1. \)

Let be \( \mathbf{g}(t) = k(t) + h^{(c)}(t) = \dot{\Lambda}(t) \mathbf{g}^c(t) \dot{\Lambda}(t), \)
\[ \mathbf{k}(t) = J \omega(t) \mathbf{h}_0 \quad \text{and} \quad \mathbf{g}^c(t) = \Lambda(t) \mathbf{g}(t) \dot{\Lambda}(t). \]

By the DL (3) there is derived the principle relation
\[ \delta = d \left( 1 - \frac{1}{e^2} \right)^{1/2} \rho, \]
where \( d, \rho \) are parameters of nonlinearity.

Then there are computed:
\[ \mathbf{h}^{(c)}(t) = \mathbf{g}(t) - \mathbf{k}(t) \implies \beta(t); \]
\[ \mathbf{g}^{(b)}(t) = -\omega(t) \times \mathbf{g}(t); \]
\[ \mathbf{h}^{(b)}(t) = \mathbf{g}^{(b)}(t) - \mathbf{k}(t) \implies \beta(t). \]

These spline onboard algorithms ensure the desirable profile smoothness for the SC motion with small level of its flexible structure oscillations. Fig. 2 briefly presents the SC programmed rotation maneuver on time interval \( t \in [0, \infty) \) sec with boundary conditions, represented in Somov and Butyrin (2002a). At low part of this figure the precession angular rate \( \dot{\beta}(t) \) values are presented for all four gyrolines.

6. NONLINEAR ROBUST CONTROL

If the error \( \delta \dot{\omega} \equiv \dot{\omega} \) in the rate vector \( \dot{\omega} \) is defined as \( \dot{\omega} = \omega - \dot{C}_e \omega^{(b)} \mathbf{p}(t), \) and the GMC's required control torque vector \( \mathbf{M}^R \) is formed as \( \mathbf{M}^R = \omega \times \mathbf{G}^{(b)} + J(\mathbf{C}_e \omega^{(b)}(t) - \omega \times [\mathbf{C}_e \omega^{(b)}(t) + \mathbf{m}]), \) then the simplest nonlinear model of the SC's attitude error is as follows:

\[ \dot{\epsilon}_0 = -\mathbf{e} \cdot \dot{\omega}/2; \quad \dot{\epsilon} = \mathbf{Q}_e \dot{\omega}/2; \quad \dot{\omega} = \dot{\mathbf{m}}. \quad (10) \]

By the relations \( \mathbf{Q}^{-1}_e \mathbf{Q}^b_e = \mathbf{C}_e; \mathbf{Q}^{-1}_e = \mathbf{Q}^b_e + \mathbf{e} \cdot \mathbf{e}^T/\epsilon_0; \)
\[ \mathbf{Q}^{-1}_e \mathbf{e} = \epsilon_0 \mathbf{e}; \mathbf{I}_3 = \epsilon_0 \mathbf{Q}^{-1} \mathbf{e}^{(b)} = \mathbf{Q}^{(b)}_e [\mathbf{e}^T] = \mathbf{Q}^{(b)}_e [\mathbf{e}], \]
which are used for \( \epsilon_0 \neq 0 \) (Somov, 1997), for model (10) a non-local nonlinear coordinate transformation is defined and used at analytical synthesis by the exact feedback linearization. This results in the NCL

\[ \dot{\mathbf{m}}(\mathbf{E}, \dot{\omega}) = -\mathbf{A}_0 \cdot \mathbf{e} \cdot \text{Sgn}(\epsilon_0) - \mathbf{A}_1 \cdot \dot{\omega}, \quad (11) \]

where \( \mathbf{A}_0 = (2\alpha_0 - \omega^2/2)/\epsilon_0 \mathbf{I}_3; \mathbf{A}_1 = \epsilon_0 \mathbf{I}_3 - \mathbf{R}_{uw}. \)
\[ \text{Sgn}(\epsilon_0) = (1, \text{if} \epsilon_0 \geq 0) \vee (-1, \text{if} \epsilon_0 < 0), \]
\[ \mathbf{R}_{uw} = (\epsilon_0 \mathbf{Q}^T_e [\mathbf{e}^T])/(2\epsilon_0), \]
and constants \( \alpha_0, \alpha_1 \) are analytically calculated on spectrum \( \mathbf{S}^\alpha_{ce} = -\alpha_0 \pm j\alpha_1. \)

Fig. 3 and Fig. 4 present some results on computer simulation of a ACS for Russian remote sensing SC by the Resource-DK type. Here the rate errors are represented at consequence of the SC spatial rotational maneuver for time \( t \in [0, 45] \) sec and the SC course motion for time \( t \in [45, 90] \) sec with a nearly-constant vector of acceleration \( \epsilon(t). \) Applied digital nonlinear control law is flexible switched at the time \( t = 45 \) sec on astatic ones with respect to the acceleration.

7. COMPUTER SIMULATION
8. CONCLUSIONS

Contemporary approaches and some new results were presented for nonlinear robust ACSs applied at the agile remote sensing SC.

REFERENCES


