A NOTE ON THE PASSIVITY–BASED CONTROL OF SWITCHED RELUCTANCE MOTORS

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Abstract: In this paper it is shown that the controller presented in (Espinosa-Perez et al., 2004) for Switched Reluctance Motors can be improved, preserving its passivity-based structure, by eliminating the necessity of solving a transcendental equation to compute the desired behavior of stator currents when nonlinear magnetic circuits are considered in the machine model. This advantage in the controller design is obtained by considering the alternative expression for the nonlinear relationship between fluxes and currents reported in (Vedagarbha et al., 1997) that, as well as the previously used, includes the saturation phenomenon presented in the motor windings. Regarding the controller structure, the main implication of introducing this modification lies in the fact that it states the possibility of implementing a control law for a general class of models for this kind of motors. The usefulness of the proposed controller is evaluated by digital simulations comparing it with that obtained when a simplified model is considered.

Keywords: Passivity-based control, Switched-reluctance motors, Nonlinear Magnetic circuits.

The importance of Switched Reluctance Motors (SRM) is a widely accepted fact due to its simple structure and its ability to produce high torque at low speeds, characteristics that are very attractive in direct–drive applications (e.g. robotics) since the use of gear boxes is eliminated. These advantages, well-recognized by the drives community (see (Krishnan, 2001) and references therein), motivated the control theory community to approach the control problem of this kind of devices by applying several nonlinear controllers, e.g. feedback linearizing (Ilic-Spong et al., 1987), sliding mode (Yang et al., 1996), backstepping (Ağırman et al., 1999) and passivity–based (Espinosa-Perez et al., 2000) control.

The main idea in the proposition of nonlinear controllers for SRM, has been to improve their dynamic performance by including in the motor model, considered in the controller design, the nonlinear behavior exhibited in the machine during basic operation, namely, magnetic saturation in the motor windings. In this sense, although precise models are currently available to describe this behavior (Ilic-Spong et al., 1987), due to the difficulty to work with them (historically) a lot of efforts were dedicated to the proposition of control

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strategies based in simplified models, as those mentioned in the previous paragraph, although it is also possible to identify a limited number of publications that deal with the control of the complete model (Filicori et al., 1993), (Vedagarbha et al., 1997), (Espinosa-Perez et al., 2004).

Unfortunately, the controller design based in the complete model for SRM has produced results that show some drawbacks. Namely, even that in (Filicori et al., 1993) a novel modelling methodology is applied to characterize the nonlinear nature of the relationship between fluxes and currents, the disadvantage of the result appears in the necessity of dealing with singularities in the proposed feedback linearizing controller. Although in (Vedagarbha et al., 1997) a singularity-free adaptive (with respect to parameters uncertainty) backstepping controller is presented, the obtained control law exhibits a high computational complexity. In (Espinosa-Perez et al., 2004), the feature of proposing a control scheme with a clear physical structure (that leads to an easy to tune controller), became unworthy due to the requirement for solving a transcendental equation to compute some controller variables.

In this brief note the controller reported in (Espinosa-Perez et al., 2004) is re-visited. Since the passivity-based methodology followed in its design showed to be useful for the case of considering a SRM simplified model, the aim of this paper is to remove the main obstacle that appears in the application of this technique to the complete model of this kind of machines. In particular, the problem approached is related with the well-known limitation of the passivity-based controller design (Ortega et al., 1998) of requiring to carry out some kind of system inversion. For SRM, this requirement is stated as the necessity to obtain an expression for stator currents in terms of the generated torque, problem whose solution strongly depends on the chosen model for the motor fluxes since, from the D'Alembert principle (Meisel, 1961), it is well-known that the structure of generated torque is obtained from the fluxes one.

In the case of the controller reported in (Espinosa-Perez et al., 2004), the structure considered to describe the behavior of the motor fluxes was originally presented in (Ilic-Spong et al., 1987). The main feature of the proposed flux model is that it fits with a high accuracy degree the actual behavior of these variables. Unfortunately, the obtained analytical expression produces a generated torque model that is not invertible, i.e. it is not possible to obtain a closed expression of stator currents in terms of torque. In this context, the main contribution of this paper is to show that this drawback can be eliminated by considering the alternative flux representation has the advantage that the derived expression for generated torque is invertible while, as well as the used in (Ilic-Spong et al., 1987), recovers very closely the actual fluxes behavior. The importance of introducing this modification on the motor model used in the controller design lies in the fact that it is possible to implement a passivity-based control, preserving all its attractive features, for the SRM complete model, fact that in its turn implies an improvement in the motor performance when compared with the simplified-based controller.

The rest of the paper is organized as follows: In section 1 the considered model of the motor, including the flux representation, is presented. The main result of the paper, an implementable passivity-based controller for the SRM complete model, is developed and numerically evaluated in section 2. A comparison between the proposed (complete) and the simplified controllers, via digital simulations, is carried out in section 3. Section 4 is dedicated to include some concluding remarks.

1. SRM COMPLETE MODEL

The usual configuration of a SRM-based drive, depicted in Fig 1, includes the electric motor itself and a static power converted (electronic commutator). The purpose of this electronic device is to excitate (by injecting electric current) each of the phases of the motor, three in the particular case of the illustration, in order to produce mechanical torque by mean of a reluctance action, i.e. one identical to the observed when two displaced opposite polarized magnets are aligned.

The main characteristics of this energy conversion mechanism are, on the one hand, that in order to maximize torque the windings must be saturated, giving as a result a highly nonlinear dynamic behavior for the drive, and on the other hand, that there exist some ripple in the mechanical variables, due to the presence of the switching device. While dealing with the nonlinear nature of the system imposes the main challenge in the control of this kind of machines, the disruption in generated torque, and then in position and speed, can be significatively reduced if smooth transition between phases is accomplished by blending the applied currents to two adjacent windings, i.e. by following a torque sharing approach (Taylor, 1992).

Under the conditions described above and considering (without loss of generality) a 3φ SRM, it is widely accepted that the dynamical behavior of the machine can be represented by the set of differential equations given by

\[
\phi = \frac{1}{2} J \frac{d^2 \theta}{dt^2}
\]
\[ \dot{\psi}_j + r\dot{i}_j = u_j; \quad j = 1, 2, 3 \]  
\[ J\ddot{\theta} = T_c(\theta, i_1, i_2, i_3) - T_L(\theta, \dot{\theta}) \]  
(1)  
(2)

where \( u_j \) is the voltage applied to the stator terminals of phase \( j \), \( i_j \) is the stator current of phase \( j \), \( \psi_j(\theta, i_j) \) is the flux linkage of phase \( j \), \( r \) is the stator winding resistance, \( \theta \) is the angular position of the rotor, \( T_L(\theta, \dot{\theta}) \) is the load torque and \( J \) is the total rotor and load inertia. The mechanical torque of electrical origin \( T_c(\theta, i_1, i_2, i_3) \), which depends on both the angular position of the rotor and stator currents, is given as the sum of torques \( T_j(\theta, i_j) \) produced by each one of the three phases, i.e.

\[ T_c(\theta, i_1, i_2, i_3) = \sum_{j=1}^{3} T_j(\theta, i_j) \]  
(3)

where each single torque is given by

\[ T_j(\theta, i_j) = \frac{\partial W_j'(\theta, i_j)}{\partial \theta} \]  
(4)

with \( W_j'(\theta, i_j) \) the magnetic co-energy function of each winding, computed as

\[ W_j'(\theta, i_j) = \int_0^{i_j} \psi_j(\theta, i_j) di_j \]

In this paper, the considered structure for the flux linkage \( \psi_j(\theta, i_j) \) is the proposed in (Vedagarhba et al., 1997) given by

\[ \psi_j(\theta, i_j) = \psi_s \arctan(\beta f_j(\theta) i_j); \quad i_j \geq 0 \]  
(5)

where \( \psi_s \) is the saturated flux linkage, \( \beta \) is an experimentally obtained positive constant and \( f_j(\theta) \), known as the winding inductance, is a strictly positive periodic function of the form

\[ f_j(\theta) = a + \sum_{m=1}^{\infty} \left\{ b_m \sin[m N_r \theta - (j - 1)2\pi/3] + c_m \cos[m N_r \theta - (j - 1)2\pi/3] \right\} \]  
(6)

where \( N_r \) is the number of rotor poles. Hence, the resulting structure for each single torque is

\[ T_j(\theta, i_j) = \frac{\psi_s}{2\beta f_j^2(\theta)} \frac{\partial f_j(\theta)}{\partial \theta} \ln \left(1 + \beta^2 f_j^2(\theta) i_j^2\right) \]  
(7)

which clearly illustrates the complexity and nonlinear nature of the developed model.

**Remark.** Contrary to the flux linkage model used in (Espinosa-Perez et al., 2004), first reported in (Ilic-Spong et al., 1987), equation (5) has the great advantage that the derived expression for generated torque, given in (7), is invertible, i.e. it is possible to obtain an expression for the motor currents in terms of the mechanical torque. In addition, this model also captures the physical features of the motor, namely, the maximum value that the flux can reach is the saturation value \( \psi_s \), the torque sign is only determined (if stator currents are positive) by the variation of \( f_j(\theta) \) with respect rotor position \( \theta \) and both flux linkage and generated torque in each phase are zero if no current is applied to stator windings.

In order to complete the mathematical structure of the SRM-based drive, following ideas reported in (Taylor, 1992), taking into account the torque sharing objective and exploiting the fact that the torque sign is only determined by the variation of the inductance \( f_j(\theta) \), it is possible to formulate the electronic commutator in the following way:

Given two sets

\[ \Theta^+_j = \{ \theta : \frac{\partial f_j(\theta)}{\partial \theta} \geq 0 \} \quad \Theta^-_j = \{ \theta : \frac{\partial f_j(\theta)}{\partial \theta} < 0 \} \]

where the superscript + and − stand for required positive and negative torque, respectively, choose any functions \( m^+_j \) and \( m^-_j \) such that

\[ m^+_j(\theta) > 0 \quad \forall \theta \in \Theta^+_j; \quad \sum_{j=1}^{3} m^+_j(\theta) = 1 \quad \forall \theta \]  
(8)

\[ m^-_j(\theta) > 0 \quad \forall \theta \in \Theta^-_j; \quad \sum_{j=1}^{3} m^-_j(\theta) = 1 \quad \forall \theta \]

Then, these sharing functions can scale each phase torque in expression (3) in order to generate a total desired torque by assigning

\[ m_j(\theta) = \{ m^+_j(\theta), T_d \geq 0 \} \quad \{ m^-_j(\theta), T_d < 0 \]  
(9)

with \( T_d \) the desired torque to be delivered.

## 2. MAIN RESULT

In this section, the main result of the paper is presented, namely, a passivity-based control for SRM complete model. Since the passivity properties of the model do not depend on the flux structure, as has been stated in (Espinosa-Perez et al., 2004), then it can be given as a fact that the methodology developed in this last reference can be applied and hence the proposed controller is directly presented in the following proposition. However, before doing this presentation, it is convenient to re-write the complete model (1-2) in the equivalent form given by

\[ \mathbf{D}(\theta, \dot{i}) \ddot{i} + \mathbf{C}(\theta, \dot{i}) \dot{i} + \mathbf{R} i = \mathbf{u} \]  
\[ J\ddot{\theta} = T_c(\theta, i) - T_L(\theta, \dot{\theta}) \]  
(10)

with \( \mathbf{u} = [u_1, u_2, u_3]^T \), \( i = [i_1, i_2, i_3]^T \), \( \mathbf{R} = r\mathbf{I}_3 \), \( \mathbf{I}_3 \) the identity matrix, \( \psi = [\psi_1, \psi_2, \psi_3]^T \), \( T_c(\theta, i) \) as in (3), while
\[ D(\theta, i) = diag \left\{ \frac{\psi_s \beta f_j(\theta)}{1 + \beta f_j^2(\theta) i_j^2} \right\}; \quad j = 1, 2, 3 \]

and

\[ C(\theta, i) = diag \left\{ \frac{\psi_s \beta \partial f_j(\theta)}{1 + \beta f_j^2(\theta) i_j^2} \right\}; \quad j = 1, 2, 3 \]

**Remark.** Notice that under the assumed machine operation with non-negative currents, each entry of \( D(\theta, i) \) is strictly positive for every bounded current. Hence, \( D(\theta, i) \) is strictly positive definite.

Once the alternative representation above has been introduced, the presentation of the proposed controller is given in the next

**Proposition.**

Consider the complete model of a SRM given by (10) and (3) in closed loop with the control law

\[ u = D(\theta, i) \frac{di_d}{dt} + C(\theta, i) \dot{\theta} i_d + R i_d - K_v (i - i_d) \]  

where \( i_d = [i_{1d}, i_{2d}, i_{3d}]^T \) is the desired current behavior and \( K_v = diag\{K_{1v}, K_{2v}, K_{3v}\} \) is such that

\[ C(\theta, i) \dot{\theta} + R + K_v > 0 \]  

The desired current behavior is computed as

\[ i_{jd} = \begin{cases} 
i_{jd}^+ & \text{if } \frac{\partial f_j(\theta)}{\partial \theta} \neq 0 \\ 0 & \text{otherwise} \end{cases} \]  

where

\[ i_{jd}^+ = \frac{1}{\beta f_j(\theta)} \sqrt{\exp \left( T_{d0}(\theta, i_j) 2 \beta f_j^2(\theta) \right) - 1} \]

with \( j = 1, 2, 3 \) and \( T_{d0} \) related to the desired torque \( T_d \) by means of the \( m_j(\theta) \) functions as

\[ T_{jd} = m_j(\theta) T_d; \quad j = 1, 2, 3 \]

The desired torque is related to the speed error \( \dot{\hat{\theta}} = \dot{\theta} - \dot{\theta}_d \) by

\[ T_d(z) = J \ddot{\hat{\theta}} - z + T_L(\theta, \dot{\theta}) \]  

with controller state

\[ \dot{z} = -az + b\dot{\hat{\theta}} \]  

and \( a, b \) positive constants.

Under these conditions, asymptotic speed tracking is insured, i.e. \( \lim_{t \to \infty} \theta = 0 \), with all internal signals bounded.

**Proof.** The proof closely follows the presented in (Espinosa-Perez et al., 2004). If the current error is defined as \( e = i - i_d \), model (10) can be equivalently written as

\[ D(\theta, i) \frac{de}{dt} + C(\theta, i) \dot{\theta} e + Re = \Phi \]  

where

\[ \Phi = u - \left\{ D(\theta, i) \frac{di_d}{dt} + C(\theta, i) \dot{\theta} i_d + Ri_d \right\} \]

Then, considering the proposed controller (12), expression (17) takes the form

\[ D(\theta, i) \frac{de}{dt} + C(\theta, i) \dot{\theta} e + Re = 0 \]

Since \( D(\theta, i) \) is a strictly positive definite matrix, this last equation can be re-arranged as

\[ \frac{de}{dt} = -D^{-1}(\theta, i) \left[ C(\theta, i) \dot{\theta} + R + K_v \right] e \]

which, due to the diagonal structure of the matrices, defines a set of three decoupled linear time-varying differential equations of the form

\[ \frac{de_j(t)}{dt} = -a_j(t)e_j(t); \quad j = 1, 2, 3 \]

If inequality (13) holds, then \( a_j(t) \) is bounded and always positive. Thus it is insured exponential convergence of the current error to zero.

Consider now the following definition, motivated by (7), for the desired generated torque

\[ T_{jd}(\theta, i_{jd}) = \frac{\psi_s}{2 \beta f_j^2(\theta)} \frac{\partial f_j(\theta)}{\partial \theta} \ln (1 + \beta^2 f_j^2(\theta) i_{jd}^2) \]

which is actually the expression that determines the structure of the current \( i_{jd} \) given by (14). Writing the difference between this variable and the generated phase torque as

\[ T_j - T_{jd} = \frac{\psi_s}{2 \beta f_j^2(\theta)} \frac{\partial f_j(\theta)}{\partial \theta} \left\{ \ln (1 + \beta^2 f_j^2(\theta) i_{jd}^2) - \ln (1 + \beta^2 f_j^2(\theta) i_{jd}^2) \right\} \]

it is shown that \( e \) tending to zero implies torque convergence due to the fact that the term in brackets on the right hand size in the above expression tends to the unity.

The final step in the proof is to show that speed error also tends to zero and that the internal stability of the overall system is guaranteed. This can be done by following the procedure developed in (Ortega et al., 1996) for the case of induction motors. 

\( \triangle \triangle \triangle \)

The following remarks are in order about the presented result:

- It is important to notice that expression (14) is always well-posed. This is due to the natural operation of the motor, namely, when the desired torque is negative then the current must be applied when \( \frac{\partial f_j(\theta)}{\partial \theta} \) is negative.
and vice versa. This behavior gives as a result a positive argument of the exponential function, which in its turn, guarantees that term into the radical could not be negative.

- In general terms, the matrix $K_v$ will depend on the rotor position $\theta$, the rotor speed $\dot{\theta}$ and stator currents $i$. In fact, in the simplest way, this matrix can be chosen as $K_v = C(\theta, 1)\dot{\theta} + K_v$ with $K_v = K_v^T > 0$.
- Although the developed controller was specialized on speed control, it must be noticed that the torque control problem is also solved when it is shown that the difference between the actual and the desired torque tends to zero. Moreover, as pointed out in (Espinosa-Perez et al., 2004), the position control problem can also be solved by defining

$$T_d(z) = \dot{J}\ddot{\theta}_d - z - f\dot{\theta} + T_L(\theta, \dot{\theta})$$

where $f > 0$ and $\ddot{\theta} = \theta - \theta_d$ is the position error.

The performance of the proposed control scheme was investigated by digital simulations. The considered motor parameters are the same as in (Taylor, 1992), $N_s = 4$, $l_0 = 30 mH$, $l_1 = 20 mH$, $r = 5 \Omega$, $J = 10^{-3} kg - m^2$ while $\psi_q = 0.25$ and $\beta = 0.6$. The load torque, for simplicity of presentation, was set to zero. With the aim to illustrate the global properties of the control, the motor was initially at standstill. Regarding the torque generation mechanism, the sharing functions were designed with a structure based on polynomial functions of the mechanical position $\theta$ as the sum of a raising polynomial (fifth order), a constant unitary function and a falling polynomial (fifth order). The structure of the raising one is

$$p_r(h) = 10\frac{h^4}{\theta_m^4} - 15\frac{h^4}{\theta_m^4} + 6\frac{h^5}{\theta_m^5}$$

where $\theta_m = \frac{\pi}{12}$ and $h = |\theta - \alpha \theta_m|$ with $\alpha = \text{int} \left( \frac{\theta}{\theta_m} \right)$. The falling polynomial is given by $p_f(h) = 1 - p_r(h)$.

Figure 2 shows the speed behavior when a square wave of $\pm 25 rad/sec$ of amplitude was used as a speed reference. The electric gain was set to $K_{\nu j} = 100$ while filter values were varied as $a = 100, 150, 200$, $b = 10$ each cycle of the reference. In this figure it can be observed how the mechanical transient response is improved as the damping injected by the controller is increased. On the other hand, to illustrate the electrical performance, in Figure 3 the current error for phase one is shown. Also concerning with the electrical performance, in Figure 4 it is shown the actual behavior for the torque error.

3. COMPLETE VS SIMPLIFIED CONTROL

With the aim to obtain a quantitative answer about the advantage of using the complete model-based controller with respect to the simplified one, several numerical experiments were carried out considering the controller proposed in this paper and the presented in (Espinosa-Perez et al., 2004). The first feature of them was that both control laws were implemented using the complete model (1-2). Since both controllers have equivalent parameters, tuning was carried out in an exactly way. Specifically, electric gain was $K_{\nu j} = 100$ while $a = 200$ and $b = 10$. The indices of evaluation included variables such that transient response, supplied and dissipated energy, steady state error among other. Due to space limitations, the whole evaluation can not be included in this document, a complete analysis will be reported elsewhere, but to illustrate this claim in Figure 5 it is presented the speed behavior, for both controllers, when the reference was set to $25 [rad/sec]$. It can be observed in this figure how the transient response is clearly improved with the complete control law. Moreover, it must be noticed that if the simplified one introduces a non zero steady state error. To confirm the superiority of the controller proposed in this paper, in Figure 6 the integral of the square speed error is presented. Again, a remarkable advantage of considering in the controller design the complete model is evident.

4. CONCLUDING REMARKS

The control problem of a saturated nonlinear model for SRM was approached in this paper. The main result is related with the proposition of a passivity-based control law that solves the torque/speed/position tracking control problem without assuming a linear relationship between fluxes and currents. The result was obtained due to the use of the flux model proposed in (Vedagarbha et al., 1997) which has the advantage to generate an invertible constitutive relationship for the generated torque in terms of motor currents. The performance achieved with the proposed controller is remarkable according to the results obtained in the presented digital simulation evaluation. Moreover, it has been shown, also via numerical evaluation, that this new control law is better than the usual scheme developed for the SRM simplified model.

REFERENCES

Fig. 1. Electronic commutator of a 3φ SRM

Fig. 2. Speed behavior with increasing damping

Fig. 3. Current error

Fig. 4. Torque error

Fig. 5. Regulation speed for complete and simplified control

Fig. 6. Integral of the square speed error for complete (solid-line) and simplified (dashed-line) control


