Abstract: In web transport systems, the main concern is to independently control speed and tension in spite of perturbations such as radius variations and changes of setting points. So far, simulation models representing web handling systems have only dealt with continuously rolling web without considering friction between web and rollers. This article aims at introducing the modeling of friction as well as the risk for the web to slide on traction rollers. The influence of several physical factors on the occurrence of such sliding is studied. In this paper, we focused on analyzing the special case of an emergency shutdown. The authors observed and analyzed the consequences of several controllers on the occurrence or not of sliding. An approach for defining the optimal command to decrease the speed as fast as possible in the case of an emergency shutdown without occurrence of sliding is proposed. Copyright © 2005 IFAC

Keywords: winding systems, large scale systems, friction, sliding, slip, reference optimization.

1. INTRODUCTION

Web handling systems are very common industrial production plants used to transport materials such as textile, paper or polymers. Although modeling and control of web handling systems have been studied for several decades, increasing requirements on performances has lead to more sophisticated mechanical and control models. The most common goal is to increase web velocity as much as possible, while controlling web tension over the entire production line. This requires decoupling between web tension and speed, so that a constant tension can be maintained during speed changes. So far, many industrial web transport systems have used decentralized PI-type controllers (as illustrated in Fig.2). However, for higher control requirements more efficient control strategies must be used, e.g. LQG or $H_\infty$ solutions. Most modern control law designs need the elaboration and validation of the plant model. A detailed description of the model we use being given in (Koç et al., 2002), we will only retain the principal laws on which it is based. Robust control has already been applied to web handling for reduced-size systems, containing no more than 3 motors, with multivariable $H_\infty$ centralized controllers (Koç et al., 2002). Due to the wide-range variation of the roller radius during the winding process, the dynamic behavior of the system changes considerably with time. Thus, a substantial improvement has been obtained by the use of LPV controllers (Koç et al., 2002). So far, simulation models representing large scale web handling systems do not consider friction between web and rollers. The present study describes the introduction of friction and the possibility of sliding in the existing web handling simulation models. Accounting for friction and sliding in the
web handling systems can lead to performance deterioration in comparison to a system without friction. The models used for simulations were validated on a three-motor setup.

Part 2 shows the development of the nonlinear model dedicated to the bench simulation. Its linearization is used for controller design. The synthesis of the controller is summarized in part 3. The modeling of friction of the master traction roll is explained in part 4. The robust control without sliding is developed in part 5. The different cases of sliding occurrence are analyzed in part 6. Part 7 will conclude the paper.

2. PLANT MODELING

A scheme of a three-motor setup with PI controllers is represented on Fig.1. The inputs to the system defined by the dashed box are the torque reference signals \( u_u, u_e, u_w \) of the brushless motors; its measurements are the web tensions \( T_u \) and \( T_e \) and the web velocity \( V \). This velocity is imposed by the master traction motor whereas the web tension is controlled by the unwinding and winding motors.

![Fig. 1. Experimental setup with 3 motors and 2 load cells](image)

**Fig. 1.** Experimental setup with 3 motors and 2 load cells

The nonlinear model (Koç et al., 2002) of a web transport system is built from the equations of web tension behavior between two consecutive rollers and the equations describing the velocity of each roll.

A. Web tension calculation:

Modeling of web transport systems is based on three laws, which allow the calculation of web tension between two rolls.

1) As stated by Hooke’s law, the tension \( T \) of an elastic web is function of the web strain \( \varepsilon \):

\[
T = E S \varepsilon = E S \frac{L - L_0}{L_0}
\]

where \( E \) is the Young elasticity modulus, \( S \) the web section, \( L \) the web length under stress and \( L_0 \) the web length without stress.

2) According to Coulomb’s law, the study of a web tension on a roll can be considered as a problem of friction between solids.

3) Equation of Continuity applied to the web gives:

\[
\frac{d}{dt} l T_{k+1} = E S (V_{k+1} - V_k) + T_k (V_{k+1} - V_k) = 2 (V_k - V_{k+1})
\]

B. Web velocity calculation:

The linear velocity \( V_k \) of roll \( k \) is obtained from the torque balance (Koç et al., 2002):

\[
\frac{d}{dt} \left[ J_k \frac{V_k}{R_k} \right] = R_k (T_k - T_{k-1}) + K_k U_k + C_f
\]

in this equation \( K_k U_k \) is the motor torque assumed equal to the reference value and \( C_f \) is the friction torque. Note that both inertia \( J_k \) and radius \( R_k \) of unwinder and winder are time dependent and vary substantially during processing.

C. State space representation (Koç et al., 2002):

The nonlinear state-space model is made of eq. (2) for the different web spans and eq. (3) for the different rolls. Under the assumption that \( J_k / R_k \) is slowly varying, which is the case for thin webs, \( V_k \) can be chosen as state variable in (3), leading to the following linear model (Koç et al., 2002):

\[
E(t) \frac{d}{dt} X = A(t) X + B(t) U \quad Y = C X
\]

where:

\[
Y = [ T_u \ V_3 \ T_e ]^T, \quad U = [ u_u \ u_e \ u_w ]^T
\]

\[
X = [ V_1 \ T_1 \ V_2 \ T_2 \ V_3 \ T_3 \ V_4 \ T_4 \ V_5 ]^T
\]

3. GAIN SCHEDULING CONTROL

If we consider the unwinder and the winder separately, while keeping a quasi-static assumption on radius variations, the transfer function between control signal and web tension appears to be inversely proportional to the radius:

\[
\lim_{s \to 0} \frac{T_1(s)}{u_u(s)} \approx \frac{K_u}{R_u} \quad \lim_{s \to 0} \frac{T_4(s)}{u_w(s)} \approx \frac{K_w}{R_w}
\]

Based on this observation, a new plant is obtained by multiplying the controller output signals \( u_u \) and \( u_w \) by the radius \( R_u \) and \( R_w \) respectively. This new plant has the advantage of making the gain at low frequency less dependent on radius and inertia. Therefore, gain scheduling improves robustness to radius variations; it is used in control strategies presented in the sequel.

4. MODELISATION OF ADHESION AND SLIDING OF THE WEB

Bastogne (2002) proposed a model representing the occurrence of sliding between a web and a single roll. Using the same procedure considering the web as a rigid body, we propose here a new formulation of the acceleration, and from the same local equations, a different way of determining whether there is sliding or not, and then, the implementation of this model in a more complex system: our three motors bench simulator.

Considering an elementary part of the web of angular value \( \theta \):

In this paper, a new approach using the combination of equations (7), (8) and (9) leads to:

\[
\frac{dT}{\mu[T(\theta) - \lambda]} = d\theta
\]

which gives:

\[
T(\theta) = (T_u - \lambda) \mu e^{\mu\theta} + \lambda
\]

where:

\[
\lambda = \frac{\rho AR}{\mu} \left[ R(S_r + S_s)^2 - R(S'_r + S'_s) \right]
\]

\(A\) is the area of the web section and \(\rho\) the density.

The evolution of the web tension is not linear but now exponential. From equation (12), it is possible, as \(T_u\) (tension for \(\theta = \alpha\)) and \(T_s\) (tension for \(\theta = 0\)) are input of the problem, to calculate a theoretical value of \(\mu\) : \(\mu_0\).

We chose to use the idea of a logical treatment introduced by (Bastogne, 2002) to determine whether the web is sliding on the master roll or not, but in our case the decision is made on the value of \(\mu_0\) instead of comparing the value of \(F_i\) to border values as made by Bastogne (2002).

The sliding state is represented by a logical parameter \(G(t)\), which is equal to 1 when there is sliding and to 0 otherwise. The logical equation governing the sliding / adhesion variable \(G(t)\) is:

\[
G(t) = \{ \text{(not } G(t-1)\} \text{ and } |\mu_0(t)| \geq \mu \} \text{ OR } [G(t-1)\text{ and } |S(t)| > \Delta]\]

Equation (7) gives:

\[
dF_n(t) = T(\theta)d\theta - \rho AR\left[R(S_r + S_s)^2 - g \cos \theta \right]d\theta
\]

and by integrating from \(\theta = 0\) to \(\theta = \alpha\), we obtain the resultant forces:

\[
F_n = \frac{T_u - \lambda}{\mu} (e^{\mu\alpha} - 1) + \lambda\alpha
\]

\[-\rho AR \left[S_r + S_s \alpha^2 - g \sin \alpha \right]
\]

the relationship (9) gives \(F_s\).

The evolution of \(G\) as a logical value governing the behaviour of the web (Bastogne 2002) generates discontinuities of physical values and therefore introduces instability in the system. It is important to implement a continuous equation (9) as made in the paper of Schraberger and Brandenburg (Schraberger et al., 2001). As a consequence, we use here a basic friction model where we considered that \(\mu = \mu_0\). A more sophisticated friction model may be necessary to account for an accurate behaviour in the sliding area.

Papers of Brandenburg and Schraberger (Schraberger et al., 2001), modelling the Stribeck effect, set the basis of a further implementation of the sliding behaviour. The integration of equation (8) gives:

\[
F_i = -\frac{T_u}{\alpha} + \frac{T_w}{\alpha} + \rho AR\left(-g(\cos \alpha - 1)\right)
\]

\[-R(S_r + S_s)\]

In equation (11), the evolution of the web tension is not linear but now exponential. From equation (12), it is possible, as \(T_u\) (tension for \(\theta = \alpha\)) and \(T_s\) (tension for \(\theta = 0\)) are input of the problem, to calculate a theoretical value of \(\mu\) : \(\mu_0\).

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\[
F_i = -\frac{T_u}{\alpha} + \frac{T_w}{\alpha} + \rho AR\left(-g(\cos \alpha - 1)\right)
\]

\[-R(S_r + S_s)\]
The possibility of sliding was neutralized.

As a result of this new model, it is now possible to detect the occurrence of sliding and to calculate the sliding speed $S$.

This model has been implemented into the three motors setup. The friction factor has been approximated to $\mu = 0.25$. Some experiments are currently carried out to identify experimentally the friction effects. The structure of the 3-motors setup simulator in Matlab/Simulink environment is presented in the Appendix.

5. ROBUST CONTROL WITHOUT SLIDING

The coupling between web velocity and tension makes the control of web transport systems inherently difficult. Several methods to suppress this coupling in a system with two driven rolls have been studied (Jeon et al., 1999) (Panda et al., 2002). For the past several years, we have developed 1DOF or 2DOF multivariable strategies based on H\(_\infty\) and LPV using different mixed sensitivity schemes (Koç et al., 2002), (Xu et al., 2003), (Knittel et al., 2003). These methods have significantly improved the tracking properties and quasi suppressed the coupling.

The controller used in the following paragraphs was calculated with H\(_\infty\) optimization using a S/KS/T synthesis scheme with model matching. In figure 4 are shown simulations with two different controller tunings: the one is the controller applied in simulations represented on figure 4, the second used the same synthesis scheme but with a faster reference model. In this figure 5, great oscillations in tensions and speed occur at 15.07s in the case of sliding for the fastest controller. The velocity tracking response of the controller synthesized in part 5 can be improved. In such case, the reference model used for the controller computation has to be faster.

Fig. 5 shows the tensions and velocity responses with two different controller tunings: the one is the controller applied in simulations represented on figure 4, the second used the same synthesis scheme but with a faster reference model. In this figure 5, great oscillations in tensions and speed occur at 15.07s in the case of sliding for the fastest controller.

The simulations are done with the following nominal values: $\rho = 500$ kg/m\(^3\), $E = 160$ Mpa, $\alpha = 3\pi/2$, web thickness = 0.26 mm, web width = 100 mm.

6. STUDY OF SLIDING OCCURRENCE

The occurrence of sliding will be detected in the particular case of a fast decreasing speed. This stands for the industrially very common case of an emergency shutdown. In most cases, when the speed set point command is rapidly set to zero, the web span will slide, and the entire system will be out of control. One of our primary concerns is to keep the control of the system during the entire phase of slow down.

A) Effect of the velocity tracking improvement

The velocity tracking response of the controller synthesized in part 5 can be improved. In such case, the reference model used for the controller computation has to be faster.

Fig. 5 shows the tensions and velocity responses with two different controller tunings: the one is the controller applied in simulations represented on figure 4, the second used the same synthesis scheme but with a faster reference model. In this figure 5, great oscillations in tensions and speed occur at 15.07s in the case of sliding for the fastest controller.

The evolution of the parameter $G$, which represents the sliding, shows that the sliding appears at that time (see Fig. 6).
Fig 6: sliding parameter $G$ in the case of sliding.

In the controller synthesis scheme, the tuning of the reference model allows to find the fastest controller without sliding.

More specific controllers for sliding reducing or avoiding are used in anti-lock brake systems (ABS) for cars (Petersen et al., 2003) (Solyom, 2002).

B) Optimization of the velocity reference slew rate
For a given controller, the occurrence of sliding is determined by the velocity slew rate. The limit slew rate can be found with our simulator, and it is affected by several physical parameters.

C) Influence of several physical parameters
For a given controller, the fastest velocity slew rate avoiding sliding depends mostly on three physical parameters $\rho$, $E$, $\mu$ and a reference $T_w$ (equal to the reference $T_u$ in our simulation case).

Figure 7 shows an example of the relationship between the maximum velocity slew rate and the friction coefficient between web and roll.

Table 1: influence of several physical factors on the limit of deceleration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$(d\Gamma / dx)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0,0004</td>
</tr>
<tr>
<td>$E$</td>
<td>0,0068</td>
</tr>
<tr>
<td>$T_w = T_u$</td>
<td>0,8867</td>
</tr>
<tr>
<td>$\mu$</td>
<td>68,67</td>
</tr>
</tbody>
</table>

Obviously, the density $\rho$ has a very little influence on the deceleration rate. We observed that for a higher Young modulus, the system can be stop faster without sliding. We have also observed that for higher web tension, the system can be shut down faster.

D) Identification method for the friction coefficient
The coefficient of friction will be calculated using the measured values of band tension $T_w$ and $T_u$ and wrap angle $\alpha$ in the case of friction on the three motors experimental setup: $\mu = \frac{1}{\alpha} \ln \left( \frac{T_w}{T_u} \right)$.

7. CONCLUSION

Web winding systems require an efficient control of the tension and the velocity during the whole process. Modern control strategies require the elaboration of an accurate plant model. Unfortunately the different web handling models published in the literature don’t consider the sliding case between web span and driving rolls. This paper presents a 3-motor plant model including a friction sliding model located on the master roll. Sliding occurrences are analysed and investigated. The influence of the velocity deceleration and the controller tracking behavior are particularly focused on. The friction coefficient is the main factor governing the limit of velocity deceleration. Its value and therefore its influences will have to be determined experimentally on our real bench. This will be the subject of future investigations.

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REFERENCES


APPENDIX

Fig 8: Structure of the 3-motor setup simulator in Matlab/Simulink environment