Stability properties for a class of reset systems, such as systems containing a Clegg integrator, are investigated. We present Lyapunov based results for verifying $L_2$ and exponential stability of reset systems. Our results generalize the available results in the literature and can be easily modified to cover $L_p$ stability for arbitrary $p \in [1, \infty]$. Several examples illustrate that introducing resets in a linear system may reduce the $L_2$ gain if the reset controller parameters are carefully tuned.

Abstract: Stability properties for a class of reset systems, such as systems containing a Clegg integrator, are investigated. We present Lyapunov based results for verifying $L_2$ and exponential stability of reset systems. Our results generalize the available results in the literature and can be easily modified to cover $L_p$ stability for arbitrary $p \in [1, \infty]$. Several examples illustrate that introducing resets in a linear system may reduce the $L_2$ gain if the reset controller parameters are carefully tuned. Copyright ©2005 IFAC.

Keywords: Hybrid systems, Lyapunov, nonlinear, stability.

1. INTRODUCTION

It is a well known fact that linear control suffers from certain fundamental performance limitations. These limitations may sometimes be alleviated by nonlinear or hybrid feedback (Feuer et al., 1997). Reset controllers are an example of nonlinear controllers that may overcome some of the fundamental performance limitations of linear controllers (Beker et al., 2001).

Reset controllers are motivated by the so-called Clegg integrator introduced in (Clegg, 1958). This device is a particular type of a nonlinear integrator that operates in the same manner as the linear integrator whenever its input and output have the same sign and it resets its output to zero otherwise (modeling of the Clegg integrator was analyzed in detail in (Zaccarian et al., 2004)). Its describing function has the same magnitude plot as the linear integrator but it has a phase lag of only $38.1^\circ$ compared to the lag of $90^\circ$ for a linear integrator (see (Clegg, 1958) for details). This feature can be used to provide more flexibility in controller design. A more general reset element is the so called First Order Reset Element (FORE). A FORE operates in the same way as the Clegg integrator except that it contains a more general first order linear filter instead of an integrator.

Early designs of reset controllers that use respectively Clegg integrators and FOREs can be found in (Krishman and Horowitz, 1974) and (Horowitz and Rosenbaum, 1975). First attempts to rigorously analyze stability of reset systems with Clegg integrators can be found in (Hu et al., 1997; Hollot et al., 1997). In particular an integral quadratic constraint was proposed in (Hollot et al., 1997)
to analyze stability of reset systems. However, the proposed condition was conservative as it was independent of reset times. Stability analysis of reset system consisting of a second order plant and a FORE was conducted in (Chen et al., 2001) (see also (Chen et al., 2000a)). Stability analysis of general reset systems can be found in (Beker et al., 2004) (see also (Hollot et al., 2001; Chen et al., 2000)). Where Lyapunov based conditions for asymptotic stability of general reset systems were presented. Moreover, the authors proposed computable conditions for quadratic stability based on linear matrix inequalities (LMIs). Bounded-input bounded-state stability of general reset systems was obtained as a consequence of quadratic stability. Finally, an internal model principle was proved for tracking of disturbance rejection.

In this paper we present Lyapunov based conditions for stability of reset systems with Clegg integrators and FOREs. For instance, the results of this paper considerably relax the Lyapunov conditions. For the development of systematic reset controller design procedures. For instance, the results of this paper are used in (Zaccarian et al., 2004) to derive LMI based tools for the construction of piecewise quadratic Lyapunov functions that establish exponential stability of reset systems with Clegg integrators and FOREs. We believe that further such developments will be made possible using the results of this paper.

The paper is organized as follows. In Sections 2 and 3 we present respectively preliminaries and the class of reset systems that we consider. Section 4 contains the main results. Examples are presented in Section 5. Summary and conclusions are given in the last section.

Notation. The sets of positive integers (including zero) and real numbers are respectively denoted as \( \mathbb{N}_0 \) and \( \mathbb{R} \). Given vectors \( x_1, x_2 \) we use the notation \([x_1^T \ x_2^T]^T\). Given an integer \( p \in [1, \infty] \) and a Lebesgue measurable function \( d : [t_1, t_2] \to \mathbb{R}^n \), we use the notation \( \|d(t_1, t_2)\|_{L_2} = \left( \int_{t_1}^{t_2} |d(\tau)|^2 d\tau \right)^{1/2} \). If \( |d(0, +\infty)|_{L_2} \) is bounded, then we write \( d \in L_2 \).

2. PRELIMINARIES

We use here the approach from (Goebel et al., 2004) to define the solutions of hybrid systems. The hybrid time domain is defined as a subset of \((0, \infty) \times \mathbb{N}_0\), given as a union of finitely or infinitely many intervals \([t_i, t_{i+1}] \times \{i\}\) where the numbers \(0 = t_0, t_1, \ldots\) form a finite or infinite nondecreasing sequence. The last interval is allowed to be of the form \([t_1, T]\) with \(T\) finite or \(T = +\infty\). Let two closed sets \( \mathcal{C} \) and \( \mathcal{D} \) be given such that \( \mathcal{C} \cup \mathcal{D} = \mathbb{R}^n \) and functions \( f : \mathcal{C} \to \mathbb{R}^n \) and \( g : \mathcal{D} \to \mathbb{R}^n \). A solution of the hybrid system \( x(\cdot) \) is a function defined on the hybrid time domain, such that

\[
\dot{x}(t, i) = f(x(t, i)) \quad \text{only if } x(t, i) \in \mathcal{C} \\
\quad \text{and } i \in (t_i, t_{i+1}) \\
x(t_{i+1}, i + 1) = g(x(t_{i+1}, i)) \quad \text{only if } x(t_{i+1}, i) \in \mathcal{D} \\
\quad \text{and } i \in \mathbb{N}_0.
\]

To shorten notation, we omit the time arguments and write (1) as:

\[
\dot{x} = f(x) \quad \text{only if } x \in \mathcal{C} \\
x^+ = g(x) \quad \text{only if } x \in \mathcal{D}.
\]

Given \((t, N)\) such that \(t \in [t_N, t_{N+1}]\) we define:

\[
\int_0^t x(\tau) d\tau := \sum_{i=0}^{N-1} \int_{t_i}^{t_{i+1}} x(\tau, i) d\tau + \int_{t_N}^t x(\tau, N) d\tau.
\]

In the next section we will use sets \( \mathcal{C} \) and \( \mathcal{D} \) of a special form that are defined next. Let \( \epsilon \geq 0 \) and \( M = M^T \) and denote

\[
\mathcal{C}_\epsilon := \{x \in \mathbb{R}^n : x^T M x + \epsilon x^T x \geq 0\} \\
\mathcal{D}_\epsilon := \{x \in \mathbb{R}^n : x^T M x + \epsilon x^T x \leq 0\}
\]

and \( \mathcal{C} := \mathcal{C}_0 \) and \( \mathcal{D} := \mathcal{D}_0 \).

3. RESET SYSTEMS

In the sequel we concentrate on the following class of hybrid models:

\[
\begin{align*}
\dot{x} &= Ax + Bd &\text{only if } x \in \mathcal{C} \\
\dot{i} &= 1 &\text{or } \tau \leq \rho \\
x^+ &= \hat{Ax} &\text{only if } x \in \mathcal{D} \\
\tau^+ &= 0 &\text{and } \tau \geq \rho \\
y &= Cx,
\end{align*}
\]
where \( x \in \mathbb{R}^n, d \in \mathbb{R}^{n+}, \tau \geq 0 \) and \( \rho > 0 \).

The role of the variable \( \tau \) to achieve "time regularization" in the sense of (Johansson et al., 1999) in order to avoid Zeno solutions. Indeed, it is obvious that the reset times satisfy \( t_{i+1} - t_i \geq \rho \) for all \( i \in \mathbb{N} \), but, therefore, Zeno solutions cannot occur.

It was shown in (Zaccarian et al., 2004) that the class of models (5), (6), (7) can be used to describe general (linear) reset systems, as the following example illustrates.

**Example 1.** (Zaccarian et al., 2004) The block diagram of the Clegg integrator controlling an integrator via a unity feedback is given in Figure 1.

![Fig. 1. Clegg integrator controlling an integrator.](image)

The model of the closed loop system can be written as follows:

\[
\begin{align*}
\dot{x} &= \tau - x \\
\dot{x} &= k \cdot x + d \quad \text{only if } (x, x_r) \in \mathcal{C} \text{ or } \tau \leq \rho \\
\dot{\tau} &= 1 \\
x^+ &= 0 \\
\tau^+ &= 0 \\n\end{align*}
\]

where \( \rho > 0; \mathcal{C} \text{ and } \mathcal{D} \text{ are defined in (3), (4)} \) with \( M = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \) and \( \epsilon = 0; x_r \text{ and } x \) are respectively the controller (reset) and plant states; \( d \) and \( r \) are the disturbance and reference inputs.

**Remark 1.** It is important to note the difference between our model and the model in (Beker et al., 2004), which has the following form:

\[
\begin{align*}
\dot{x} &= A_{cl} x + B_{cl} d \quad \text{if } x \notin \mathcal{M}(t) \\
x^+ &= A_R x \quad \text{if } x \in \mathcal{M}(t),
\end{align*}
\]

where \( \mathcal{M} := \{ x : C_{cl} x = 0, (I - A_R)x \neq 0 \} \) for some matrix \( C_{cl} \in \mathbb{R}^{p \times n} \). There are three main differences between our model (5), (6) and the model (8), (9):

1. In the model (8), (9) resets are only possible on the hyperplane \( C_{cl} x = 0 \) (as long as some flow has occurred since the last reset), whereas in our model (5), (6) resets are enforced on a sector \( \mathcal{D} \).

2. Our model (5), (6) uses time regularization to avoid Zeno solutions whereas there is no time regularization in the model (8), (9). Instead, (Beker et al., 2004, Theorem 1) states existence of solutions for (8), (9). Despite this result, it is not clear what they mean by solution for some states. Indeed, for the reset system (8), (9) without disturbances it is not clear how to define solutions for the initial conditions satisfying \( C_{cl} x_0 = 0, (I - A_R)x_0 = 0 \) and, following the differential equation for arbitrarily small time yields \( C' x(t) = 0 \) and \( (I - A_R)x(t) \neq 0 \). As an example, consider the initial condition \( x_0 = (0, a, 0), a > 0 \) for the system with \( A_{cl} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & -1 \\ 1 & 0 & 0 \end{bmatrix}, A_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \).

3. The set \( \mathcal{M} \) and its complement are not closed whereas the sets \( \mathcal{C} \text{ and } \mathcal{D} \) are always closed. Moreover, the sets \( \mathcal{M} \) and its complement are disjoint, whereas the sets \( \mathcal{C} \text{ and } \mathcal{D} \) have a common boundary and, hence, they overlap.

**4. MAIN RESULTS**

In this section we state our main results. Sufficient \( \mathcal{L}_2 \) and exponential stability conditions for the system (5), (6) are presented respectively in Theorems 1 and 2. In all our results we will rely on the following assumption:

**Assumption 1.** For the system (5), (6), the reset map \( \tilde{A} \) is such that

\[
x \in \mathcal{D} \implies \tilde{A} x \in \mathcal{C}.
\]

Condition (10) is quite natural to assume for reset systems. This condition guarantees that after each reset time the solutions will be mapped to the set \( \mathcal{C} \) where the dynamics are governed by the differential equation (5) so that flow is possible from there. Without this condition, due to the time regularization, defective trajectories may correspond to solutions that keep flowing and jumping within the set \( \mathcal{D} \), so that it would be impossible to establish that all solutions flow only in the set \( \mathcal{C} \). This last property is a key tool for exploiting the advantages of resets within the Lyapunov framework, thereby establishing our main results.

**Theorem 1.** Suppose that Assumption 1 holds and that there exists a locally Lipschitz Lyapunov function \( V : \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0} \), strictly positive numbers \( a_1, a_2, a_3, a_4, a_5, \epsilon \) and a matrix \( M := MT \) such that the following holds for all \( d \in \mathbb{R}^{n+} \):

\[
\begin{align*}
\dot{V}(x) &= \sum_{i=1}^{5} a_i V(x)^{-\gamma_i} + \epsilon \sum_{i=1}^{5} a_i V(x)^{-\gamma_i} \\
\end{align*}
\]
\begin{align}
a_1 |x|^2 & \leq V(x) \leq a_2 |x|^2, \forall x \in \mathbb{R}^n; \\
\frac{\partial V}{\partial x} (Ax + Bd) & \leq -a_3 |y|^2 + \gamma |d|^2, \text{ for a.a. } x \in \mathcal{C}, \tag{12} \\
V(\tilde{A}x) - V(x) & \leq 0 \quad \forall x \in \mathcal{D}; \\
\left| \frac{\partial V}{\partial x} \right| & \leq a_4 |x|, \text{ for a.a. } x \in \mathbb{R}^n. \tag{14}
\end{align}

Then, for any \( L > 1 \) there exists \( \rho^* > 0 \) such that \(^4\) for all \( \rho \in (0, \rho^*) \) the solutions of the system (5), (6), (7) satisfy:

\[
\int_0^t |y(\tau)|^2 d\tau \leq \frac{L a_2}{a_3} |x_0|^2 + \gamma \frac{t}{a_3} \int_0^t |d(\tau)|^2 d\tau,
\]

for all \( t \geq 0, \tau(0,0) = \tau_0 \geq 0, x(0,0) = x_0 \in \mathbb{R}^n \) and \( d \in \mathcal{L}_2 \).

Remark 2. A results similar to Theorem 1 can be stated for the case of \( \mathcal{L}_p \) stability for arbitrary \( p \in [1, \infty] \). The conditions of Theorem 1 need to be changed slightly and the proofs modified in a straightforward manner. We did not state this result due to space constraints and for simplicity.

Remark 3. Sufficient conditions for \( \mathcal{L}_\infty \) (bounded input bounded state) stability of reset systems were presented in (Beker et al., 2004) for general reset systems. Theorem 1 presents for the first time results on \( \mathcal{L}_2 \) stability of reset systems.

It is instructive to note that \( \mathcal{L}_p \) stability from \( w \) to \( y \) for some \( p \in [1, \infty] \) implies exponential stability of the system in the absence of disturbances. Therefore, if we have an appropriate \( \mathcal{L}_p \) detectability from \( y \) to \( x \), we can conclude \( \mathcal{L}_p \) stability from \( w \) to \( x \) from Theorem 1. Then, under mild technical conditions this implies exponential stability in the absence of disturbances. This result can be proved using results of (Teel et al., 2002) and it is very similar to (Nesic and A.R.Tee, 2004). A special case of the required detectability property is when there exists \( \mu > 0 \) such that \( \mu^2 |x|^2 \leq |y|^2 \). We formally state this case in the next theorem, while additional results relying on more general detectability conditions will be not covered here.

**Theorem 2.** Consider the system (5), (6) without disturbances. Suppose that Assumption 1 holds and that there exists a locally Lipschitz Lyapunov function \( V : \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0} \), strictly positive numbers \( a_1, a_2, a_3, a_4, \epsilon \) and a matrix \( M = M^T \) such that the following holds:

\[
a_1 |x|^2 \leq V(x) \leq a_2 |x|^2, \forall x \in \mathbb{R}^n; \\
\frac{\partial V}{\partial x} Ax \leq -a_3 |x|^2, \text{ for a.a. } x \in \mathcal{C}; \tag{16} \\
V(\tilde{A}x) - V(x) \leq 0, \forall x \in \mathcal{D}; \\
\left| \frac{\partial V}{\partial x} \right| \leq a_4 |x|, \text{ for a.a. } x \in \mathbb{R}^n. \tag{18}
\]

Then, there exist \( \rho^*, K > 0 \) such that for all \( \rho \in (0, \rho^*) \) the solutions of the system (5), (6) satisfy:

\[
x(t, i) \leq K \exp \left( -\frac{a_3}{2a_2} t \right) |x_0|,
\]

for all \( t \in [t_i, t_{i+1}], i \geq 0, \tau(0,0) = \tau_0 \geq 0 \) and \( x(0,0) = x_0 \in \mathbb{R}^n \).

**Remark 4.** Note that conditions (12) and (16) need to hold only on the set \( \mathcal{C}_e \), which is a subset of \( \mathbb{R}^n \). Moreover, the closure of \( \mathcal{C}_e \) is typically a proper subset of \( \mathbb{R}^n \); hence conditions (12) and (16) are much weaker than requiring stability of \( \dot{x} = Ax + Bd \) that was required in (Beker et al., 2004, Theorem 1) to guarantee stability of the reset system. Hence, Theorems 1 and 2 relax the stability conditions used in (Beker et al., 2004). Finally, we note that in general we can not replace \( \mathcal{C}_e \) by \( \mathcal{C} \) in (12). However, when \( V() \) has extra properties (as in the following Proposition 1), this can be possible.

**Remark 5.** Our conditions (14) and (18) allow for non-differentiable Lyapunov functions \( V() \), which is another relaxation of the conditions in (Beker et al., 2004, Theorem 1), where continuous differentiability of \( V() \) was required. This generalization allows us, among other things, to consider piecewise quadratic Lyapunov functions which were not possible to handle using the results of (Beker et al., 2004, Theorem 1). It turns out that piecewise quadratic Lyapunov functions are a key tool for exploiting convex optimization tools such as LMIs when trying to obtain tight estimates of \( \mathcal{L}_2 \) gains for this class of systems, as illustrated in (Zaccarian et al., 2004).

Theorems 1 and 2 provide a theoretical framework for analysis and design of reset systems. A typical analysis problem consists in finding an appropriate Lyapunov function satisfying the conditions of the theorems for a given system (5), (6). Computational approaches via LMIs that use piecewise quadratic Lyapunov functions are given in (Zaccarian et al., 2004). For instance, Theorem 1 can be used to prove the following result on \( \mathcal{L}_2 \) stability via quadratic Lyapunov functions \( V(x) = x^T P x \).

**Proposition 1.** (Zaccarian et al., 2004) Consider the reset control system (5), (6), (7), where the sets \( \mathcal{C} \) and \( \mathcal{D} \) are defined by the matrix \( M \) via (3), (4). If the following linear matrix inequalities in the variables \( P = P^T > 0, \tau_F, \tau_R \geq 0, \gamma > 0 \) are feasible:

\[
\begin{pmatrix}
A^T P + PA + \tau_F M & PB \\
-\gamma I & -\gamma I
\end{pmatrix} < 0,
\]

\[
A^T P A - P - \tau_R M \leq 0,
\]

\[
\begin{pmatrix}
P A^T & P A & \tau_F M & PB & C^T \\
* & -\gamma I & 0 \\
* & * & -\gamma I
\end{pmatrix} < 0
\]

The explicit value of \( \rho^* \) is omitted due to space constraints.
then, there exists $\rho^*$ such that, for all $\rho \in (0, \rho^*)$, the reset system (5), (6), (7) has a finite $L_2$ gain from $d$ to $y$ that is smaller than $\gamma$.

We note that using quadratic Lyapunov functions is often too restrictive for reset systems and more general theorems based on piecewise quadratic Lyapunov functions from (Zaccarian et al., 2004) are often needed.

**Sketch of Proof of Theorem 1:** The proof is based on Lemmas 1-3 (see below) that are stated without a proof. Denote the reset times as $t_i$ where we use the convention that $t_0 = 0$ and $t_N = t$ even though the times 0 and $t$ may not be reset times. Lemma 2 gives us an appropriate bound on the time interval $[t_i, t_{i+1}]$. Because of Assumption 1 we have that $x(t, i) \in \mathcal{C}$ for all $i \geq 1$ and Lemma 1 gives us appropriate bounds on the intervals $[t_i, t_{i+1}]$ for $i = 1, \ldots, N - 1$. Lemma 3 guarantees that the value function does not increase at reset times. Hence, by concatenating the intervals $[t_i, t_{i+1}]$ we can add the bounds in Lemmas 1 and 2 to prove the result. More details can be found in the journal version of this paper (Nešić et al., 2004).

**Lemma 1.** Suppose that the conditions of Theorem 1 hold. Then, there exists $\rho^* > 0$ such that for all $\rho \in (0, \rho^*)$ we have that if $x(t_i, i) \in \mathcal{C}$ and $d \in \mathcal{L}_2$ then $a_3 \int_{t_i}^{t_{i+1}} |y(\tau, i)|^2 d\tau \leq V(x(t_i, i)) - V(x(t, i)) + \gamma \int_{t_i}^{t_{i+1}} |d(\tau)|^2 d\tau$ for all $t \in [t_i, t_{i+1}]$.

**Lemma 2.** Suppose that the conditions of Theorem 1 hold. Then, for any $L > 1$ there exists $\rho^* > 0$ such that for any $\rho \in (0, \rho^*)$, $x(0, 0) = x_0$, $\tau(0, 0) \geq 0$ and $d \in \mathcal{L}_2$ we have that $a_3 \int_{t_0}^{t_{N+1}} |y(\tau, 0)|^2 d\tau \leq LV(x(t_0, 0)) - V(x(t_0)) + \gamma \int_{t_0}^{t_{N+1}} |d(\tau)|^2 d\tau$ for all $t \in [t_0, t_1]$.

**Lemma 3.** Under the conditions of Theorem 1, for any $i \geq 0$ we have that $V(x(t_{i+1}, i + 1)) \leq V(x(t_{i+1}, i + i))$.

5. EXAMPLES

Constructing Lyapunov functions for general reset systems that satisfy the conditions of Theorems 1 and 2 is typically hard. It is easier to do so for systems containing FOREs. In (Zaccarian et al., 2004) we presented a method based on Linear Matrix Inequalities to construct piecewise quadratic Lyapunov functions to check $L_2$ stability for a class of reset systems containing FOREs. In this section, we use results from (Zaccarian et al., 2004) to analyze the $L_2$ stability of systems with reset controllers. In particular, we show how changing parameters in the FORE affects the gain of the reset closed-loop system.

**Example 2.** Consider an integrator (plant) controlled by a FORE:

\[
\begin{align*}
\dot{x}_1 &= x_2 + d \\
\dot{x}_2 &= -x_1 + \beta x_2 \\
y &= x_1.
\end{align*}
\]

and assume that the output is $y = x_1$. Here, $x_1$ and $x_2$ respectively denote the state of the scalar plant and of the FORE. We computed the $L_2$ gain from $d$ to $y$ for the system (20), (21) using the LMI method from (Zaccarian et al., 2004). The gain has been computed for the limit case as $\rho \to 0$. (Larger values of $\rho$ correspond, in general, to larger gains due to the fact that $C_\epsilon$ would be larger.) The gain is plotted as a function of the parameter $\beta$ that determines the pole of the FORE. This plot is represented by the dashed line in Figure 2. Moreover, we considered the linear system without resets:

\[
\begin{align*}
\dot{x}_1 &= x_2 + d \\
\dot{x}_2 &= -x_1 + \beta x_2 \\
y &= x_1.
\end{align*}
\]

The full line in Figure 2 shows the $L_2$ gain of the linear system (22) as a function of the parameter $\beta$. Note that adjusting the parameter $\beta$ in the linear controller can not produce a gain smaller than $\approx 1.5$. Moreover, as $\beta \to 0$, the $L_2$ gain of the linear system tends to infinity. For positive values of $\beta$ the linear system (22) is unstable and does not have a well defined $L_2$ gain.

**Example 3.** We also address a second example borrowed from (Hollot et al., 2001). In this example, a FORE element whose linear part is characterized by the transfer function \( \frac{X(s)}{E(s)} = \frac{s+1}{s(s+0.2)} \) controls via a negative unitary feedback a SISO plant whose transfer function is $\frac{X(s)}{E(s)} = \frac{s+1}{s(s+0.2)}$. For this...
example, the control system involving the FORE is shown in (Hollot et al., 2001) to behave more desirably than the linear control system. It was shown in (Hollot et al., 2001) that the reset system had only about 40% overshoot of the linear closed loop system while retaining the rise time of the linear design. This example can be further interpreted using our results. Indeed, when computing the $L_2$ gain from the plant input to the plant output, the linear closed-loop system has an $H_{\infty}$ norm around 5, while using the construction in (Zaccarian et al., 2004, Theorem 3) and the main results of our paper we obtain that the $L_2$ gain of the reset system is 3.82.

Figure 3 reports the $L_2$ gains for the linear closed-loop and the reset closed-loop as a function of the pole of the FORE. Once again, for positive values of $\beta$ (unstable fores) the linear closed-loop is unstable, while the reset closed-loop guarantees smaller gains. The case studied in (Hollot et al., 2001) corresponds to the horizontal coordinate $\beta = -1$ in Figure 3.

![Fig. 3. $L_2$ gains of linear and reset closed loops for Example 3, as a function of the pole of the FORE.](image)

6. CONCLUSIONS

We provided Lyapunov like conditions that guarantee $L_2$ stability and exponential stability of a class of reset systems, such as systems containing Clegg integrators. Our results provide a theoretical framework for systematic analysis and controller design of reset systems and they generalize the corresponding results in (Beker et al., 2004). Examples illustrate that it is possible to improve the $L_2$ gain of a linear controller by a simple introduction of resets.

REFERENCES


