SMOOTH SLIDING MODE CONTROL FOR CONSTRAINED MANIPULATOR WITH JOINT FLEXIBILITY

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Abstract: This paper presents a smooth sliding mode control plus backstepping method for the motion/force control of a constrained robotic manipulator with flexible joints. The control scheme is presented based on that overall system parameters are with uncertainties whereas only the constrained force, positions and velocities of links and rotors are measurable. The smooth sliding controller can achieve zero motion and force tracking errors. The simulation results for a planar robot illustrate the expected satisfactory performance. Copyright © 2005 IFAC

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1. INTRODUCTION

In many industrial tasks such as deburring, writing, grinding, painting, etc, it is necessary to drive the robot’s end-effector to move along the environmental constraint and track both the desired force and the desired motion. The initial study on this issue was conducted in (Hemami and Wyman, 1979), and then a number of approaches were proposed. These theoretical frameworks and stability analysis for constrained robots have stemmed from the reduced-state position/force control described by (McClamroch and Wang, 1988). In (Su, et al., 1995; Kwan, 1996; Yu and Lloyd, 1997), adaptive control has been proposed to deal with parametric uncertainty in the form of a robust approach. However, in most schemes the concerned force tracking error is either arbitrarily small at best by using high gain of force feedback or steered to zero under the condition of persistent excitation (PE). In order to achieve zero motion/force errors, some methods shown in (Lian and Lin, 1998; Yuan, 1997; Yao, et al., 1994) use different error space decomposition techniques, i.e., the motion/force decomposition or the nonlinear curvilinear coordination transformation, etc.

Most papers on the manipulator control neglect the actuator dynamics. Actually, robot manipulators are usually with flexible joints due to gear elasticity, shaft windup, etc. It was pointed out in (Spong and Vidyasagar, 1989; Spong, 1989) that the joint flexibility must be considered in designing the control law if high performance is to be achieved. For a constrained flexible joint robot (FJR), the rigid manipulator subsystem is driven by the rotor subsystem. The dynamics form performs a strictly feedback structure. Based on this feature, the integrator backstepping control technique would be straightforward applied in the problem. However, the full-order model with parametric uncertainty results that the controller is difficult designed by only using reasonable measurements, e.g. constrained force, positions and velocities of the manipulator and rotor.

2. KINEMATICS, DYNAMICS, AND RELATED PROPERTIES

In the section, we first give the kinematic relation of constrained motion and the dynamics of
constrained FJRs. Then, some properties are addressed. Here we assume that all end-effectors are rigidly attached to the environment. Hence the motion of a robot is governed by the constrained dynamics, i.e., a dynamic system subject to algebraic constraint equations.

2.1 Kinematics and Dynamical Models

Consider the system of an $n$-link rigid robot with environmental constraints. Let $q \in \mathbb{R}^n$ be the vector of generalized coordinates and let $\sigma(q) = 0$ , where $\sigma : \mathbb{R}^n \mapsto \mathbb{R}^n$ , denote the environmental constraints. In order to perform the subsequent derivation, the following assumptions concerning the constraint equations $\sigma(q) = 0$ are made:

**Assumption 1:** The constraint are assumed in the absence of friction and other disturbance.

**Assumption 2:** The environmental constraints can be described by a smooth and exactly known equation $\sigma(q) = 0$. There exists an open set $\Theta \subset \mathbb{R}^{n\times m}$ and a function $\Omega : \Theta \mapsto \mathbb{R}^n$ ; $\dot{q}_i = \Omega(q_i)$ , so that $\sigma(q_i, q_j) = \sigma(\Omega(q_i), q_j)$. Moreover, the function $\frac{\partial \Omega}{\partial q_i}$ and $\frac{\partial \Omega}{\partial q_j}$ are assumed bounded in the working space.

Since the dimension of the constraint is $m$, the robot losses $m$ degrees of freedom and is left with $(n-m)$ degrees of freedom. One can always partition the generalized coordinates to $q_1 \in \mathbb{R}^n$ and $q_2 \in \mathbb{R}^{m-n}$ such that the Jacobian matrix of the constraint equations $A(q) = \partial \sigma(q)/\partial \dot{q}$ is denoted in the form:

$$A(q) = \begin{bmatrix} \frac{\partial \Omega(q_1)}{\partial q_1} & \frac{\partial \Omega(q_2)}{\partial q_2} \end{bmatrix} = \begin{bmatrix} A_1(q_1) & A_2(q_2) \end{bmatrix},$$

where $q_1$ has been substituted by the kinematics $q_1 = \Omega(q_2)$. From Assumption 2, that the environmental constraints $\sigma(q) = 0$ are always satisfied, the manipulator dynamics is constrained by an invariant submanifold defined by $\{ (q, \dot{q}) : \sigma(q) = 0, A(q)\dot{q} = 0 \}$. Then velocities $\dot{q}_1 = -A_1^{-1}A_2\dot{q}_2 = \frac{\partial \Omega(q_2)}{\partial q_2}\dot{q}_2$ is well-defined according to that $A_1 \in \mathbb{R}^{n\times n}$ is nonsingular, i.e., $A(q)$ is full row rank, and the boundedness of $\frac{\partial \Omega(q_2)}{\partial q_2}$ in working space. Consequently, we have the following kinematics:

$$q = (\Omega(q_1), q_2);$$

$$\dot{q} = \begin{bmatrix} \frac{\partial \Omega(q_1)}{\partial q_1} & 0 \end{bmatrix} \dot{q}_2 = J_2 \dot{q}_2$$

Accordingly, one has $A(q)\dot{q} = A(q)J_2\dot{q}_2 = 0$. Since $\dot{q}_2$ are independent variables, it follows that:

**Fact 1:** $A_1 J_2^T A_2^T = 0$ .

Consider the flexible joint constrained robot which has been modeling as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = K(q_m - q) + A_2^T \lambda$$

where $q \in \mathbb{R}^n$, $q_m \in \mathbb{R}^r$ are generalized coordinates of the robot and the rotor subsystem, respectively; $M(q)$ is an $n \times n$ inertia matrix, which is symmetric and positive definite for all $q \in \mathbb{R}^n$; $C(q, \dot{q})\dot{q}$ is the Coriolis/centripetal force vector; $G(q)$ and $K(q_m - q)$ are $n \times 1$ vectors containing gravity force and elastic force transmitted through joints, respectively; $K$ is an $n \times n$ diagonal matrix representing the joint stiffness; $J_2$ is actuator inertia matrix; $\tau \in \mathbb{R}^r$ is the input force vector, $\tau_r$ is the external disturbance of the server system; and $\lambda \in \mathbb{R}^n$ is a vector that represents the generalized force multipliers associated with the constraints, namely the constrained force. Then the term $f = A_2^T (q_2) \lambda$ ; $f \in \mathbb{R}^n$ denotes the contact force due to of the environmental constraints. It is reasonable that the constrained force is required to be always positive under proper control.

2.2 Reduced Dynamics and Some Properties

Since motion equations (2) is subjected to the kinematic constraints (1), the reduced dynamics for the rigid manipulator subsystem is derived to facilitate the control design. Let $[E_1^T : E_2^T]$ be a partition of an identity matrix $I_n$ with $E_1 = [I_{m \times n} \ 0] \in \mathbb{R}^{m \times n}$. Then so some facts about the transformation $T$ and Jacobian matrix $J$ are stated below:

**Fact 2:** $T^T E_1^T = E_1^T$.

**Fact 3:** $T^T A^T = \begin{bmatrix} A_1 & 0_{n \times (n-m)} \end{bmatrix}^T$.

**Fact 4:** Matrix $J$ can be factorized as $J = TE_2^T$. It is also noted that $J^T E_2^T = I_{n\times n}$.

Then (2) can be rewritten in the form:

$$MJ_2 \ddot{q}_2 + (MJ + CJ)q_2 + G = K(q_m - q) + A_2^T \lambda$$

where

$$B = Kq_2.$$

Applying the kinematics (1), the equations of motion (2) can be rewritten as

$$J^T M J_2 \ddot{q}_2 + J^T (MJ + CJ)q_2 + J^T G = J^T K(q_m - q) - J^T B$$

which, by multiplying $T^T$ on the both sides, further results in

$$E_2^T (\tilde{M} E_2^T \ddot{q}_2 + \tilde{C} E_2^T q_2 + \tilde{G}) = E_2^T (T^T K q_m - T^T B),$$

where

$$\tilde{M} = T^T M T; \quad \tilde{C} = T^T (M T + C T); \quad \tilde{G} = T^T G.$$
According to Fact 1, 3 and 4, the reduced dynamics can be written in the following form after multiplying $E_2^T$ on the both sides of (5):
\[
M\ddot{q}_2 + \dot{C}q_2 + \dot{G} = J^T K q_m - J^T B
\]
where $M = E_2^T M E_2 = J^T M J$, $C = E_2^T C E_2 = J^T C J$, $G = E_2^T G = J^T G$.
Now, to be beneficial for the stability analysis and control design, some useful properties are addressed below:

**Property 1**: $\tilde{M}$ is a symmetric and positive definite matrix. Also, $\tilde{M}^{-1}(q_2)$ is uniformly bounded.

**Property 2**: $\tilde{M}(q_2)$ and $\tilde{M}^{-1}(q_2)$ are uniformly bounded. Further, matrix $\tilde{M}(q_2)$ is bounded by $M_u I_{n-m} \leq \tilde{M} \leq M_u I_{n-m}$, with $M_u, M_u > 0$.

**Property 3**: $\tilde{C}(q_2)$ and $\tilde{M}(q_2, q_2)$ are uniformly bounded functions if $q_2$ and $\dot{q}_2$ are uniformly bounded. If $q_2$, $\dot{q}_2$, and $\ddot{q}_2$ are uniformly bounded, $\tilde{C}(q_2, q_2, \dot{q}_2)$ is a uniformly bounded function.

**Property 4**: For an appropriate choice of C, $\tilde{M} - 2\tilde{C}$ can be a skew-symmetric matrix. This means that $z'(\tilde{M} - 2\tilde{C})z = 0$, $\forall z \in R^n$.

**Property 5 (Linear Parameterization)**: It follows that
\[
\tilde{M}q_2 + \dot{C}q_2 + \dot{G} = T^T [M(Tq_2 + \dot{T}q_2) + C(Tq_2) + G] = T^T Y(q_2, \dot{q}_2, v, v) \phi
\]
where $q_2$ will be defined later, $v = Tq_2$, the regression matrix $Y \in R^{n\times n}$ is a known function and $\phi \in R^n$ is unknown but constant.

The hyperbolic tangent function has the following property:

**Property 6**: The hyperbolic tangent function defined as $\tanh(\mu) = (e^\mu - e^{-\mu})/(e^\mu + e^{-\mu})$ satisfies the following properties:
(a) $\mu \tanh(\mu a) \geq 0$, for $a > 0$, this further implies for any odd positive integer $b$ with $\mu \tanh(\mu a^b) \geq 0$
(b) $|\tanh(\mu)| = \tanh(\mu) \geq 0$
(c) if $\tanh(c |\mu|) \geq d$ , for $c > 0$ , $0 \leq d < 1$ , then $|\mu| \geq 2\ln(1+d)/2d$ must be satisfied.

Let $z = K q_m$ such that the complete system (2) (3) is expressed in the new coordinates as
\[
\begin{align*}
M\ddot{z}_2 + \dot{C}z_2 + \dot{G} &= J^T z_1 - J^T B \\
J q_m^{-1}\ddot{z}_2 + z_1 - B &= \tau + \tau_d
\end{align*}
\]

### 3. SMOOTH SLIDING MODE CONTROL FOR A CONSTRAINED FJR

For a constrained FJR, the rigid manipulator subsystem is driven by the actuator subsystem. Hence the dynamical equations (1) and (2) perform the strictly feedback structure. In this section, a two-stage approach, or namely the integrator backstepping approach is applied to design the controller.

#### 3.1 Design a Virtual Control Input $z_i$ for the First Stage

Consider the stiffness matrix $K$ is uncertainty and $M, C, G$ is with the unknown parameters.

Let us define the following error signals:
\[
e_p = q_{z_2d} - q_2, \text{ and } \dot{\lambda} = \dot{\lambda}_d - \lambda
\]
Define a force filter as
\[
\dot{e}_p + a\dot{e}_p = \Lambda_j \dot{\lambda}
\]
where $e_j \in R^n$ and $\Lambda_j \in R^{m\times m}$ is positive definite.

Then the error measure is
\[
s = E_2^T (\dot{e}_p + a\Lambda e_p) + E_1^T e_1 = \begin{bmatrix} e_1 \\ \dot{e}_p + a\Lambda e_p \end{bmatrix}
\]
\[
= q_2 - E_2^T \dot{q}_2
\]
where $q_2 = E_2^T (\dot{q}_{z_2d} + a\Lambda e_p) + E_1^T e_1 = \begin{bmatrix} e_1 \\ \dot{q}_{z_2d} + a\Lambda e_p \end{bmatrix}$.

Therefore, if the error measure can be converge to zero, the error signal $e_1$, $\dot{e}_p$, and $e_p$ will converge to zero and thus our objective is achieved. To this end, the error dynamics of the error measure is rewritten as
\[
\dot{M} \dot{s} = \tilde{M}q_2 + \tilde{C}q_2 + \tilde{G} - T^T z_1 - T^T B - T^T A^T \dot{\lambda}
\]
If let $z_i$ as a virtual control input, and consider the feedback control $z_i = z_{i_d}$ with $z_{i_d}$ defined
\[
z_{i_d} = (T^T)^{-1} \left[ M\ddot{q}_i \tilde{C}q_i + \tilde{G} - T^T B - T^T A^T \dot{\lambda} \right] + \psi s
\]
Because the system has the parametric uncertainty, the equation can not be implemented. Hence the LP property is used for solve above problems. Therefore, if assume the uncertain parameters can be partition into $\phi = \phi_0 + \Delta \phi$ which are known priori, then the virtual control input $z_{i_d}$ can be rewritten as
\[
z_{i_d} = (T^T)^{-1} \left[ T^T Y \phi_0 + T^T Y \Delta \phi + T^T B - T^T A^T \dot{\lambda} + \psi s \right]
\]
where the control parameter $\psi$ is positive defined and $\phi_0 = [\phi_{i1}, \phi_{i2}, \cdots, \phi_{i_r}]^T$ is the robust function designed as follows:
\[
\phi_{i_d} = (\Delta \phi + \eta_{i_d}) \tanh \left( (a + \nu r) \sum_{j=1}^{\sigma} s(T^T j) Y_{i,j} \right)
\]
\[
i = 1, 2, \cdots, r \quad \text{with } \eta_{i_d}, a, \nu > 0 \text{ and } \Delta \phi, \eta_{i_d} \geq 0 \]
Further, using the Lyapunov function
\[
V_i = \frac{1}{2} s^T \bar{M} s
\]
is positive.
\[
V_i = s^T \bar{M} s + \frac{1}{2} s^T \bar{M} \bar{s}
\]
\[
\dot{V}_i = -s^T \psi s + \sum_{j=1}^{\sigma} \left( \sum_{j=1}^{\sigma} s(T^T j) Y_{i,j} \right) \Lambda \phi
\]
\[
- \sum_{j=1}^{\sigma} \left( \sum_{j=1}^{\sigma} s(T^T j) Y_{i,j} \right) \Delta \phi, \eta_{i_d} \tanh \left( (a + \nu r) \sum_{j=1}^{\sigma} s(T^T j) Y_{i,j} \right)
\]
\[ V_i \leq -s^T \psi + \sum_{j,k} \sum_{i,j} s_j(T^i)Y_{jk} \Delta \phi_{jk} \]
\[ -\sum_{j,k} \sum_{i,j} s_j(T^i)Y_{jk} (\Delta \phi + \eta_0) \tanh [(\alpha + \nu) \sum_{j,k} \sum_{i,j} s_j(T^i)Y_{jk}] \]
Since \( V_i \) is positive definite and \( V_i' \leq 0 \) if (8) sustains, it can be concluded that all error signals \( s, e, \hat{e}, \hat{\lambda} \) are all uniformly bounded.
\[ \tanh [(\alpha + \nu) \sum_{j,k} \sum_{i,j} s_j(T^i)Y_{jk}] \leq \frac{\| \Delta \phi \|}{\Delta \phi_{ij} + \eta_0} \]
Equation (9) can be rewritten in the following form
\[ V_i' = -s^T \psi \nu S \]
where the Property 3 had been used and integrating both sides of the above inequality with respect to time. We obtain \( V_i \leq V_0 e^{-2\lambda_\nu(\psi)0} M_u \| \nu \| \) and this means using the property again, the reaching transient response is shaped by
\[ \| s(t) \| \leq \frac{M_u}{M_n} \| s(0) \| e^{-2\lambda_\nu(\psi)0} \]
As a result, the motion tracking error exponentially decay if \( \| s(t) \| \geq b_i \). Obviously, the ball \( b_i \) will converge to zero linearly in time. Therefore, the error signals \( s \) will asymptotically converge to zero.

3.2 Regulate \( z_i \) Converge to \( z_{i,d} \) for the Second Stage.

Step 2: We now have the error dynamics of \( \dot{z}_i \) as
\[ \dot{z}_i = z_{i,d} - \dot{z}_i \]
where \( z_{i,d} \) is with the uncertainty of the manipulator, so \( \dot{z}_{i,d} \) is not realizable. To let the control input be realized by position, velocity and force measurements only, the following relations are introduced:
\[ \dot{\hat{s}} = M^{-1}(T^T \Delta \phi - T^T \eta_0 + T^T \tilde{z} - \tilde{C} \hat{S} - \psi \nu) \]
\[ \dot{\tilde{z}}_i = M^{-1}(J^T \dot{z}_i - J^T B - C \dot{q}_2 - G) \]
Then the terms \( Y \), where \( \dot{q}_2 \) and \( \dot{s} \) replaced by \( q_2, \dot{q}_2 \) signals, yields
\[ \ddot{\hat{s}}(q_i, \dot{q}_i, \dot{q}_2, \dot{q}_2, \dot{q}_i, \dot{q}_i) = \ddot{\hat{s}}(q_i, \dot{q}_i, \dot{q}_2, \dot{q}_2, \dot{q}_i, \dot{q}_i, \Delta \phi, \Delta \phi) \]
Additionally, the time derivative of the virtual control input can be partitioned into the known part \( N_0 \) and unknown part \( \Delta N \), and is expressed as
\[ \dot{\tilde{z}}_{i,d} = N_0 + \Delta N \]
where
\[ \Delta N = T(q_i, \dot{q}_i, \dot{q}_i, \dot{q}_2, \dot{q}_2, \dot{q}_i, \dot{q}_i, \Delta \phi, \dot{x}) \]
Next, let us consider the error dynamics (12) again, yields
\[ \ddot{z}_i = \dot{z}_{i,d} - \dot{z}_i = N_0 + \Delta N - \dot{z}_i \]
Now, we assume \( \dot{z}_i \) as the virtual control input in the form:
\[ \dot{z}_i = \dot{z}_{i,d} = N_0 + \Delta N - \dot{z}_i \]
with
\[ \Delta N = diag[\Delta N_1, \ldots, \Delta N_n] ; \eta_0 = diag[\eta_0, \ldots, \eta_0] \]
Then (13) can be further rewritten as
\[ \dot{z}_i = \dot{z}_{i,d} = -T \dot{s} \Delta N - (\Delta N + \eta_0) \tanh [(\alpha + \nu) \dot{s}] \]
Consider \( V_i = V_i + \frac{1}{2} \dot{z}_i^T \dot{z}_i \) as a Lyapunov function candidate for the error system (15) with the virtual input (14). Let \( e = [s^T \dot{z}_i^T]^T \). We have the time derivative of \( V_i \) as
Therefore let us consider the Lyapunov function 
\[
\frac{1}{2} \sum_{i=1}^{n} \eta_i \Xi_i^T \Xi_i \|1\|_{max} (\ln)_{2} (\|N_{\eta}+\eta_{\nu}\|_{max}) \]
and the state 
\[
\|z_{\nu}\|\frac{1}{2} \sum_{i=1}^{n} \eta_i \Xi_i^T \Xi_i \|1\|_{max} (\ln)_{2} (\|N_{\eta}+\eta_{\nu}\|_{max}) \]
Note that \( w_0 \) is a known part in the error system and the term \((u+v)\) is bounded has been shown in step 2. Once the error \( z_{\nu} \) can be driven to zero, which is achieved in this step. Moreover, the uncertain term \( \Delta w \) is assumed with a upper bound \( |\Delta w_{\nu}(1)| \leq |\Delta w_{\nu}| \). Let the control input as \( r = z_{\nu} \) and \( + \beta \) \( z_{\nu} \) \( + \beta \) \( J_{\omega} K^{-1} \nu \), where \( \nu \) will be defined later. Thus we obtain the overall error system:
\[
\begin{align*}
\dot{z}_{\nu} &= -(W_{\nu}+\Delta +\Delta w) + \beta \Xi_i^T \Xi_i \Xi_{\nu} + (u + v) z_{\nu} + (u + v) \Xi_i^T \Xi_i \Xi_{\nu} \[ \dot{z}_{\nu} \] \\
\text{where} \quad &\theta = \sum_{i=1}^{n} \eta_i \Xi_i^T \Xi_i \|1\|_{max} (\ln)_{2} (\|N_{\eta}+\eta_{\nu}\|_{max}) \end{align*}
\]
In the same way, if both the inequality (8) and (16) are satisfied, then we obtain the ultimate bound defined by (8) and (16) as follows:
\[
\|z_{\nu}\|\frac{1}{2} \sum_{i=1}^{n} \eta_i \Xi_i^T \Xi_i \|1\|_{max} (\ln)_{2} (\|N_{\eta}+\eta_{\nu}\|_{max}) \]
Obviously, the ball \( \|z_{\nu}\|\frac{1}{2} \sum_{i=1}^{n} \eta_i \Xi_i^T \Xi_i \|1\|_{max} (\ln)_{2} (\|N_{\eta}+\eta_{\nu}\|_{max}) \) will converge to zero linearly in time. Therefore, the error signals \( \epsilon \) will asymptotically converge to zero as \( t \rightarrow \infty \). The remaining of the proof is to show the control law is smooth as \( t \rightarrow \infty \). From the facts that \( \dot{V}_2 \) satisfies the inequality as
\[
\begin{align*}
\dot{V}_2 &= -(c_2 \|t\|^{2} + c_1 \|t\|^2) \\
\text{where} \quad &c_1 = \lambda_{w}(M_{\omega}), c_1 = \lambda_{w}(M_{\omega}), \text{and} \quad c_1 = \lambda_{w}(M_{\omega}) \\
\text{we can easily show that the error signal } \epsilon \text{ is shaped by } \|t\| \leq \sqrt{\frac{c_1}{c_2}} \|t\|^{2} \sqrt{c_1} \%
\end{align*}
\]
In Step 3: Before applying the same design scheme as in Step 2, let us reformulate \( z_{\nu} \) into a nominal part and a uncertain part to obtain actual control input only form position, velocity, and force. Assume \( J_{\omega} \) is nominally known as \( J_{\omega} \) and the unknown part \( J_{\omega} \). Thus the dynamics of \( z_{\nu} \) is
\[
\begin{align*}
J_{\omega} K^{-1} z_{\nu} &= J_{\omega} K^{-1} (z_{\nu} - z_{\nu}) \\
&= J_{\omega} K^{-1} (w_0 + \Delta w + E \Delta N) + z_{\nu} - \beta \nu \\
\text{where} \quad &w_0, \Delta w, E, \Delta N, \beta \\
\text{this means that all error signals are uniformly bounded, and the signal } \sigma \text{ will exponentially converge to the ultimate ball as}
\end{align*}
\]
\[ \|f\| = \max(\|b_1, b_2, b_3\|). \]  Since the ball shrinks to the origin \(\bar{e}\) will converge to zero along the ball as \(t \to \infty\). Applying this fact, we have

\[ d_1 \|f\| \leq V_1 \leq d_2 \|f\| \]

\[ V_1 = -d_3 \|f\| \]

where \(d_1 = \min(M_n,1), d_2 = \max(M_n,1)\)

\[ d_3 = \lambda_{\min} \left[ \begin{array}{ccc} \psi & T^T & 0 \\ -T & \beta_1 I_n & I_n \\ 0 & -I_n & \beta_2 I_n \end{array} \right] \]

Hence \( \|f(t)\| \leq \sqrt{d_2/d_1 \|f(0)\| e^{-\frac{d_3}{2}}}, \) this meaning that the control law is smooth and bounded as \(t \to \infty\). Therefore the results of step1 is achieved such that the \(e_x, e_\theta\) will converge to zero as \(t \to \infty\). Moreover, the contact force asymptotically tracks to \(\lambda_x(t)\).

4. SIMULATION RESULTS

Here a two-link constrained robotic manipulator with revolute flexible joint, as shown in Fig.1. Take \(m_1 = m_2 = a_1 = a_2 = 1\) and gravity \(g = 9.8\) m/s\(^2\), and the actuator inertial, stiffness matrices of the rotor subsystem are set as:

\[ J = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad K = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}. \]

The constraint which is indeed by restricting the robot end-effector to keep contact with the environment can be described in terms of the Cartesian coordinate as \(x=1\). Through the forward kinematics, we can obtain the implication

\[ x = 1 \Rightarrow \cos(q_1) + \cos(q_2 + q_3) = 1 \]

The control design is performed in assumption that \(m_1, m_2\) are unknown. The initial states are chosen to be \(q_1(0) = 0.5, q_2(0) = 0, \hat{m}_1 = 1.2, \hat{m}_2 = 1.2\). Select

\[ \psi = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}, \quad \beta_1 = \begin{bmatrix} 30 & 0 \\ 0 & 35 \end{bmatrix}, \quad \beta_2 = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, \]

\[ \Lambda = 20, \eta = 0.001. \]

The desired joint trajectory and force trajectory are chosen as \(\lambda_j = 5 + \sin(\pi t), q_{2d} = \pi(\frac{1}{2} + \frac{1}{2}\sin(\pi t))\). The desired trajectory \(q_{2d}\) and the actual joint position \(q_j\) of the simulation are shown in Fig. 2. The desired velocity \(q_{2d}\) and the joint velocity \(\dot{q}_j\) are shown in Fig. 3. While the desired \(\lambda_j\) and the constraint force \(\lambda\) are shown in Fig. 4. The toques of robot are shown in Fig. 5.

The expected performance can be verified from these numerical results.

5. CONCLUSIONS

An approach based on a smooth sliding-mode controller has been successfully developed for motion control and force control of constrained robots having parametric uncertainty. Some properties of the reduced dynamics of constrained robots have been presented and exploited to demonstrate the motion and tracking errors to zero.

The numerical simulation of a two-link constrained manipulator driven through flexible shafts has been set up to demonstrate the performance of the proposed controllers.

REFERENCES


