Abstract: The paper illustrates the design of a power system voltage control scheme based on the model reference adaptive approach. Firstly, for an assigned operating condition, a power system approximated linear model is build-up which represents the system dynamics as seen from the busbar at which the controller is connected. Then, by imposing a desired reference model, the controller parameters are obtained by solving the model-following problem which ensures that the controlled voltage tracks the voltage reference. In presence of unknown operating conditions changes, the regulator parameters are varied according to an adjustment laws designed on the basis of the gradient rule. The task of the adaptation mechanism is to counteract the effects of such changes. Referring to a Static VAR System (SVS) controllable compensator device the adopted model-reference adaptive control scheme has been designed and its performance analyzed by accurate numerical time simulation studies in the case of both unknown load variations and changes in the power system structure, in particular line opening. Copyright © 2005 IFAC

Keywords: Power system control, model-reference adaptive control, model approximation.

1. INTRODUCTION

Secure and reliable operation of electric power transmission and distribution systems is guaranteed by active power control and by voltage/reactive power control. In theory, voltage amplitude must be regulated at all nodes, but applying direct nodal voltage control at all system busbars is impracticable and uneconomical. Then, some devices, called compensators, are used to support an adequate voltage profile in the whole system by injecting reactive power at some key nodes. The compensators are mainly synchronous or static machines; they vary the reactive power injection to regulate the voltage amplitude of the busbar at which they are connected. The local nodal voltage regulator follows a reference signal which is determined by another centralized control system. However, the performance of the compensator is strictly related to the power system operating conditions and topology which change unpredictably and unexpectedly due to various factors, e.g., variations of the loads and of the generation, forced outages of components, etc..

In these circumstances the compensator voltage regulation may worsen its performance and, in extreme cases, may become unstable. An effective strategy to counteract these problems lead to the adoption of adaptive control techniques. Closed-loop adaptive control methods can be divided into model reference adaptive systems and self-tuning regulators. The former has lower computing complexity of the adaptation algorithm and speed of the adaptation process. It is especially important for improving transient stability in presence of fault, see for example (Gao et al., 1992; Wang et al., 1994). In these cases, the whole power system is assumed to be represented by a single machine infinite bus model. In the case of voltage regulation, the self-tuning technique has been adopted in papers (Chengxiong et al., 1990; Fusco et al., 2001; Fusco and Russo, 2003) to design an adaptive control scheme starting from a steady-state power system model represented by the Thevenin circuit (Kundur, 1994). Following the adaptive approach, this paper presents a control scheme designed on the basis of the model...
reference adaptive control systems (Åström and Wittenmark, 1989) to control the rms (root-mean-square) busbar voltage \( v(t) \). According to such technique, the regulator parameters are adjusted without the help of any identification procedure. The first step consists of building-up an approximate dynamic linear model of the power system as seen from the controlled busbar in a given operating condition. Then a controller based on the solution of the model-following problem is designed to ensure that \( v(t) \) tracks \( v_m(t) \) which represents the desired output of a reference model. In presence of power system operating conditions variations, the regulator parameters are modified according to an adaptation mechanism. The adjustment laws are designed on the basis of the gradient approach. The proposed adaptive voltage regulation control scheme is then applied to the case of a Static VAR System (Cigre, 1992) connected to a High Voltage (HV) power system. Accurate numerical-time simulations, both in the case of load step variations and line opening, have been run to evaluate the performance achievable by the controller.

2. MODEL-REFERENCE ADAPTIVE CONTROL DESIGN

The model-reference adaptive control scheme adopted for nodal voltage control is represented in Figure 1. Starting from a power system model, see Subsection 2.1, the regulator parameters values are determined by resorting to the model-following approach (Åström and Wittenmark, 1989) which ensures that the closed-loop transfer function is close to the reference model, see Subsection 2.2. In this way the output of the power system model is close to the output of the reference model and the error

\[
e(t) = v(t) - v_m(t)
\]  

is small. The task of the adjustment mechanism is to vary the controller parameters in order to make the error small in presence of operating conditions variations. The adjustment laws are described in Subsection 2.3.

2.1 Plant model

A general plant is formed by controllable actuator devices and by the power system which mainly includes transformers, transmission lines, synchronous generators with their field excitation, static and dynamic loads. It represents a nonlinear system with variable operating conditions. In literature various power system models have been proposed according to the design problem. In the voltage control problem, since the short-circuit power of the regulation node is finite, the single machine infinite bus model cannot be adopted. In this context, power system steady-state frequency models (Cigre, 1992; Kundur, 1994) are usually used, such as the ones represented by a simple short-circuit impedance or by the Thevenin equivalent circuit referred to the regulation node. These models account for the power system nonlinearities but neglect its dynamics. Another power system model is represented by the power-flow balance equations (Kundur, 1994; Sauer and Pai, 1998) describing the system behavior in normal steady state operating condition. However such models were linearized around an assigned operating point to produce a linear dynamic model for control design, see for example (Chaudhuri et al., 2004). Concerning the adopted actuators, approximated models are usually available. To solve the model-following problem it is necessary to build-up a plant linear model. For this aim let’s consider the following model

\[
M(z) \left( v(k) - v_0(k) \right) = N(z) u(k - d) 
\]  

in which \( v_0(k) \) is an unknown constant bias representing the no-load voltage (Kundur, 1994). The goal consists in choosing the degrees \( n_M \) and \( n_N \) of polynomials \( M(z) \) and \( N(z) \) as well as in identifying their coefficients starting from data obtained by field measurements or by accurate numerical time simulations of the overall plant in an assigned operating condition. In the case of this paper we will refer to data output by simulations. The value of the delay \( d \) is usually known since it depends on the choice of the electronic device used as actuator. In details, the considered power system is shown in Figure 2 in which the electronic device is represented by a SVS compensator, constituted by a Fixed Capacitor-Thyristor Controlled Reactor, see Figure 3, whose time delay is approximatively equal to \( T_D = 0.0034 \) s (IEEE Working Group, 1994). The three-phase 132 kV - 50 Hz power system is assumed to be balanced in all its components. The transmission lines are represented through the series of elementary cells, each one representing the equivalent circuit for a length of a 10 km. In particular each cell is constituted by a series resistance, series inductance and shunt capacitance. Loads are represented by means of shunt resistors and inductances. The 10 MVAR SVS is simulated in the time domain, including active losses and detailed modeling of the thyristors. Concerning voltages, reference is
made in the following to the phase voltage rms values expressed in per unit on a 100 kV base; consequently, the rated phase voltage is equal to 0.76. In the assumed operating condition loads \( L_3 \) and \( L_4 \) are equal to 100 MW and 67 MW, respectively, both with a lagging power factor equal to 0.9. To collect the input/output data necessary to identify model (2) let consider the open-loop scheme shown in Figure 4 in which the nonlinear function

\[
f(\alpha) = \frac{2\alpha}{\pi} - \sin(2\alpha) - 1.
\]

The values of \( u(t) \) and \( v(t) \) at the sampling instants \( t = kT \), have been stored with a sampling period equal to \( T_s = 1 \) ms with reference to a step variation of \( u(t) \) from 0 to 0.4. Since \( T_s = 1 \) ms the delay \( d \) is equal to 3. When \( u(k) \) is equal to zero, the mean value of \( v(k) \) coincides with the amplitude \( D \) of the no-load voltage \( v_0(k) \). In this case \( D = 0.757 \) has been determined. For a correct identification of \( n_M, n_N, M(z) \) and \( N(z) \), the value of \( D \) has been subtracted to the stored data of \( v(k) \). To estimate \( n_M \) and \( n_N \) a classical Least-Squares technique has been then applied to calculate the residual prediction error, that is

\[
R_0 = \frac{1}{N} \sum_{k=1}^{k=N} e(k)^2.
\]

If the condition

\[
R_0, n_M + 1 > 0.8 R_0, n_M
\]

is verified, the degree \( n_M \) is no longer increased. The same criteria stands for \( n_N \) (Landau, 1990). According to this procedure one has

\[
\begin{align*}
n_M &= 2 \\
n_N &= 0 \\
M(z) &= z^{-2} - 1.831 z + 0.8404 \\
N(z) &= 0.0008866.
\end{align*}
\]

and model (2) is particularized as

\[
(1 - 1.831 z^{-1} + 0.8404 z^{-2}) v(k) - 0.0008866 z^{-3} u(k) \]

\[
= \frac{1 - 1.831 z^{-1} + 0.8404 z^{-2}}{1 - z^{-1}} 0.757.
\]

At this point mapping model (3) in the \( s \) plane one has (Franklin et al., 1990)

\[
V(s) = \frac{966.6 e^{-0.0034 s}}{(s^2 + 173.8 s + 9860)} U(s) + \frac{0.757}{s}
\]

where \( V_0(s) \) is expressed as

\[
A_d(s) V_0(s) = D
\]

being \( A_d(s) = s \). It can be easily recognized that model (4) presents two dominant complex poles modeling the power system dynamics, while the electronic actuator, whose dynamics are faster than the dominant poles, is modeled only with a delay term. By resorting to the first order Padé approximation for the delay term \( e^{-0.0034 s} \) yields

\[
\begin{align*}
V(s) &\approx \frac{966.6(-s + 2/0.0034)}{(s^2 + 173.8 s + 9860)(s + 2/0.0034)} U(s) \\
&+ \frac{0.757}{s} \frac{B(s)}{A(s)} U(s) + V_0(s).
\end{align*}
\]

Figure 5 shows a comparison between the step response of model (6) and of the scheme depicted in Figure 4. The approximated linear model gives an acceptable step response; the main difference from the actual response appears at the end of the rising edge. When the power system operating conditions change, the coefficients of polynomials \( A(s) \) and \( B(s) \) will vary as well as the bias amplitude \( D \). Conversely, the delay terms will be obviously unchanged. In the remainder it will be set \( K_P = 966.6 \). For control design purpose, the polynomial \( B(s) \) appearing in (6) is factorized as follows

\[
B(s) = B^-(s) B^+(s)
\]

in which \( B^-(s) \) contains the unstable zeros of \( P(s) \) while \( B^+(s) \) contains the remaining factors of \( B(s) \). It is immediate to recognize that in this case \( B^-(s) = B(s) \) and, obviously, \( B^+(s) = 1 \).

\subsection{Model-following design}

The regulator designed according to the model following approach is such that the relation between the reference voltage \( V_{\text{ref}} \) and the desired voltage is given by

\[
V_m(s) = \frac{B_m(s)}{A_m(s)} V_{\text{ref}}(s)
\]

where \( B_m(s) \) and \( A_m(s) \) are assigned polynomials. The regulator law assumes the following form

\[
F(s) U(s) = H(s) V_{\text{ref}}(s) - G(s) V(s)
\]
This equation has solution with
Fig. 6. Voltage regulator block scheme.
which includes a feedback term with transfer operator \(-G(s)/F(s)\) and a feedforward term with transfer operator \(H(s)/F(s)\). The block scheme representing the regulator structure is shown in Figure 6. By combining equations (6) (7) and (9) it is simple to obtain the closed-loop expression of the voltage \(V\) which is given by

\[
V(s) = \frac{B(s)H(s)}{A(s)F(s) + B^-(s)G(s)} V_{\text{ref}}(s) + \frac{A(s)F(s)}{A(s)F(s) + B^-(s)G(s)} \left( \frac{D}{A_d(s)} \right). \tag{10}
\]

To take into account the presence of the voltage bias \(V_0\) the regulator should include its dynamic expression (5), that is

\[
F(s) = \tilde{F}(s) A_d(s). \tag{11}
\]

According to (11) the expression (10) becomes

\[
V(s) = \frac{B(s)H(s)}{A(s)A_d(s)\tilde{F}(s) + B^-(s)G(s)} V_{\text{ref}}(s) + \frac{A(s)\tilde{F}(s)}{A(s)A_d(s)\tilde{F}(s) + B^-(s)G(s)} D. \tag{12}
\]

The polynomials \(\tilde{F}(s)\) and \(G(s)\) are obtained by solving the following Diophantine equation (Aström and Wittenmark, 1989)

\[
A(s) A_d(s) \tilde{F}(s) + B^-(s) G(s) = A_0(s) A_m(s), \tag{13}
\]

in which \(A_0(s)\) is an assigned observer polynomial. This equation has solution with

\[
dg G(s) < dg A(s) + dg A_d(s) = 4.
\]

where \(dg\) stands for degree. In the following it is assumed

\[
G(s) = g_3 s^3 + g_2 s^2 + g_1 s + g_0.
\]

In addition, the following conditions must be fulfilled

\[
B_m(s) = B^-(s) B_m', \tag{14}
\]

\[
dg A_m(s) - dg B_m(s) \geq dg A(s) - dg B^-(s) \tag{15}
\]

\[
dg A_0(s) \geq 2 dg A(s) + dg A_d(s) - dg A_m(s) - 1 \tag{16}
\]

Condition (14) imposes that \(B^-(s)\) divides \(B_m(s)\). The simplest structure for the polynomial \(B_m(s)\) satisfying the imposed constrain is

\[
B_m(s) = b_{1m}s + b_{0m}.
\]

At this point, having in mind the expression of \(B^-(s)\), see model (6) and factorization (7), one obtains that condition (14) is fulfilled if

\[
b_{0m}/b_{1m} = -2/T_d.
\]

Furthermore, to assure an unitary dc gain of the transfer function in (8) \(b_{0m}\) must be equal to \(A_m(0)\). According to (14) one has

\[
B_m = \frac{T_d A_m(0)}{2K_p}.
\]

Since the degree of \(B_m(s)\) is unitary, condition (15) is satisfied if \(A_m(s)\) has a degree greater or equal to the one of \(A(s)\), see model (6). In the reminder \(A_m(s)\) is assumed of the fourth order. Eventually, condition (16) is satisfied if the degree of polynomial \(A_0(s)\) is greater than two; in the reminder, it will be set \(dg A_0(s) = 3\). According to the degrees of polynomials \(A_m(s)\) and \(A_0(s)\) a third order polynomial is assumed for \(\tilde{F}(s)\), see equation (13), that is

\[
\tilde{F}(s) = s^3 + f_2 s^2 + f_1 s + f_0.
\]

The coefficients of the polynomials \(\tilde{F}(s)\) and \(G(s)\) are calculated by solving a set of 7 linear equations which are obtained by substituting all the polynomials into (13). Once that \(\tilde{F}(s)\) has been determined the polynomial \(F(s)\) is obtained through (11). It is simple to recognize that \(F(s)\) is of fourth degree and it posses a root in \(s = 0\). Finally, the polynomial \(H\) is given by

\[
H(s) = A_0(s) B_m' = h_3 s^3 + h_2 s^2 + h_1 s + h_0.
\]

2.3 \textbf{Regulator parameters adjustment laws}\n
The adjustment mechanism has the objective to vary the coefficients of the regulator polynomials on the basis of the error \(e\) defined in (1) when changes in power system occur. To pursue this task the adaptation law is designed employing the gradient approach (Aström and Wittenmark, 1989) to reduce the function

\[
J(t) = \frac{1}{2} e^2(t).
\]

This is accomplished by changing the regulator parameters in the direction of negative gradient of \(J\)

\[
\frac{\text{d}}{\text{d}t} f_i = -\gamma_{f,i} \frac{\partial J}{\partial f_i} = -\gamma_{f,i} \frac{\partial e}{\partial f_i} \quad i = 0, 1, 2 \tag{17}
\]

\[
\frac{\text{d}}{\text{d}t} g_j = -\gamma_{g,j} \frac{\partial J}{\partial g_j} = -\gamma_{g,j} \frac{\partial e}{\partial g_j} \quad j = 0, 1, 2, 3
\]

\[
\frac{\text{d}}{\text{d}t} h_k = -\gamma_{h,k} \frac{\partial J}{\partial h_k} = -\gamma_{h,k} e \frac{\partial e}{\partial h_k} \quad k = 0, 1, 2, 3
\]

where \(\gamma_{f,i}\), \(\gamma_{g,j}\), \(\gamma_{h,k}\) are adequate positive rating factors. The implementation of laws (17) requires the calculation of the sensitivity derivatives. Based on (1), by subtracting (8) to (12) one obtains the expression in the Laplace domain of the error \(E(s)\) as

\[
E(s) = \frac{B(s)H(s)}{A(s)A_d(s)\tilde{F}(s) + B^-(s)G(s)} V_{\text{ref}}(s) + \frac{A(s)\tilde{F}(s)}{A(s)A_d(s)\tilde{F}(s) + B^-(s)G(s)} D
\]
The derivatives of $E(s)$ with respect to the regulator parameters are then

$$\frac{\partial E(s)}{\partial f_i} = - \frac{s^i B^-(s)A(s)A_d(s)}{A(s)A_d(s)\dot{F}(s) + B^-(s)G(s)} U(s)$$

$$\frac{\partial E(s)}{\partial g_j} = - \frac{s^j B^-(s)}{A(s)A_d(s)\dot{F}(s) + B^-(s)G(s)} V(s)$$

$$\frac{\partial E(s)}{\partial h_k} = - \frac{s^h B^-(s)}{A(s)A_d(s)\dot{F}(s) + B^-(s)G(s)} V_{\text{ref}}(s)$$

Finally, by expressing the derivatives (19) in term of the differential operator $p$ and substituting in (17) yields

$$\frac{d}{dt} f_i \approx \tilde{\gamma}_{f,i} c(t) \left[ \frac{p^{i+1}(-p + 2/T_d)}{A_0(p)A_m(p)} u(t) \right]$$

$$\frac{d}{dt} g_j \approx \tilde{\gamma}_{g,j} c(t) \left[ \frac{p^j(-p + 2/T_d)}{A_0(p)A_m(p)} v(t) \right]$$

$$\frac{d}{dt} h_k \approx -\tilde{\gamma}_{h,k} c(t) \left[ \frac{p^h(-p + 2/T_d)}{A_0(p)A_m(p)} v_{\text{ref}}(t) \right]$$

where the gain $K_{p,n}$ has been absorbed into $\tilde{\gamma}_{f,i}$ and $\tilde{\gamma}_{g,j}$ respectively.

3. SIMULATION CASE STUDY

To analyze the performance achievable by the voltage regulator control scheme designed on the basis of model reference adaptive approach, accurate time-domain simulations have been run with reference to a case study. The simulation is performed in MATLAB/SIMULINK environment by resorting also to Power System Blockset. Concerning the model-following design, the assumed reference model (8) is characterized by real poles and a step response without overshoot, a rising time between 10 and 90% equal to about 50 ms (that is, 2.5 times the fundamental cycle) and a settling time at $\pm 2\%$ equal to about 90 ms (that is, 4.5 times the fundamental cycle). In the adaptation laws (20) the rating factors $\tilde{\gamma}_{f,i}$, $\tilde{\gamma}_{g,j}$ and $\tilde{\gamma}_{h,k}$ have been chosen as follows:

$$\tilde{\gamma}_{f,0} = \tilde{\gamma}_{g,0} = \tilde{\gamma}_{h,0} = 0$$

$$\tilde{\gamma}_{f,1} = \tilde{\gamma}_{g,1} = \tilde{\gamma}_{h,1} = 1$$

$$\tilde{\gamma}_{f,2} = \tilde{\gamma}_{g,2} = \tilde{\gamma}_{h,2} = 2$$

$$\tilde{\gamma}_{f,3} = \tilde{\gamma}_{g,3} = \tilde{\gamma}_{h,3} = 3$$

where $\gamma_0 = 10^5$, $\gamma_1 = 10^{-1}$, $\gamma_2 = 10^{-6}$ and $\gamma_3 = 10^{-8}$. It should be noted that this choice for the rating factors is adopted because in (20) the variables $\gamma_{ref}(t)$, $u(t)$, $v(t)$ and $e(t)$ have been expressed in p.u. values. The presented simulation case study aims at showing the performance achieved by the proposed adaptive voltage control scheme when the power system operating condition is unknown. In particular, the operating condition is different from the one assumed to identify the parameter values of model (4) necessary to solve the model-following problem, because the load $L_4$ is increased by $20\%$, and the voltage reference is assumed equal to 0.75. Obviously, in this operating point the amplitude of no-load voltage is unknown. Starting from this condition, the power system is firstly subject to a load variation, then to a topological change and finally a new voltage reference value is imposed. In particular, at time instant $t = 0.3$ s a $20\%$ step increase of the load $L_4$ is considered. It must be noted that the amplitude of such variation is comparable to the rated power of the considered SVS. Due to this overload, the transmission line between busbar 3 and busbar 4 is disconnected at time $t = 0.6$ s. Then a $20\%$ step increase of the voltage reference signal is imposed. The time evolution of the controlled voltage at busbar 4 is reported in Figure 7. For the sake of comparison, in the same Figure it is also shown the time evolution of the voltage at busbar 4 in the case of classical design (IEEE Working Group, 1994) which requires the adoption of a simple gain with an integral action. The gain value has been set equal to 170 according to the worst short-circuit power of the electrical system. From the analysis of Figure 7 it can be recognized that the adaptive control scheme is able to restore the voltage to its reference value equal to 0.75 in about 0.12 s after the perturbation occurred. Contrary, the classical scheme exhibits a slower response with amplitude variations larger than the ones given by the adaptive scheme. In Figure 8 the time evolution of the error given by (1) is shown. To analyze this figure, it is worthwhile to note that during the structural and topological changes of the power system the reference model output is constant and equal to 0.75; consequently, the time evolution of the error exactly reproduces the time evolution of the voltage. On the contrary, during the voltage
reference step variation, the error evolution shows the effectiveness of the adaptation task, which guarantees a voltage response very close to the one of the reference model in spite of the different operating conditions of the system. After all, from Figure 7 it can be easily verified that, in the case of adaptive design, the rising time and the settling time of the step response corresponds to the ones characterizing the reference model. Eventually, in Figure 9 the time evolution of the firing angle $\alpha$ is reported, showing the command sent to the SVS thyristors by the controller.

4. CONCLUSIONS

This paper has presented a nodal voltage control scheme in power systems based on the adoption of model-reference adaptive approach. Using a simplified power system model, the voltage regulator parameters have been obtained by solving the model-following problem. To counteract the effects of unexpected and unpredictable changes of power system operating conditions, an adjustment mechanism varies the controller parameters to guarantee robustness and an adequate performance of the designed control scheme. The proposed model-reference adaptive control system has been applied to a Static VAR System connected to a HV power system. Accurate numerical simulations have been run to validate the performance achievable by the proposed model-reference adaptive control scheme.

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