Abstract: Output stabilization of uncertain discrete-time nonlinear models via an observer is a problem that can be considered through a Takagi-Sugeno framework. This work provides systematic design procedures by using direct Lyapunov’s method with a non quadratic Lyapunov’s function. This leads to LMI conditions that can be efficiently solved. This result always includes the quadratic case. Copyright © 2005 IFAC

Keywords: fuzzy, output stabilization, observer design, stabilization, robust control.

1. INTRODUCTION

The stabilization of uncertain nonlinear models is a large domain. A way to get systematic procedures to derive control laws is to use the model in one of its Takagi-Sugeno (TS) (Takagi and Sugeno 1985) form. This way commonly uses the direct Lyapunov’s method to get LMI (Linear Matrix Inequalities) conditions. Solutions that can satisfy such a problem can be found efficiently by using the interior point algorithm (Boyd et al. 1994). The first works used a quadratic Lyapunov function and a parallel distributed compensation (PDC) control law (Wang et al. 1996; Tanaka et al. 1998). Other Lyapunov functions were also proposed. In the continuous case a fuzzy Lyapunov function were used (Blanco, et al. 2001; Tanaka et al. 2001) but the presence of the derivative of the activation function makes the results “poor”. Interesting works are related to piecewise Lyapunov functions (Johansson et al. 1999). Finally in the discrete case, some results are presented in (Guerra and Vermeiren 2004) which use a non quadratic fuzzy Lyapunov function that shares the same rules than the models. The use of this Lyapunov function ensures that the solutions obtained in the quadratic case are included.

The stabilization of uncertain models received an increasing attention. Several results are available in the continuous and the discrete case. Examples of results using norm bounded linear models are given in (Xie and De Souza 1992). For uncertain Takagi Sugeno models some results are available too. An example, in the continuous case with the utilization of a quadratic Lyapunov function, the stabilization via state feedback is given in (Lee et al. 2001). And finally in the discrete case with or without state delays with the utilization of a non quadratic Lyapunov function, some results are given in (Guerra et al. 2004).

In this work, we are interested in the uncertain discrete Takagi Sugeno’s fuzzy models case. A non quadratic Lyapunov’s function is designed to obtain LMI conditions for the output stabilization of uncertain TS fuzzy models via an observer and a non PDC control law.

The paper is organized as follows. A first part presents the notations and some basic properties. These ones are mainly useful matrix properties that allow dealing with uncertainties for discrete models. A relaxation scheme for LMI conditions is also proposed (Liu and Zhang 2003). The main result is then presented. This one gives the LMI conditions to be satisfied for a control law together an observer to stabilize an uncertain discrete fuzzy model. They are obtained using a non quadratic Lyapunov function. The obtained results are shown to always include the quadratic case. At last an example is provided to show the effectiveness of the proposed approach.
2. NOTATIONS AND MATERIAL

We consider the following notations, with $h_i(\cdot) \geq 0$ scalar positive functions and matrices of the same dimension $Y_i$, $i \in \{1, \ldots, r\}$:

$$Y_r = \sum_{i=1}^{r} h_i(z(t))Y_i, \quad Y_{r,r} = \sum_{i=1}^{r} h_i(z(t+1))Y_i,$$

$$Y_i^{-1} = \left( \sum_{i=1}^{r} h_i(\cdot)Y_i \right)^{-1}. $$

A star (*) indicates a transpose quantity in a symmetric matrix. Congruence of a symmetric definite positive matrix $P = P^T > 0$ with $Y$ corresponds to the following quantity: $YPY^T > 0$.

The next lemmas will be useful in the paper. The first one is well known (Xie and De Souza, 1992). The second one corresponds to a property given in (De Oliveira et al. 1999) and modified in the context of non-quadratic stabilization of discrete TS models (Guerra and Vermeiren 2004).

Lemma 1: With $X$, $Y$ and $F = F^T > 0$ matrices of appropriate dimension the following inequality holds.

$$X^TY + Y^TX \leq X^TFX + Y^TF^{-1}Y $$

(1)

Lemma (Schur’s complement): With $P = P^T > 0$, $R > 0$ and $X$ matrices of appropriate dimensions, the two following properties are equivalent:

(i) $P - X^TR^{-1}X > 0 \iff (ii) P(X^*) > 0$

Lemma 2 (extension to the Schur’s complement): With $P = P^T > 0$, $R > 0$ and $X$ matrices of appropriate dimensions, the two following properties are equivalent.

(i) $P - X^TRX > 0$ \hspace{1cm} (2)

(ii) There exists $\Psi$ a matrix of appropriate dimensions such that:

$$
\begin{bmatrix}
P & (\Psi) \\
\Psi X & \Psi + \Psi^T - R
\end{bmatrix} > 0
$$

(3)

Proof: Sufficiency: If $P - X^TRX > 0$, then $P - X^TR^{-1}RX > 0$ and with Schur’s complement (3) is obtained with $\Psi = R$. Necessity: Using the congruence with the row full rank matrix $[I - X^T]$ on the expression (3) gives immediately the result.

Remark 1: If $\Psi$ is under constraint, the sufficiency may not be more true.

The models considered are nonlinear affine models. Takagi-Sugeno’s models can exactly represent such models in a compact set of the state variables (Tanaka and Wang 2001). There exists a systematic way to put affine nonlinear models in the TS form:

$$x(t+1) = f(x(t)) + g(x(t))u(t) $$

(4)

$$x(t+1) = \sum_{i=1}^{r} h_i(z(t)) (A_i x(t) + B_i u(t)) $$

(5)

With $z(t)$ the premise vector which depends on state vector $x(t)$ and $r$ represents the number of linear models. This last number grows exponentially according to the number of nonlinearities involved in the nonlinear model (Tanaka et al. 1998; Taniguchi et al. 2001). Note also that the TS representation of (4) is not unique (Taniguchi et al. 2001). By using this representation of the nonlinear models, the goal is to derive output stabilization conditions that only depend on the parameters of the linear models and the parameters of the control law and the observer. Excepted the convex sum property, no specific property of the membership functions $h_i(z(t))$ are used. This means that the conditions will always be sufficient ones. Then, another goal is to get the less conservative conditions, i.e.: find conditions that solve the biggest set of problems.

The obtained conditions derived from the stabilization of TS models are usually in the following form:

$$Y_{\alpha}^* = \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} \sum_{l=1}^{r} h_i(z(t))h_j(z(t))h_k(z(t))h_l(z(t+1))Y_{\alpha}^i > 0 \hspace{1cm} (6)$$

A way to get LMI conditions from (6) is to use the following relaxation scheme (Lai and Zhang 2003) based on a first one due to (Kim and Lee 2000) which is a good compromise between conservatism and complexity. This one will be used in all the theorems for the stabilization conditions. In our case, it corresponds to the following writing, with:

$$Q_i^* = (Q_i^*)^T > 0, $$

$$Q_i^* = (Q_i^*)^T, \hspace{0.5cm} i,j,k \in \{1, \ldots, r\}, \hspace{0.5cm} j > i$$

$$Y_{\alpha}^* > Q_i^* $$

(7)

$$Y_{\alpha}^* + Y_{\beta}^* > Q_i^* + Q_j^* \hspace{0.5cm} j > i, i,j,k \in \{1, \ldots, r\} $$

(8)

$$\Psi^{\alpha} = \begin{bmatrix} Q_i^* & Q_{i1} & \cdots & Q_{i,r} \\ Q_{i1} & Q_i^* & \cdots & \vdots \\ \vdots & \vdots & \ddots & Q_{i,j} \\ Q_{i,r} & \cdots & Q_{i,j} & Q_i^* \end{bmatrix} > 0 $$

(9)

3. STATEMENT OF THE PROBLEM AND MAIN RESULT

Consider the following TS uncertain fuzzy model with measurable premises:

$$\begin{align*}
\dot{x}(t+1) &= (A_x + \Delta A_x)x(t) + (B_x + \Delta B_x)u(t) \\
y(t) &= C_x x(t)
\end{align*}$$

With the uncertainties written as:

$$\Delta A_x \ \Delta B_x = \begin{bmatrix} Ha & Hb \end{bmatrix} \begin{bmatrix} \Delta a(t) 0 \\ 0 \Delta b(t) \end{bmatrix} \begin{bmatrix} E_{a_x} \\ E_{b_x} \end{bmatrix} $$

$$\begin{bmatrix} \Delta a(t) \Delta a(t) 0 \\ 0 \Delta b(t) \Delta b(t) \end{bmatrix} < \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} $$

(11)

Consider also the non PDC control law:

$$u(t) = -F_x G_x^{-1} \dot{x}(t) $$

(12)
and the following TS fuzzy observer:
\[
\hat{x}(t+1) = A_x \hat{x} + B_x u(t) + S_x^{-1} K_x (y(t) - \hat{y}(t)) \quad (13)
\]

With: \(\hat{x} = x - \hat{x}\) the error of prediction we can write:
\[
\hat{x}(t+1) = \left( A_x - S_x^{-1} K_x C_x + \Delta B_x F_x G_z^{-1} \right) \hat{x}(t) + \left( \Delta A_x - \Delta B_x F_x G_z^{-1} \right) x(t) \quad (14)
\]

The closed loop (TS model + control law + observer) is written as:
\[
\begin{bmatrix}
  x(t+1) \\
  \hat{x}(t+1)
\end{bmatrix} = 
\begin{bmatrix}
  \bar{A}_x - \bar{B}_x F_x G_z^{-1} \\
  \Delta A_x - \Delta B_x F_x G_z^{-1}
\end{bmatrix}
\begin{bmatrix}
  x(t) \\
  \hat{x}(t)
\end{bmatrix} + \begin{bmatrix}
  (B_x + \Delta B_x) F_x G_z^{-1} \\
  S_x A_x - K_x C_x + S_x \Delta B_x F_x G_z^{-1}
\end{bmatrix} \delta(t) \quad (15)
\]

with \(\bar{A}_x = A_x + \Delta A_x\) and \(\bar{B}_x = B_x + \Delta B_x\)

The following Lyapunov function which is clearly a non quadratic one is chosen:
\[
V(x, \hat{x}) = x^T(t) \Gamma x(t) + \hat{x}^T(t) \check{P} \hat{x}(t) \quad (16)
\]

Remark 2: This function is a Lyapunov one and the proof can be found in (Guerra and Vermeiren 2004).

The variation of the Lyapunov function (16) is negative if (19) holds (bottom of the page).

After congruence with
\[
\begin{bmatrix}
  G_z^T & 0 \\
  0 & 1
\end{bmatrix}
\]

\(\Delta V(x(t)) < 0\) is true if (20) holds (bottom of the page). Then by using the lemma 2, (20) is verified if (21) is verified (bottom of the page).

The goal is to remove all terms that cannot be put in a linear form. First the terms with \(G_z^{-1}\) is treated. Consider from (21) the following terms:

\[
\begin{bmatrix}
  A_x + \Delta A_x - (B_x + \Delta B_x) F_x G_z^{-1} \\
  \Delta A_x - \Delta B_x F_x G_z^{-1}
\end{bmatrix}
\begin{bmatrix}
  x(t+1) \\
  \hat{x}(t+1)
\end{bmatrix} =
\begin{bmatrix}
  (B_x + \Delta B_x) F_x G_z^{-1} \\
  S_x A_x - K_x C_x + \Delta B_x F_x G_z^{-1}
\end{bmatrix} \delta(t) \quad (19)
\]

\[
\begin{bmatrix}
  (A_x + \Delta A_x) G_z - (B_x + \Delta B_x) F_x G_z^{-1} \\
  \Delta A_x G_z - \Delta B_x F_x G_z^{-1}
\end{bmatrix}
\begin{bmatrix}
  x(t+1) \\
  \hat{x}(t+1)
\end{bmatrix} =
\begin{bmatrix}
  (B_x + \Delta B_x) F_x G_z^{-1} \\
  S_x A_x - K_x C_x + \Delta B_x F_x G_z^{-1}
\end{bmatrix} \delta(t) \quad (20)
\]

\[
\begin{bmatrix}
  \begin{bmatrix}
    P_z \\
    \bar{P}_z
  \end{bmatrix} \\
  \begin{bmatrix}
    0 \\
    0
  \end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
  (A_x + \Delta A_x) G_z - (B_x + \Delta B_x) F_x \\
  \Delta A_x G_z - \Delta B_x F_x
\end{bmatrix}
\begin{bmatrix}
  \begin{bmatrix}
    P_z \\
    \bar{P}_z
  \end{bmatrix} \\
  \begin{bmatrix}
    0 \\
    0
  \end{bmatrix}
\end{bmatrix} + \begin{bmatrix}
  (B_x + \Delta B_x) F_x G_z^{-1} \\
  S_x A_x - K_x C_x + \Delta B_x F_x G_z^{-1}
\end{bmatrix} \delta(t) \quad (21)
\]

\[
\begin{bmatrix}
  \begin{bmatrix}
    -P_z \\
    -\bar{P}_z + G_z^{-T} (\Gamma + \Gamma_z) G_z^{-1}
  \end{bmatrix} \\
  \begin{bmatrix}
    0 \\
    0
  \end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
  (A_x + \Delta A_x) G_z - (B_x + \Delta B_x) F_x \\
  \Delta A_x G_z - \Delta B_x F_x
\end{bmatrix}
\begin{bmatrix}
  \begin{bmatrix}
    -P_z \\
    -\bar{P}_z + G_z^{-T} (\Gamma + \Gamma_z) G_z^{-1}
  \end{bmatrix} \\
  \begin{bmatrix}
    0 \\
    0
  \end{bmatrix}
\end{bmatrix} + \begin{bmatrix}
  (B_x + \Delta B_x) F_x G_z^{-1} \\
  S_x A_x - K_x C_x + \Delta B_x F_x G_z^{-1}
\end{bmatrix} \delta(t) \quad (22)
\]

Then using lemma 1 on \(N + N^T\), (21) is verified if (22) is satisfied (bottom of the page) with \(T > 0\) and:
\[
\Omega_{(3,3)} = -G_z + G_z^T + P_z
\]

\[
\Omega_{(4,4)} = -S_z + S_z^T + \bar{P}_z + S_z \Delta B_z F_z \Gamma_z + F_z^T \Delta B_z^T S_z^T
\]

Using lemma 2 for the expression \(G_z^{-T} (\Gamma + \Gamma_z) G_z^{-1}\) (with \(X = G_z^{-1}\) and \(\Psi = G_z\)) and the Schur’s complement for the last parts of terms \(\Omega_{(3,3)}\) and \(\Omega_{(4,4)}\) leads to the condition (23) (top of the next page).
The matrix in the inequality (23) can be split into two terms. The second one recovers all the uncertainties and the term $-R_i S_i^{-1} K_i C_i$. Let us define the second term noted $\Delta M + \Delta M^T$ where:

$\Delta M_{(4,0)} = [(R_i + I) H \Delta a][E a, G_i] - [(R_i + I) H \Delta b][E b, F_i]$

$\Delta M_{(4,3)} = [-R_i][S_i^{-1} K_i C_i]$

$\Delta M_{(5,4)} = [H \Delta a][E b, F_i] + [R_i H \Delta b][E b, F_i]$

$\Delta M_{(6,3)} = [S_i H \Delta a][E a, G_i] - [S_i H \Delta b][E b, F_i]$

$\Delta M_{(7,6)} = [F_i^T E b^T \Delta b_i^T][H b H^T S_i^T]$

Using lemma 1 on $\Delta M + \Delta M^T$ the following inequality is obtained, $\Delta M + \Delta M^T \preceq \Theta$ with $\Theta$ defined equation (26) (next page) and with $\varepsilon > 0$, $\mu > 0$, $\eta > 0$, $\lambda > 0$, $\gamma > 0$, $X > 0$ and $\delta > 0$.

So the inequality (23) is verified if (27) (next page) is satisfied with:

$\Theta_{1,1} = -P_i + e G_i^T E a^T E a G_i + \eta F_i^T E b^T E b F_i$

$\Theta_{4,4} = -G_i + G_i + P_i + \lambda^{-1} R_i H b H b^T R_i^T + R_i X_i^{-1} R_i^T$

$+ \eta^{-1} (I + R_i) H b H b^T (I + R_i)^T + \delta H b H b^T$

$+ \varepsilon^{-1} (I + R_i) H a H a^T (I + R_i)^T$

$\Theta_{5,5} = -\Gamma_i + \delta^{-1} F_i^T E b^T E b F_i + \lambda F_i^T E b^T E b F_i$

$\Theta_{6,6} = -S_i - S_i + \tilde{P}_{i,1} + \varepsilon^{-1} S_i H a H a^T S_i$

$+ \eta^{-1} S_i H b H b^T S_i^T + \mu^{-1} S_i H b H b^T S_i^T$

$\Theta_{2,2} = \Gamma_i + \mu F_i^T E b^T E b F_i$

Using the Schur’s complement on the terms of $\Theta$ excepted the expression $-\tilde{P}_{i,1} + C_i^T K_i S_i^{-1} X S_i^{-1} K_i C_i$

on which the lemma 2 is applied, (27) is satisfied if:

$\gamma_y = \gamma_y = \begin{bmatrix} \Xi_{1,1} & (\ast) & (\ast) & 0 \\ \Xi_{2,1} & \Xi_{2,2} & (\ast) & 0 \\ \Xi_{3,1} & \Xi_{3,2} & \Xi_{3,3} & 0 \\ 0 & 0 & 0 & \Xi_{4,4} \end{bmatrix} < 0$

Where all the matrices are given in (28), (29), (30) and (31) next page.

At last, notice that $\delta > 0$, $\Gamma_i > 0$, $\Gamma_i > 0$ and $X > 0$ are only multiplied with terms who are not depending on a LMI variable. Then they can be considered as unspecified sums of matrices as $\delta_{xx}^{i,1}$, $\Gamma_{xx}^{i,1}$, $\Gamma_{xx}^{i,2}$ and $X_{xx}^{i,2}$. These sums have to be positive definite but not each of their elements. Let us define:

$\beta = \text{diag}
\begin{bmatrix}
\varepsilon G_i^T E a^T E a G_i + \eta F_i^T E b^T E b F_i \\
C_i^T K_i S_i^{-1} X S_i^{-1} K_i C_i \\
0 \\
\varepsilon^{-1} (I + R_i) H a H a^T (I + R_i)^T + \eta^{-1} (I + R_i) H b H b^T (I + R_i)^T \\
+ \delta H b H b^T + \lambda^{-1} R_i H b H b^T R_i^T + R_i X_i^{-1} R_i^T \\
\delta^{-1} F_i^T E b^T E b F_i + \lambda F_i^T E b^T E b F_i \\
\varepsilon^{-1} S_i H a H a^T S_i + \eta^{-1} S_i H b H b^T S_i^T + \mu^{-1} S_i H b H b^T S_i^T \\
\mu F_i^T E b^T E b F_i
\end{bmatrix}$

(26)
\[
\begin{bmatrix}
(\Theta_{1,1}) - \hat{P}_i + C_i^T \hat{S}_i^T \hat{X} \hat{S}_i \hat{K}_i \hat{C}_i \\
0 - \hat{P}_i + C_i^T \hat{S}_i^T \hat{X} \hat{S}_i \hat{K}_i \hat{C}_i \\
0 - G_i - G_i^T + \Gamma_1 + \Gamma_2 \\
A_x G_x - B_x F_x \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
-\hat{P}_i + C_i^T \hat{S}_i^T \hat{X} \hat{S}_i \hat{K}_i \hat{C}_i \\
0 - \Gamma_1 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \end{bmatrix}
\]

\[
\begin{bmatrix}
G_i - G_i^T + \Gamma_1 + \Gamma_2 \\
-\hat{P}_i + C_i^T \hat{S}_i^T \hat{X} \hat{S}_i \hat{K}_i \hat{C}_i \\
0 - \Gamma_1 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \\
0 0 0 \end{bmatrix}\]

Theorem 1: Consider the uncertain discrete fuzzy model (10) together the control law (12) and the observer (13). With \( Y_0^k \) defined in (25) if there exists for given scalars \( \varepsilon > 0 \), \( \mu > 0 \), \( \eta > 0 \) and \( \lambda > 0 \) matrices \( P_i = P_i^T > 0 \), \( \hat{P}_i = \hat{P}_i^T > 0 \), \( X_i^k \), \( \Gamma_1^k \), \( \Gamma_2^k \), \( Q_i^k = (Q_i^k)^T > 0 \), \( Q_i^k = (Q_i^k)^T > 0 \), \( S_i \), \( K_i \), \( F_i \), \( R_i \) and \( G_i \), \( i, j, k \in \{ 1, \ldots, r \} \), \( j > i \) and scalars \( \delta_i^k \) such that the problem defined in (7), (8) and (9) is satisfied, then the closed loop fuzzy model is globally asymptotically stable.

Proof: Existence of the \( G_i^{-1} \). Let us notice that with \( Y_0^k \) defined in and condition (7) we have \( G_i + G_i^T > 0 \). Therefore \( \sum_{i=1}^{r} h_i \left( G_i + G_i^T \right) > 0 \) which ensures that \( G_i^{-1} \) exists. Then we must check that the function defined in (16) is a Lyapunov function candidate (See Guerra & Vermeiren 2004). Then, according to (24) and to the expression of the \( Y_0^k \), the variation of the Lyapunov function is negative if:

\[
\sum_{i=1}^{r} h_i \left( \sum_{j=1}^{r} \sum_{i=1}^{r} \sum_{j=1}^{r} h_j \left( Y_j^k + Y_j^k \right) \right) < 0
\]

With equations (7) and (8), (32) is verified if:

\[
\sum_{i=1}^{r} h_i \left( \sum_{j=1}^{r} \sum_{i=1}^{r} \sum_{j=1}^{r} h_j \left( Q_j^k + Q_j^k \right) \right) < 0
\]

and with the definition of \( \Psi_k \) and condition (9), (33) is true.

4. EXAMPLE

Considering the following nonlinear model in one of its Takagi-Sugeno's form:

\[
\begin{align*}
\dot{x}(t+1) &= (A_x + \Delta A_x)x(t) + (B_x + \Delta B_x)u(t) \\
y(t) &= C_x x(t)
\end{align*}
\]

\[
A_x = \begin{bmatrix} -0.1 & -0.2 \\ -1 & 0.9 \end{bmatrix}, \quad \Delta A_x = \begin{bmatrix} -0.5 & -1.2 \\ -0.09 & -0.9 \end{bmatrix}, \quad B_x = \begin{bmatrix} 4.1 \\ 0.8 \end{bmatrix}, \\
C_x = \begin{bmatrix} 3 \\ 0.1 \end{bmatrix}, \quad \Delta C_x = \begin{bmatrix} 0.5 & 2.5 \\ 0.5 & 2 \end{bmatrix}, \quad \Delta B_x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\
E_{A_x} = E_{B_x} = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \quad H_b = 0, \quad E_b = E_{b_2} = 0
\]
\begin{align*}
h_1(z) = 1 - h_2(z) = (1 - \cos(x_i(t))) / 2 \in [0, 1]
\end{align*}

The LMI solver SEDUMI gives a solution in the non quadratic case but not in the quadratic one (i.e. 
\[ P = G_i \text{ and } S_i = \hat{P} = \hat{P} \]) and this solution gives 
the following observer and control law parameters: 
\[ F_1 = \begin{bmatrix} -0.3395 & -0.7796 \end{bmatrix}, 
F_2 = \begin{bmatrix} -1.999 & -2.820 \end{bmatrix} \]
\[ G_1 = \begin{bmatrix} 12.61 & 9.3 \\ 4.609 & 10.94 \end{bmatrix}, 
G_2 = \begin{bmatrix} 9.629 & 2.132 \\ 1.888 & 9.312 \end{bmatrix} \]
\[ K_1 = \begin{bmatrix} -0.1084 \\ 0.1128 \end{bmatrix} \text{ and } K_2 = \begin{bmatrix} -0.2471 \\ 0.0191 \end{bmatrix}. \]

The next figures show the evolution of the state and its estimate, figure 1 and the evolution of the estimation error for each state, figure 2, for the following initial conditions: \( \hat{x}(0) = \begin{bmatrix} 5 \\ 3 \end{bmatrix} \) for the model and \( \hat{x}(0) = \begin{bmatrix} -3 \\ 5 \end{bmatrix} \) for the observer. 

Equipotentials of the Lyapunov function are also plotted figure 1 to show they are far to be ellipsoids.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{state_estimation.png}
\caption{State trajectory and its estimate.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{estimation_error.png}
\caption{Evolution of the estimation errors}
\end{figure}

5. CONCLUSION

In this paper, conditions for the output stabilization of a class of uncertain nonlinear discrete models are given. They are developed using a Takagi-Sugeno fuzzy model representative of the nonlinear model, this one being exact in a compact set of the state variables. These conditions were obtained by the utilization of a non quadratic Lyapunov function and some matrix properties and are given in the form of a LMI problem. This work seems to be the first to propose a non quadratic approach for the output stabilization via a nonlinear observer and a non PDC control law. This approach includes the classical quadratic approach. An example is given to show the effectiveness of the approach.

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