DEVELOPING D-OPTIMUM EXPERIMENTAL CONDITIONS FOR MODEL-BASED FAULT DETECTION SYSTEMS

Marcin Witczak *

* Institute of Control and Computation Engineering,
University of Zielona Góra,
ul. Podgórna 50, 65–246 Zielona Góra, Poland,
e-mail: M.Witczak@issi.uz.zgora.pl

Abstract: The main objective of this paper is to show an importance of developing suitable experimental conditions while designing and utilizing model-based fault detection systems. In particular, the paper shows the possibilities of exploiting the theory of optimum experimental design in parameter-estimation-based fault detection schemes. More precisely, a novel scheme for measuring and diagnosing an impedance is proposed. Copyright©2005 IFAC.

Keywords: parameter estimation, non-linear systems, experimental design, fault diagnosis, fault detection.

1. INTRODUCTION

A fault detection process (Chen and Patton, 1999; Korbiçz et al., 2004; Witczak, 2003) can be perceived as a two-stage procedure, i.e. residuals generation and symptom evaluation based on these residuals. The residual can be defined in many different ways depending on the choice of the fault diagnosis scheme, e.g. in the parameter-estimation-based scheme the residual is defined as a difference between the nominal and estimated values of the parameters while in most of the schemes the residual is perceived as difference between the output of the model and that of the system. The residual should ideally carry an information regarding a fault only. Under such an assumption, the faults can be detected by setting a fixed threshold on the residual. The fundamental difficulty with this kind of symptom evaluation is that the residuals are normally uncertain, corrupted by noise, disturbances and modelling uncertainty. That is why it is necessary to assign a threshold (significantly) larger than zero in order to avoid false alarms. This usually implies a reduction of fault detection sensitivity. Thus, threshold selection should be performed in such a way so as to attain a compromise between the fault detection sensitivity and the false-alarm rate. An obvious remedy to the above mentioned problems is to utilize the knowledge regarding model uncertainty. One of the possible approaches to gather such knowledge is to use statistical techniques (Atkinson and Donev, 1992; Uciński, 2005; Walter and Pronzato, 1997) to obtain parametric uncertainty of the model. This knowledge makes it possible to design the so-called adaptive threshold (Frank et al., 1999) that allows robust fault detection. Apart from the possibilities of generating an adaptive threshold, the knowledge regarding model uncertainty makes it possible to formulate suitable criteria of the optimum experimental design (OED) (Atkinson and Donev, 1992; Uciński, 2005; Walter and Pronzato, 1997) that allow minimization of model uncertainty. This means that more accurate models can be obtained resulting in an increase of fault sensitivity as well as an increase in a general relia-
bility of the fault diagnosis scheme. Another kind of solutions that may increase the performance of the fault diagnosis scheme is based on an appropriate scheduling of the control test signals in such a way so as to gain as much information as possible about the system being supervised (Delebecque et al., 2003).

The paper shows the possibilities of exploiting the theory of OED in parameter-estimation-based fault detection schemes. The paper is organized as follows. In the second section, a novel scheme for measuring and diagnosing an impedance is proposed. The final part of the paper is devoted to the numerical simulations and conclusions.

2. IMPEDANCE MEASUREMENT AND DIAGNOSIS

The objective of this section is to propose a new impedance measurement and diagnosis scheme. The idea of measuring the impedance with the so-called virtual bridge was introduced by (Dutta et al., 1987). The virtual bridge is composed of two arms, namely a real (hardware) arm, as shown in Fig. 1 and a virtual arm that is implemented with the help of a computer. Dutta et al. (1987) formulated the problem of balancing the bridge as a non-linear parameter estimation one. To solve such a problem they employed the gradient descent algorithm. Other researchers (Awad et al., 1994) developed some modifications of the algorithm proposed in (Dutta et al., 1987) that increased its convergence. Unfortunately, none of the authors has provided an analytical rules for estimating the accuracy of the obtained impedance, i.e. knowledge about model uncertainty. Moreover, they have not provided any analytical rules that can be used for obtaining an auxiliary resistance $R_r$ and the sampling time. Another question that arises while analysing (Dutta et al., 1987; Awad et al., 1994) where the non-linear parameter estimation techniques were employed for obtaining $R$ and $C$, it is proposed to use the classical recursive least-square (RLS) algorithm for estimation of $p_1$ and $p_2$. Such an algorithm can be given as follows:

$$\dot{p}_{k+1} = \hat{p}_k + k_{k+1}\varepsilon_{k+1},$$

$$k_{k+1} = P_k r_{k+1} (1 + r_{k+1}^T P_k r_{k+1})^{-1},$$

$$\varepsilon_{k+1} = y_{k+1} - f(\hat{p}_k, u_{k+1}),$$

$$P_{k+1} = [I_{n_p} - k_{k+1} r_{k+1}^T] P_k,$$

where stands for the so-called forgetting factor, $y_k$ is the $k$-th measurement of $v_k$, $f(\hat{p}_k, u_{k+1}) = p_{1,k} u_{1,k+1} + p_{2,k} u_{2,k+1}$, $r_k = [u_{1,k}, u_{2,k}]^T$, and $P_k = [\hat{p}_{1,k}, \hat{p}_{2,k}]^T \in \mathbb{R}^{n_p}$ denotes the $k$-th estimate of $p$. Thus, knowing $\hat{p}_k$ it is possible to obtain estimates of $R$ and $C$ according to the following equations:

$$\dot{R} = -\frac{R_r(\hat{p}_1^2 + \hat{p}_2^2)}{\hat{p}_1^2 + \hat{p}_2^2 - \hat{p}_r^2},$$

$$\dot{C} = -\frac{R_r(\hat{p}_2^2 + \hat{p}_1^2)}{R_r \omega^2 (\hat{p}_1^2 + \hat{p}_2^2)},$$

obtained by solving (4) with respect to $R$ and $C$. It should be also pointed out that when there is no need for on-line estimation of the impedance then the classical, non-recursive least-square algorithm can be employed. The well-known advantage of this algorithm, comparing with its recursive counterpart, is that the highest estimation accuracy can be attained with a smaller $n_r$. In this case, estimates of $p_1$ and $p_2$ can be computed as follows:

$$\hat{p}_1 = \frac{\gamma_1 \eta - \beta_2 \gamma_1}{\eta^2 - \beta_1 \beta_2}, \quad \hat{p}_2 = \frac{\gamma_1 \eta - \beta_1 \gamma_2}{\eta^2 - \beta_1 \beta_2},$$

Fig. 1. An impedance measurement scheme

presented in Fig. 1 the following current equality can be established:

$$C \frac{dv(t)}{dt} + v(t) = \frac{u(t) - v(t)}{R_r},$$

Assuming that $u(t) = U \sqrt{2} \sin(\omega t)$, the steady-state solution of (1) can be written as:

$$v(t) = \rho U \sqrt{2} R ((R + R_r) \sin(\omega t) + R_r R C \cos(\omega t)), \quad (2)$$

where $\rho = \frac{R_r^2 + 2 R R_r + R^2 (1 + \omega^2 R^2 C^2)}{R_r^2}$.

Equation (2) can be transformed into a discrete-time form and written as follows:

$$v_k = p_1 u_{1,k} + p_2 u_{2,k},$$

where

$$p_1 = \rho R (R + R_r), \quad p_2 = \rho R_r C R^2,$$

and $u_{1,k} = U \sqrt{2} \sin(\omega k \tau)$, $u_{2,k} = U \sqrt{2} \cos(\omega k \tau)$ where $\tau$ stands for the sampling time. In this paper it is assumed that $u_{1,k}$ and $u_{2,k}$ are available. This is a mild assumption since it is not difficult to design a hardware providing such signals. Another important fact that can be observed while analysing (3) is that it can be perceived as a linear-in-parameter model with respect to $p_1$ and $p_2$. Contrary to (Dutta et al., 1987; Awad et al., 1994) where the non-linear parameter estimation techniques were employed for obtaining $R$ and $C$, it is proposed to use the classical recursive least-square (RLS) algorithm for estimation of $p_1$ and $p_2$.
where

\[ \gamma_i = \sum_{k=1}^{n_r} u_{1,k} y_k, \quad \eta = \sum_{k=1}^{n_r} u_{1,k} u_{2,k}, \]

\[ \beta_i = \sum_{k=1}^{n_r} u_{1,k}^2. \]  

(12)

2.1 Initialization of the RLS algorithm

As can be found in the literature (Walter and Pronzato, 1997) regarding the RLS algorithm, the initial matrix \( P_0 \), i.e. \( P_0 \) should be set as \( P_0 = \gamma I \) where \( \gamma \) stands for a sufficiently large positive constant (usually \( 10^3 \)–\( 10^5 \)). When some rough values of \( R \) and \( C \) are known then \( \hat{p}_0 \) should be initialized according to (4). Otherwise, it can be observed from (9) that \( \hat{p}_2 + \hat{p}_2 - \hat{p}_1 < 0 \) and hence:

\[ \frac{1}{2} - \frac{1}{2} \sqrt{1 - 4\hat{p}_2^2} < \hat{p}_1 < \frac{1}{2} + \frac{1}{2} \sqrt{1 - 4\hat{p}_2^2}. \]  

(13)

Since \( \hat{p}_2 \) should satisfy \( 1 - 4\hat{p}_2^2 > 0 \) and (10) indicates that \( \hat{p}_2 < 0 \) then it is clear that:

\[ -\frac{1}{2} < -\frac{1}{2} \hat{p}_2 < 0. \]  

(14)

Thus, when no knowledge is available about \( R \) and \( C \) then \( \hat{p}_0 \) should be set so as to satisfy (13)–(14).

2.2 Confidence region and fault detection

The solutions presented in the subsequent part of this paper are based on the following assumption:

\[ y_k = v_k + \epsilon_k, \]

(15)

where \( \epsilon \) stands for the zero-mean, uncorrelated, Gaussian noise sequence. In other words, \( \epsilon \) represents the difference between the output of the model (3) and \( y_k \) that represents the actual measurements of \( v_k \) (cf. Fig. 1).

Since estimates of \( R \) and \( C \) are known, the next problem being considered is to obtain a set of all possible \( \hat{R} \) and \( \hat{C} \) that are consistent with the measurements. Such a set can be easily obtained with the use of (1–\( \alpha \))100\% confidence region (Walter and Pronzato, 1997) for \( p \) and equations (4). As a result the following inequality is given:

\[ d_k^T P_k^{-1} d_k \leq 2\hat{p}_k^2 F_{n_r-2, n_r} \]  

(16)

where

\[ d_k = \hat{p}_k - \rho \left[ R \left( R + R_r \right), R_r C \omega R^2 \right]^T, \]

(17)

and \( F_{n_r-2, n_r} \) stands for the F-Snedecor distribution quantile with \( 2 \) and \( k - 2 \) degrees of freedom, and \( \hat{\sigma} \) is the estimate of the standard deviation.

Thus, the problem of fault detection can be transformed into the task of testing the hypotheses. This means that, at the \( \alpha \)-level, the hypothesis:

\[ H_0 : (R, C) = (R_0, C_0) \]

vs.

\[ H_1 : (R, C) \neq (R_0, C_0), \]  

(18)

where \( R_0, C_0 \) are the required values of \( R \) and \( C \), is rejected when the inequality (16) is violated. The acceptance of hypothesis \( H_1 \) denotes the faulty behaviour of the impedance.

2.3 Application of OED

As we can see from (16), the size of the confidence region depends on the so-called Fisher Information Matrix (FIM) \( P^{-1} \). On the other hand, FIM depends on the experimental conditions, e.g. \( \xi = [u_1, \ldots, u_m] \). Thus, optimal experimental conditions can be obtained by optimising some scalar function \( \Phi(P^{-1}) \). Such a function can be defined in several different ways (Atkinson and Donev, 1992; Uciński, 2005; Walter and Pronzato, 1997).

In this paper, the so called D-optimality criterion is used, i.e. \( \Phi(P^{-1}) = \det(P^{-1}) \) is maximised. It should be also pointed out that the experimental conditions are developed for \( R \) and \( C \) but not for \( p_1 \) and \( p_2 \). This means that all dependencies among \( R, \omega, \tau, R \), and \( C \) that provide additional source of knowledge are exploited. First, let us define FIM:

\[ P^{-1} = \sum_{k=1}^{n_r} r_k r_k^T, \quad r_k = \left[ \frac{\partial v_k}{\partial R} \frac{\partial v_k}{\partial C} \right]^T, \]  

(19)

The purpose of further consideration is to obtain D-optimum values of \( R \) and \( \tau \), i.e. \( R \) and \( \tau \) that maximise \( \det(P_k^{-1}) \).

It can be observed that:

\[ r_k = P_1 r_{1,k} \]

(20)

\[ P_1 = \sqrt{2} U R \rho \text{diag}(1, \omega R^2), \]

\[ r_{1,k} = [a \sin(\omega k \tau) + b \cos(\omega k \tau), \]

\[ b \sin(\omega k \tau) - a \cos(\omega k \tau)], \]

\[ a = R^2 + 2R R + R^2 (1 - \omega^2 R^2 C^2), \]

\[ b = -2 \omega^2 R^2 + 2 RR C^2. \]

Bearing in mind that:

\[ \sqrt{a^2 + b^2} \sin(\omega k \tau + \arctan(a/b)) \]

\[ = a \sin(\omega k \tau) + b \cos(\omega k \tau), \]

\[ \sqrt{a^2 + b^2} = \rho^{-1}, \]  

(20)

it is possible to write:

\[ r_k = P_2 r_{2,k}, \quad P_2 = \sqrt{2} U R \rho \text{diag}(1, \omega R^2), \]

\[ r_{2,k} = [\sin(\omega k \tau + \arctan(a/b)), \]

\[ \sin(\omega k \tau + \arctan(-b/a))]^T. \]  

(21)

Using equations (21), now FIM can be given as follows:

\[ P^{-1} = P_2 \sum_{k=1}^{n_r} r_{2,k} r_{2,k}^T P_2 \]  

(22)
The solution of (25) is given as follows:

\[ R^* = \frac{R}{\sqrt{1 + \omega^2 R^2 C^2}}. \] (28)

2.4 Other properties

The objective of this section is to investigate the influence of the experimental conditions (27) and (28) on the estimation accuracy of \( p \). First let us define FIM for \( p \):

\[ P^{-1} = \sum_{k=1}^{n} \mu_k r_{2,k} r_{2,k}^T. \] (30)

Substituting (27) into (30) it can be shown that:

\[ P^{-1} = 2U^2 \text{diag}(\mu_1 + \mu_3, \mu_2 + \mu_4). \] (31)

From (31) it can be observed that FIM is diagonal. A design satisfying this property is called the orthogonal design. Its appealing property is that the covariance between the parameters \( p_1 \) and \( p_2 \) equals zero, which means that they are estimated independently. The remaining task is to check if the experimental conditions (27)–(28) are D-optimum for \( p \). In order to do that the following useful criterion can be used (Atkinson and Donev, 1992; Walter and Pronzato, 1997):

\[ r_k P r_k \leq n_p. \] (32)

When the equality holds for \( r_k \) satisfying the experimental conditions (27) and (28). Substituting \( n_p = 2 \) and then (31) into (32) it can be shown that:

\[ \sin(\frac{\pi}{2} k)^2 + \cos(\frac{\pi}{2} k)^2 \leq 2. \] (33)

Setting \( \mu_k = 1/4, k = 1, \ldots, n_e = 4 \) in (33) implies that the experimental design (27)–(28) is D-optimum and orthogonal for \( p \).

3. EXPERIMENTAL RESULTS

Let us consider a numerical simulation example for the following parameters: \( R = 5000\,\text{Ohm}, C = 300\,\text{nF}, \omega = 1000\pi, \tau = \tau^* \) (for \( i = 0 \)). For the purpose of simulation \( v_k \) was disturbed with noise generated according to the uniform distribution \( U(-3 \times 10^{-4}, 3 \times 10^{-4}) \). Two different experiments
were performed for two different values of $R_r$, i.e. $R_r = R_r^*$ and $R_r = 5\, \text{kOhm}$. Figure 2 presents the obtained confidence regions and the associated estimates of $R$ and $C$ (assuming $\alpha = 0.01$, i.e. 99% confidence region). From these results it is clear that the proposed solution provides more accurate estimates with considerably smaller uncertainty than those obtained without it. Undoubtedly, this will result in an increased fault sensitivity and in a general improvement of the reliability of the proposed fault detection scheme.

Let us assume that the non-faulty $R$ and $C$ are $R = 500.03\, \Omega$ and $C = 300.6\, \text{nF}$. Thus, the problem of fault detection boils down to the task of testing:

$$
\mathcal{H}_0 : (R, C) = (500.03\Omega, 300.6\, \text{nF})
$$

vs.

$$
\mathcal{H}_1 : (R, C) \neq (500.03\Omega, 300.6\, \text{nF}).
$$

(34)

It can be observed from Fig. 2 and inequality (16) that hypothesis $\mathcal{H}_0$ is rejected when $R_r = R_r^*$ which means that a fault occurs. Contrary, hypothesis $\mathcal{H}_0$ is accepted when $R_r = 5\, \text{kOhm}$ which means that there is no fault. These results clearly indicate that the application of the D- optimum experimental conditions increases the fault sensitivity, i.e. it makes the proposed fault diagnosis scheme more reliable.

### 3.1 Accuracy analysis

The main objective of this section is to estimate the measurement accuracy provided by the considered approach. For that purpose a set of different impedances were selected (similar to that of (Angrisani et al., 1996)). The relative measurement errors were defined as:

$$
\delta_R = \frac{R - \bar{R}}{R} \times 100\%, \quad \delta_C = \frac{C - \bar{C}}{C} \times 100\%.
$$

(35)

Each measurement was repeated 50 times and then the mean measured values $\bar{R}$ and $\bar{C}$ were calculated and for each of them a coefficient of variation $\bar{\sigma}$ was computed:

$$
\bar{\sigma} = \frac{\sigma_R}{R} \times 100\%, \quad \bar{\sigma} = \frac{\sigma_C}{C} \times 100\%.
$$

(36)

where $\sigma_C$ (or $\sigma_R$) stands for the standard deviation of 50 measurements. Table 1 shows the achieved results. From this results it is clear that the proposed approach provides a high measurement accuracy. It should be also pointed out that these measurements were achieved for $n_t = 4000$ which implies that the measurement time was $1\, \text{[s]}$. Figure 3 shows the evolution of the relative

<table>
<thead>
<tr>
<th>True value</th>
<th>Mean measured value</th>
<th>$\bar{\sigma}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 0.75[nF]</td>
<td>0.7502[nF]</td>
<td>0.41</td>
</tr>
<tr>
<td>R 500[Ω]</td>
<td>499.9997[Ω]</td>
<td>9.2252 $\times 10^{-4}$</td>
</tr>
<tr>
<td>C 150[nF]</td>
<td>150[nF]</td>
<td>0.0018</td>
</tr>
<tr>
<td>R 570[Ω]</td>
<td>569.999[Ω]</td>
<td>9.3179 $\times 10^{-4}$</td>
</tr>
<tr>
<td>C 320[nF]</td>
<td>320[nF]</td>
<td>4.6464 $\times 10^{-4}$</td>
</tr>
<tr>
<td>R 48[kΩ]</td>
<td>47.999[kΩ]</td>
<td>0.0425</td>
</tr>
<tr>
<td>C 1[nF]</td>
<td>0.9999[nF]</td>
<td>0.1441</td>
</tr>
<tr>
<td>R 1[kΩ]</td>
<td>999.99[Ω]</td>
<td>0.0010</td>
</tr>
<tr>
<td>C 30[nF]</td>
<td>50[nF]</td>
<td>0.0028</td>
</tr>
<tr>
<td>R 1.1[kΩ]</td>
<td>1.1[kΩ]</td>
<td>8.7803 $\times 10^{-4}$</td>
</tr>
<tr>
<td>C 160[nF]</td>
<td>160[nF]</td>
<td>5.1671 $\times 10^{-4}$</td>
</tr>
<tr>
<td>R 97[kΩ]</td>
<td>96.968[kΩ]</td>
<td>0.0391</td>
</tr>
<tr>
<td>C 840[pF]</td>
<td>840[pF]</td>
<td>0.0313</td>
</tr>
<tr>
<td>R 5[kΩ]</td>
<td>5[kΩ]</td>
<td>8.6657 $\times 10^{-4}$</td>
</tr>
<tr>
<td>C 15[nF]</td>
<td>15[nF]</td>
<td>0.0018</td>
</tr>
<tr>
<td>R 5.7[kΩ]</td>
<td>5.7[kΩ]</td>
<td>9.1961 $\times 10^{-4}$</td>
</tr>
<tr>
<td>C 32[nF]</td>
<td>32[nF]</td>
<td>4.5225 $\times 10^{-4}$</td>
</tr>
<tr>
<td>R 296[kΩ]</td>
<td>296[kΩ]</td>
<td>0.0301</td>
</tr>
<tr>
<td>C 540[pF]</td>
<td>540[pF]</td>
<td>0.0220</td>
</tr>
<tr>
<td>R 10[kΩ]</td>
<td>10[kΩ]</td>
<td>8.8802 $\times 10^{-4}$</td>
</tr>
<tr>
<td>C 13[nF]</td>
<td>13[nF]</td>
<td>8.8790 $\times 10^{-4}$</td>
</tr>
<tr>
<td>R 17[kΩ]</td>
<td>17[kΩ]</td>
<td>0.0012</td>
</tr>
<tr>
<td>C 16[nF]</td>
<td>16[nF]</td>
<td>5.3046 $\times 10^{-4}$</td>
</tr>
<tr>
<td>R 495[kΩ]</td>
<td>495.01[kΩ]</td>
<td>0.0245</td>
</tr>
</tbody>
</table>

4. CONCLUSION

It was shown that the experimental conditions do not concern an appropriate input selection only but can also provide rules for selecting other parameters, e.g. sampling time, auxiliary resistances, etc. In particular, the problem of impedance measurement was transformed into the parameter estimation task. Contrary to the approaches presented in the literature, parameter estimation was realized with the use of the linear least-square method (or the recursive least-square method when an on-line measurement is
Fig. 3. Relative errors in the subsequent iterations of the proposed algorithm

required) which enables fast convergence rate. Another important contribution of this paper was the development of the D-optimum experimental conditions that make it possible to enhance the measurement accuracy. In particular, explicit formulae for selecting the reference resistance \( R_r \) and the sampling time \( \tau \) were provided. It was also shown that the proposed approach can effectively be applied for fault detection which is very important from the point of view of modern control and fault diagnosis. The numerical experiments performed with the proposed impedance measurement scheme confirm that the application of OED is very profitable and leads to the decreased modelling uncertainty and an increased fault sensitivity.

ACKNOWLEDGMENTS

The work was supported by State Committee for Scientific Research in Poland (KBN) under the grant 4T11A01425 Modelling and identification of non-linear dynamic systems in robust diagnostics.

The author would like to express his sincere gratitude to the referees, whose efforts have significantly improved the paper’s quality.

REFERENCES


