1. INTRODUCTION

A hot strip mill is composed of a roughing mill, a finishing mill, and a coiler. In the finishing mill, a sheet bar transported from the roughing mill is further reduced to the final thickness. The resulting strip is then coiled to form the finished coil of steel strip.

In the finishing mill, to achieve the required reduction, final qualities and tolerances, several passes of rolling are executed by tandem rolling with 6 or 7 successive stands in the presence of interstand tension. The tension indispensable to prevent the strip from skidding between stands and to diminish edge waves or center buckles of the strip which may cause pinching or rupturing of the strip. Therefore tension control is the key to successful mill operations. The looper implemented between each pair of adjacent stands fulfills an important role in tension control. Processing of each strip starts up with threading of the strip through the stands. The looper is raised above the passline just after the leading end of the strip passes through the downstream stand so that the looper comes into contact with the strip and eventually forms a loop of the stored strip between the stands. Both tension and looper angle control in this phase is normally performed in an ad hoc manner; a constant value is given as the looper motor torque reference and the feedback control does not start until the looper comes into contact with the strip. Feedback control of the tension and looper angle starts only after the transient behaviour in this start-up phase has settled.

There exist mutual interactions between the tension and looper angle, which has been considered the main problem in tension and looper control. Several multivariable control schemes have been applied to this problem. Among them are interaction decoupling (Kotera and Watanabe, 1981), optimal control (Seki, et al., 1987), H∞ control (Imanari, et al., 1997) and decentralized control (Asano, et al., 2000). All of them are, however, intended only for feedback control after the start-up phase.

This paper presents a new control design method for tension control in the start-up phase. In the proposed method, strip tension and looper trajectories are simultaneously optimized throughout the start-up phase.
which consists of the non-contact and contact modes. First, a discrete-time piecewise affine model is derived to describe the discontinuous dynamics of this system. Next, the optimal manipulated variables are calculated within a model predictive control framework. An approximate solution method is also presented for ease of online optimization. Furthermore, the proposed method is applied to generate the optimal feedforward control input which can be used as a good alternative to the conventional constant torque reference. This is very practical because the optimization can be done offline prior to rolling operations. Simulation results are shown with respect to the offline optimization to demonstrate the control performance compared to the conventional ad hoc control.

This research was conducted within the hybrid system working group of the Control Forum in the Division of Instrumentation, Control and System Engineering, the Iron and Steel Institute of Japan (ISIJ).

2. HYBRID SYSTEM MODELING

2.1 Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>J</td>
<td>Looper inertia</td>
</tr>
<tr>
<td>θ</td>
<td>Looper angle</td>
</tr>
<tr>
<td>σ</td>
<td>Interstand tension</td>
</tr>
<tr>
<td>q</td>
<td>Looper torque</td>
</tr>
<tr>
<td>q_ref</td>
<td>Looper torque reference</td>
</tr>
<tr>
<td>D</td>
<td>Looper damping constant</td>
</tr>
<tr>
<td>T_ACR</td>
<td>Time constant of looper motor ACR</td>
</tr>
<tr>
<td>h</td>
<td>Strip thickness</td>
</tr>
<tr>
<td>b</td>
<td>Strip width</td>
</tr>
<tr>
<td>β</td>
<td>Strip angle with passline</td>
</tr>
<tr>
<td>ρ</td>
<td>Strip density</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational constant</td>
</tr>
<tr>
<td>l</td>
<td>Half of length between stands</td>
</tr>
<tr>
<td>r</td>
<td>Looper arm length</td>
</tr>
<tr>
<td>W_L</td>
<td>Looper weight</td>
</tr>
<tr>
<td>r_L</td>
<td>Distance between axis and center of gravity of looper</td>
</tr>
<tr>
<td>θ_G</td>
<td>Offset angle between center of gravity of looper and looper angle</td>
</tr>
<tr>
<td>E</td>
<td>Young’s modulus of strip</td>
</tr>
</tbody>
</table>

2.2 Dynamic Equations

The looper dynamics shown in Fig.2 is described by the following equations:

\[ J \ddot{\theta} = q - \delta[K_\sigma(\theta)\sigma + K_\beta(\theta)] - K_L(\theta) - D\dot{\theta}. \]  (1)

\[ \dot{q} = -\frac{1}{T_{ACR}}(q - q_{ref}), \]  (2)

where \( K_\sigma, K_\beta \) and \( K_L \) denote the looper load torque by the tension, strip weight and looper weight, respectively, and are given as follows:

\[ K_\sigma(\theta) = 2hhr \cos \theta \sin \beta, \]
\[ K_\beta(\theta) = 2pbg \frac{1}{r \cos \theta}, \]
\[ K_L(\theta) = W_L g r_L \cos(\theta + \theta_G). \]

\( \delta \) is a 0-1 variable which denotes the two modes shown in Fig. 3; \( \delta = 1 \) in the contact mode (C-mode) and \( \delta = 0 \) in the non-contact mode (N-mode). The mode transition rule is given as follows:

\[ \delta = \begin{cases} 
0 & \text{if } \theta \leq \theta_{min} \\
1 & \text{if } \theta > \theta_{min} 
\end{cases}. \]
where \( \theta_{\text{min}} \) is the looper angle when the looper is raised to the passline.

The tension dynamics is governed by the following equations:

\[
\dot{\sigma} = \frac{E}{2l} \left[ -\left(1 + f(\sigma)\right) V_R + \frac{\partial L}{\partial \theta} \right],
\]

\[
\dot{V}_R = -\frac{1}{T_{\text{ASR}}} (V_R - V_{\text{Ref}}),
\]

The looper angular velocity and the tension at the transition from the N-mode to the C-mode is assumed as follows:

\[
\dot{\theta}(t) = \varepsilon_1 \dot{\theta}(t_\text{C}), \text{ if N-mode } \rightarrow \text{ C-mode},
\]

\[
\sigma(t) = \sigma(t_\text{C}) + \varepsilon_2 \dot{\theta}(t_\text{C}), \text{ if N-mode } \rightarrow \text{ C-mode},
\]

where \( \varepsilon_1 \) and \( \varepsilon_2 \) are each an appropriately estimated constant, \( \dot{\theta}(t_\text{C}) := \lim_{t \rightarrow t_\text{C}} \dot{\theta}(t) \) and \( \sigma(t_\text{C}) := \lim_{t \rightarrow t_\text{C}} \sigma(t) \).

### 2.3 Piecewise Affine Model

The dynamics equations are linearized around an operating point of each mode. Then both linear models are unified based on the coordinate system of the C-mode to obtain a piecewise affine model. Let \( x \) and \( x_c \) denote the value of \( x \) at the operating point of the C-mode and N-mode, respectively. Then the operating point of each mode is described by \((\theta_c, \theta, \sigma, q_c, q, V_R)\) and \((\theta_c, \theta, \sigma, q_c, q, V_R)\), respectively, for a set of state variables \((\theta, \dot{\theta}, \sigma, q, V_R)\).

C-mode: Let \( \bar{x} \) denote the variation of \( x \) at the operating point of the C-mode, namely, \( \bar{x} = x - x_c \). The following equations are assumed to hold at the operating point:

\[
q_c = q_{\text{ref}} = K_c(\theta_c) \sigma + K_s(\theta_c) \dot{\theta},
\]

Then the following equations are derived by linearizing Eqs. (1)-(5) with \( \delta = 1 \) around the operating point:

\[
\ddot{\sigma} = F_1(\sigma) \ddot{V}_R + F_2(\sigma, V_R) \dot{\sigma} + F_3(\theta) \dot{\sigma},
\]

\[
\dot{\bar{x}} = -\frac{1}{T_{\text{ACR}}} (\bar{x} - \bar{x}_\text{ref}),
\]

\[
\dot{V}_R = -\frac{1}{T_{\text{ASR}}} (V_R - V_{\text{Ref}}),
\]

\[
\dot{\bar{x}} = \varepsilon_1 \dot{\theta}(t_\text{C}), \text{ if N-mode } \rightarrow \text{ C-mode},
\]

\[
\bar{\sigma}(t) = \bar{\sigma}(t_\text{C}) + \varepsilon_2 \ddot{\theta}(t_\text{C}), \text{ if N-mode } \rightarrow \text{ C-mode},
\]

where

\[
K(\theta, \sigma) := \sigma \frac{\partial K_c}{\partial \sigma} + \frac{\partial K_s}{\partial \sigma} \dot{\theta} + \frac{\partial K_s}{\partial \theta} \dot{\theta}.
\]

N-mode: Let \( \bar{\chi} \) denote the variation of \( x \) at the operating point of the N-mode, namely, \( \bar{\chi} = x - x_c \). The following equations are assumed to hold at the operating point:

\[
q_n = q_{\text{ref}} = K_c(\theta_c) \sigma + K_s(\theta_c) \dot{\theta},
\]

Then the following equations are derived by linearizing Eqs. (1)-(5) with \( \delta = 0 \) around the operating point:

\[
\ddot{\sigma} = F_1(\sigma) \ddot{V}_R + F_2(\sigma, V_R) \dot{\sigma} + F_3(\theta) \dot{\sigma},
\]

\[
\dot{\bar{x}} = -\frac{1}{T_{\text{ACR}}} (\bar{x} - \bar{x}_\text{ref}),
\]

\[
\dot{V}_R = -\frac{1}{T_{\text{ASR}}} (V_R - V_{\text{Ref}}),
\]

Substituting the relationship between both coordinate systems, \( \bar{x} = \chi + (x - x_c) \), into Eq. (7) yields the following representation:

\[
\ddot{\sigma} = F_1(\sigma) \ddot{V}_R + F_2(\sigma, V_R) \dot{\sigma} + F_3(\theta) \dot{\sigma},
\]

\[
\dot{\bar{x}} = -\frac{1}{T_{\text{ACR}}} (\bar{x} - \bar{x}_\text{ref}),
\]

\[
\dot{V}_R = -\frac{1}{T_{\text{ASR}}} (V_R - V_{\text{Ref}}),
\]

where

\[
\Delta S_{\theta} = q_c - q_{\text{ref}} - \frac{\partial K_c}{\partial \theta} (\theta_c - \theta),
\]

\[
\Delta S_{\sigma} = F_1(\sigma)(V_{R_n} - V_{\text{Ref}}) + F_2(\sigma, V_{R_n}) \dot{\sigma} + F_3(\theta) \dot{\sigma} - \sigma_c - \sigma_{\text{ref}}.
\]

The tension is measured by a tensiometer mounted on the looper, so it is unmeasurable in the N-mode. Therefore, it is assumed that the roll velocity is not manipulated and thus its reference \( V_{\text{Ref}} \) is kept constant in the N-mode. As for the C\( \rightarrow \)N mode transition, the following is assumed:

\[
V_{\text{Ref}}(t) = V_{\text{Ref}}(t_\text{C}), \text{ if C-mode } \rightarrow \text{ N-mode}.
\]

A block diagram representation of the linear model for each mode is shown in Figs. 4 and 5. Fig. 5 clearly shows that the N-mode model does not contain any interactions between the tension and looper dynamics, because the looper and the strip do not have contact.
obtained from Eqs. (6) and (8), and where Fig. 5  Linearized model for the N-mode

with each other.

State Space Model; Let the state and input vectors be denoted by $x$ and $u$, respectively:

$$x = \begin{bmatrix} \theta, \dot{\theta}, \sigma, \dot{\sigma}, \bar{V}_R \end{bmatrix}^T, \quad u = \begin{bmatrix} \bar{q}_{1\text{ref}}, \bar{V}_{R_{\text{ref}}} \end{bmatrix}^T.$$  \hfill (10)$$

Then, the piecewise affine model is derived from Eqs. (6) and (8) as follows:

N-mode  
$$\dot{x} = A_n^0 x + B_n^0 u + a_n^0, \quad \text{if } Cx + c \leq 0,$$

C-mode  
$$\dot{x} = A_c^0 x + B_c^0 u, \quad \text{if } Cx + c > 0,$$

N → C-mode  
$$x(t) = E_w x(t_\text{c}) + e_w, \quad \text{if } Cx(t_\text{c}) + c = 0 \text{ and } N \rightarrow C\text{-mode},$$

C → N-mode  
$$x(t) = x(t_\text{c}), \quad \text{if } Cx(t_\text{c}) + c = 0 \text{ and } C \rightarrow N\text{-mode},$$

where $A_n^0$, $B_n^0$, $A_c^0$, $B_c^0$ and $E_w$ contain constants obtained from Eqs. (6) and (8), and

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad c = \theta - \theta_{\text{min}}.$$  

The N → C-mode reflects the jump of the state vector $x$ at the N → C-mode transition assumed in Eq. (6).

Discretizing Eq. (11) with the sampling period $dT$ and adding a discrete variable $I \in [0,1]$ yield the following discrete-time piecewise affine model:

N-mode:  
$$\begin{align*}
  x(k+1) &= A_n x(k) + B_n u(k) + a_n \\
  I(k+1) &= I(k)
\end{align*} \quad \text{if } Cx(k) + c \leq 0, I(k) = 0 \quad (6),$$

C-mode:  
$$\begin{align*}
  x(k+1) &= A_c x(k) + B_c u(k) \\
  I(k+1) &= I(k)
\end{align*} \quad \text{if } Cx(k) + c > 0, I(k) = 1 \quad (7),$$

N → C-mode:  
$$\begin{align*}
  x(k+1) &= A_n x(k) + B_n u(k) \\
  I(k+1) &= 1
\end{align*} \quad \text{if } Cx(k) + c \leq 0, I(k) = 0 \quad (8),$$

C → N-mode:  
$$\begin{align*}
  x(k+1) &= A_c x(k) + B_c u(k) \\
  I(k+1) &= 0
\end{align*} \quad \text{if } Cx(k) + c > 0, I(k) = 1 \quad (9),$$

where

$$A_n = e^{c_n dT}, \quad B_n = \int_0^{dT} e^{c_n \tau} d\tau B_n^c, \quad a_n = \int_0^{dT} e^{c_n \tau} d\tau a_n^c,$$

$$A_c = e^{c_c dT} \text{ and } B_c = \int_0^{dT} e^{c_c \tau} d\tau B_c^c.$$  

3. MODEL PREDICTIVE CONTROL

This section proposes a new method for optimizing the input vector within a model predictive control framework based on the discrete-time piecewise affine model. The control problem considered here is to raise the looper from the initial position in the N-mode to the operating point of the C-mode. A faster response of the looper is preferred, but the looper should not bump the strip not to disturb the strip tension. Let $J$ denote the following cost function over the control horizon $[k, k_N + 1]$:

$$J(x(k), u) = \sum_{i=k}^{k_N} \left[ x_i^T Q x_i + u_i^T R u_i + \lambda N_i \right], \quad (13)$$

where $N_i$ is the time corresponding to the N → C-mode transition, $\gamma$ is a weight $\gamma > 0$, $Q$ and $R$ are matrices $Q \succeq 0$ and $R > 0$, and $P$ is a positive definite matrix satisfying the following algebraic Riccati equation:

$$P = A_n^0 P A_n - A_n^0 P B_n (R + B_n^c P B_n^c)^{-1} B_n^c P A_n + Q.$$  

The third term in Eq. (13) is the penalty for staying within the N-mode and the N → C-mode transition time can be adjusted by tuning its weight $\gamma$.

If $N$ is sufficiently large, the state $x$ enters an invariant set including the origin in the C-mode within $[k, k_N + 1]$ and thus the following equation holds in the
cost function (13):

\[
x^T(k + N)Px(k + N) = \\
\min_{u} \sum_{k=1}^{N} \{x^T(i)Qx(i) + ud^T(i)Ru(i)\}.
\]

Consequently, the optimal control input is given by the following equation after time \( k+N \):

\[
u(k) = -(B_k^T P B_k + R_k)^{-1} B_k^T P A_k x(k) \quad \text{if C-mode}.
\]

The optimal control law for the C-mode which minimizes the cost function (13) becomes nonlinear, complicated and impractical. Therefore, for more practical control laws, Eq. (14) is imposed as the input constraint in the C-mode and the optimization problem is subject to it.

In addition, the following assumption is imposed: Assumption 3.1 With the input restriction imposed by Eq. (14), the region of the C-mode, \( C + c > 0 \), is a invariant set.

With the above assumption, the \( N \rightarrow C \)-mode transition occurs only once at time \( N \) in the optimal mode sequence:

\[

\begin{align*}
C(x(i)) + c &< 0, \quad i = k, \ k+1, \ldots, \ k + N, - 1, \\
C(x(i)) + c &> 0, \quad i = k + N, \ k + N, + 1, \ldots, \ k + N - 1.
\end{align*}
\]

Fixing \( N \) yields the following optimization problem:

\[

\begin{align*}
\min_{q_{opt}(i)} \ & J(x(k),q_{opt}) \\
\text{s.t.} & \quad (12), (14), (15),
\end{align*}
\]

which results in a strictly convex quadratic program. Let the optimal control input sequence and the cost function obtained by solving the quadratic problem be denoted by \( q_{opt}(i), x(k), \) and \( J^*(x(k),N) \), respectively. The optimal \( N \rightarrow C \)-mode transition time \( N_c \) is obtained by solving the quadratic program \( N - 1 \) times as follows:

\[

N_c = \arg \min_{N \in [0, 1, \ldots, N - 1]} J^*(x(k),N)
\]

In accordance with the receding horizon strategy, the first control input \( q_{opt}(k, x(k), N_c) \) is implemented on the real plant at time \( k \) to obtain \( x(k + 1) \), which is used to update the optimization problem (17) as a new initial condition.

However, this control law is still impractical for the tension control, which requires the sampling period of 0.02 (sec). Therefore, the following approximate solution method is proposed for ease of online optimization.

Step 1: Solve the optimization problem (17) at time \( k = 0 \) to obtain \( N_c(0) \).

Step 2: Solve the optimization problem (16) for \( N_c = N_c(0) - 1 \) to obtain \( q^*(i, x(k), N_c) \), \( i = k, k+1, \ldots, k + N_c \), and implement \( q^*(k, x(k), N_c) \).

Step 3: Repeat Step 2 at time \( k + 1 \). For \( k > N_c + 1 \), use the control input given by Eq. (14).

If the \( N \rightarrow C \) mode transition does not occur at time \( k = N_c(0) \) due to modeling errors or disturbances, \( N_c \) is reset to 1 in Step 2 at time \( k = N_c(0) \). On the contrary, if the mode transition occurs at time \( k > N_c(0) \), the control input given by Eq. (14) is applied for time \( k > k_c \). In this algorithm, the number of the quadratic programs to be solved at each sampling time is reduced to one by fixing the optimal mode transition time \( N_c(0) \). The least time-consuming and still meaningful way of utilizing this problem formulation is to use the optimal control input sequence at time \( k = 0 \), \( q(i, x(k), N_c(0)) \), \( i = 0, 1, \ldots, N_c \), as a feedforward input sequence in the N-mode and the control input given by Eq. (14) after the \( N \rightarrow C \)-mode transition. In this case, the optimization can be done offline prior to rolling operations, which is very practical and feasible with usual process computers.

4. SIMULATION RESULTS

Performance of the conventional and proposed control schemes is compared by simulations. The simulator consists of the discrete-time piecewise affine model (12) and the two control schemes, which are alternatively used in the simulations. In the conventional control, a constant torque reference is applied in the N-mode and the optimal control law (14) is applied after the \( N \rightarrow C \) mode transition. In the proposed control, the optimal control input sequence \( q_{opt}(i, x(k), N_c(0)) \), \( i = 0, 1, \ldots, N_c \), is applied in a feedforward manner in the N-mode and the optimal control law (14) is applied after the \( N \rightarrow C \) mode transition. Since the same \( Q \) and \( R \) are used for both control schemes, the difference between the two control schemes is only the looper motor reference \( q_{opt} \) in the N-mode. No modeling errors or disturbances are considered in the simulations, so the control input in the proposed control coincides with that obtained by solving the optimization problem (17) at each sampling time. Therefore, it is relevant to evaluate the performance of the proposed control scheme by these simulations.

It is assumed that the strip is 1000 mm in width and 3 mm in thickness in the adjacent stands to be considered. \([\theta_c, \sigma_c] = [0.350(\text{rad}), 9.8(\text{MPa})] \) is assumed for the operating point of the C-mode. \([\theta_m, \sigma_m] = [0.175(\text{rad}), 9.8(\text{MPa})] \) is assumed for the operating point of the N-mode. The initial looper angle \( \theta(0) \) is assumed to be 0 rad. Therefore, the control problem considered here is to raise the looper from the initial angle \( \theta(0) = 0 \) (rad) in the N-mode through the operating point of the N-mode \( \theta_c = 0.175 \) (rad) to the operating point of the C-mode \( \theta_m = 0.350 \) (rad) while keeping the tension at \( \sigma_m = 9.8 \) (MPa). The initial state vector is given as follows:

\[
\begin{bmatrix}
T(0) = T_c(0) , \ t(0), L_c(0) , \ x(0), \ v_r(0)
\end{bmatrix}^T
\]

\[
= \begin{bmatrix}
-0.350 \ 0 \ 0 \ 0 \ 0
\end{bmatrix}^T
\]

The length of control horizon \( N \) is 100 and the sampling period \( dT \) is 0.02 (sec).
Fig. 6, 7 and 8 summarize the simulation results. The N→C mode transition occurs at the same time 0.18 sec ($N_1 = 9$) in both cases, which results from adjusting the constant looper torque reference in the conventional control and $\gamma$ in the cost function (13) in the proposed control. The results can be summarized as follows:

1. **Looper motor torque in the conventional control** increases due to a big control move by the optimal control in the C-mode. In the proposed control, the looper motor torque, on the contrary, decreases after the initial rise.

2. **As a result, in the conventional control, the looper keeps accelerating while in the N-mode and bumps against the strip and pushes it up, resulting in a rapid increase of the tension. In the proposed control, the looper decelerates before the mode transition, which alleviates the bump and decreases the increase in the tension by approximately 40%.

3. **The settling time of the tension and looper angle is slightly longer in the proposed control scheme**, but it is within the acceptable range.

In the proposed control, the looper motor torque is optimized throughout the transient response in the start-up phase, which enables a smooth transition from the N-mode to the C-mode. The optimal control input sequence can be a good alternative to the conventional constant torque reference.

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**5. CONCLUSIONS**

A new control design method has been proposed for tension control in the start-up phase. In the proposed method, based on a hybrid system approach, strip tension and looper trajectories are simultaneously optimized throughout the start-up phase which consists of the non-contact and contact modes. First, a discrete-time piecewise affine model is derived to describe the discontinuous dynamics of this system. Next, the optimal manipulated variables are calculated within a model predictive control framework. An approximate solution method is also presented for the ease of online optimization. Furthermore, the proposed method is applied to generate the optimal feedforward control input instead of the conventional ad hoc control input. Simulation results in the case of the feedforward control have shown that the transient responses of the tension and looper angle can be improved. Online optimization is still challenging for this control problem which requires a short sampling period of 0.02 (sec). The assumptions imposed in the problem formulation and solution should be assessed from a theoretical and practical point of view. Robustness of the control scheme to modeling errors and disturbances needs to be clarified. Further investigations would be devoted to these open questions within the hybrid system working group in the ISIJ.

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