AN IMPROVED ARCHITECTURE FOR NETWORKED CONTROL SYSTEMS

Daniel E. Quevedo * Graham C. Goodwin *

* School of Electrical Engineering & Computer Science, The University of Newcastle, Callaghan, NSW 2308, Australia

Abstract: This paper describes a novel networked control systems architecture in which the down-link communication system is designed to operate in two different modes. This achieves improved trade-offs between error probabilities and transmission delays. The method is illustrated for the case of a physical channel which can either function as an error-free delay (virtual) channel or as a delay-free erasure (virtual) channel with a given probability of error. It is shown how this architecture outperforms existing methods which make use of only one communication mode. Copyright © 2005 IFAC

Keywords: predictive control, quantized signals, communication control applications, digital control, multirate, multilevel control.

1. INTRODUCTION

Networked Control Systems (NCSs), where controller(s) and plant(s) are connected via a communication network have attracted significant attention, see e.g. (Antsaklis and Baillieul, 2004; Tiptswan and Chow, 2003; Thompson, 2004) and references therein. It turns out that the network itself is a dynamical system, which in general needs to be accounted for in the overall design of an NCS. In particular, digital networks can only transport quantized data and introduce time delays in both the down-link, i.e. between controller and actuator nodes, and up-link, i.e. between sensor and controller nodes.

In (Quevedo et al., 2004b) the idea of sending filtered versions of plant input signals through the down-link channel was investigated. It was shown that this source coder design problem is governed by a trade-off between channel utilization (i.e. bit-rates used) versus immunity of the NCS with respect to transmission errors.

In particular, if the communication system deployed is such that error free reception can be ensured, then the Δ-Modulation based approach proposed in (Quevedo et al., 2003; Goodwin et al., 2004) minimizes bit-rates. In this method, incremental information of plant inputs is sent. At the actuator nodes, plant input signals are reconstructed through integration in the source decoders. The resultant NCS can follow references and compensate for step-like disturbances. One drawback, however, with this source coding method, is that, if transmission errors occur, then performance is degraded significantly. Practical physical channels are always affected by noise, see e.g. (Proakis, 1995; Liu and Yen, 2004; Paulraj et al., 2004). To achieve low error rates, the communication system needs to deploy sophisticated channel coding methods. Their purpose is to overcome the effects of channel noise via introduction of redundancies in the signal transmitted. Larger delays and lower sampling rates are a necessary consequence of these strategies aimed at achieving lower error rates. In a closed loop control setting there is a hard real time requirement and...
hence these delays are highly undesirable, see e.g. (Tipsuwan and Chow, 2003; Srinivasagupta et al., 2004).

On the other hand, if faster channel coding methods are deployed, then delays can be kept low and sampling rates high, but random transmission errors are more likely to occur. In this case, the integrating source decoder should be replaced by a stable one, which gives more robustness with respect to transmission errors, as shown in (Quevedo et al., 2004b). This is achieved at the expense of increased channel utilization. Also, with a stable source decoder, the reconstructed plant inputs will be artificially restricted in magnitude (due to quantization of all signals transmitted), and regulating capacities of the NCS may be compromised.

To overcome the limitations described above, in the present work, an improved NCS architecture is proposed. In it, the down-link communication system is allowed to operate in two different modes, which are situated at different locations in the performance-complexity spectrum of communication systems for a given physical channel. One mode essentially ensures “error-free transmission”, but at the expense of significant time delays and at a reduced sampling rate. The other mode is faster (no time delay, higher sampling rate) but is affected by transmission errors.

Within this architecture, a novel controller is developed, which can access both of the virtual and at a reduced sampling rate. The other mode faster down-link architecture is applicable to a variety of specifications, but at the expense of significant time delays and at the reduced sampling rate. This situation can be modeled via the (noiseless) delay channel model:

\[
\hat{u}_D^{\text{out}}(\ell + \Delta) = \hat{u}_D(\ell), \quad \ell \in \mathbb{N}. 
\]

If data is allowed to be delayed enough, i.e. a sufficiently large timeout value \( \Delta \) is used, and sampling rates are low, then dropouts will be very rare and can be said to essentially not occur. This model can be modeled via the (noiseless) delay channel model:

\[
\hat{u}_D^{\text{out}}(\ell + \Delta) = \hat{u}_D(\ell), \quad \ell \in \mathbb{N}. 
\]

In (2), the channel input \( \hat{u}_D(\ell) \) is sub-sampled:

\[
\hat{u}_D(\ell) = 0, \quad \forall (\ell/L) \notin \{N \cup \{0\}\}, 
\]

where \( L \in \mathbb{N} \) is a given sub-sampling factor.

When focusing on Ethernet networks, the two-channel concept described above leads to the down-link architecture illustrated in Fig. 1. As can be seen in that figure, a networked controller utilizes up-link information and a reference signal \( r(\ell) \) to calculate its control actions \( \hat{u}_D(\ell) \) and \( \hat{u}_E(\ell) \). Each of the two virtual channels is equipped with a source decoder, denoted as \( H_D(z) \) and \( H_E(z) \), which can be tuned following the procedure outlined in (Quevedo et al., 2004b).

Note 1. (Relationship to PI-controller). The proposed architecture is conceptually related to a PI-controller implemented in parallel form, see e.g. (Aström and Hägglund, 1995), where the P-part is fast and is affected by noise while the I-part has less bandwidth and serves essentially for regulation purposes.

1 This model does not include any delay. It is assumed that small delays are directly incorporated into \( G(z) \). More sophisticated erasure-channel models can also be developed, if \( \epsilon(\ell) \) is not assumed independent, but governed by a Markov chain which models the network load.

2 For this, the delay \( \Delta \) should be chosen larger than the so-called “maximum roundtrip time” of the network.
3. THE TWO-CHANNEL NETWORKED CONTROLLER

A key point associated with an NCS such as the architecture depicted in Fig. 1 is that the controller needs to satisfy communication constraints imposed by the network.

3.1 Communication Constraints

Since the network is digital, all data sent through the down-link is restricted to be quantized. Moreover, the data sent through the delay channel needs to satisfy the sub-sampling constraint (3). These two communication constraints can be captured via a finite set constraint on the controller outputs:

\[ \tilde{u}(\ell) \triangleq \begin{cases} \tilde{u}_D(\ell) \in U(\ell), & \ell = 0, 1, 2, \ldots \end{cases} \]

where: \( U(\ell) \triangleq U_D(\ell) \times U_E \).

In the above expression, \( U_E \) is a given finite set and \( U_D(\ell) \) is defined via:

\[ U_D(\ell) = \begin{cases} \{0\}, & \text{if } (\ell/L) \notin \mathbb{N} \cup \{0\}, \\ U_d, & \text{otherwise,} \end{cases} \]

where \( U_d \) is a given finite set. Note that \( U(\ell) \) is \( L \)-periodic, i.e.:

\[ U(\ell + L) = U(\ell), \quad \forall \ell \geq 0 \]

and that a controller which respects (4) implicitly incorporates source encoding.

Having established that the networked controller needs to satisfy the constraint (5), finite set constrained receding horizon control (Quevedo et al., 2004a) can be used to optimize performance.

3.2 Cost-function

At each time instant \( \ell = k \) the Two-channel Networked Controller, minimizes the finite horizon cost function:

\[ V_N = \sum_{\ell=k+1}^{k+N} \| y'(\ell) - r(\ell) \|^2 + \sum_{\ell=k}^{k+N-1} \| u'(\ell) - u_r(\ell) \|^2_R, \]

which weights predictions of future plant inputs and outputs. In this cost function, \( u_r(\ell) \) is the input reference corresponding to \( r(\ell) \). The scalar \( R \) is the control weighting and \( N \in \mathbb{N} \) is the prediction horizon. Both serve as tuning parameters, see e.g. (Maciejowski, 2002; Mayne et al., 2000).

To minimize \( V_N \), the following (noise-free) prediction model, see (1), (2) and Fig. 1, is used:

\[ \begin{align*}
    y'(\ell) &= G(z)u'(\ell) \\
    u'(\ell) &= [z^{-\Delta} H_D(z) \ H_E(z)] \tilde{u}(\ell) 
\end{align*} \]

In (9) the decision variables satisfy, see (4):

\[ \tilde{u}(\ell) \in U(\ell), \quad \forall \ell \in \{k, k+1, \ldots, k+N-1\}. \] (10)

Rather than sending the entire optimizing control sequence corresponding to \( \ell \in \{k, k+1, \ldots, k+N-1\} \), following the Receding Horizon Principle, the controller output is set equal to the first element (corresponding to \( \ell = k \)). This leads ultimately to the true plant input \( u(\ell) \). The optimizing procedure is then repeated ad infinitum.

Fig. 2 illustrates this idea for a horizon \( N = 4 \). Also included in this figure is the constraint set \( U_D(\ell) \) for a delay channel with sub-sampling factor \( L = 3 \), see (6).

4. COMPUTATIONAL ASPECTS

The deployment of two virtual down-link channels certainly increases the complexity of the networked controller when compared to a controller which uses only the erasure channel. However, since the delay channel is down-sampled, the increase of complexity will only be moderate.

Fig. 2. Receding horizon principle and constraint sets for \( U_D(\ell) \), given \( L = 3 \) and \( N = 4 \).
More precisely, to find the optimizing sequence to \( V_N \) in (8), in general an (explicit or implicit) exhaustive enumeration technique will be used. Thus, the on-line computation time of the controller is essentially proportional to the cardinality of the search set of the decision variables. Due to the sub-sampling constraint (6), the effective number of decision variables \( \tilde{u}_D(\ell) \) examined within the horizon \( k \leq \ell < k + N \) depends upon \( k \). This is illustrated in Fig. 2, where at time instants \( k = 0 \) and \( k = 3 \) two decision variables need to be optimized, while at \( k = 1 \) and \( k = 2 \), only one variable falls inside the horizon.

An exact expression for the cardinality of the search set is given in Lemma 2 below. It uses the following definition:

**Definition 1.** (Division with remainder). For any \( N, L \in \mathbb{N} \), the integers floor \( (N/L) \) and \( \text{rem} (N/L) \) are defined implicitly as:

\[
N = L \text{ floor} (N/L) + \text{rem} (N/L),
\]

where:

\[
(N/L) - 1 < \text{floor} (N/L) \leq N/L, \quad 0 \leq \text{rem} (N/L) < L.
\]

**Lemma 2.** The cardinality of the search set is:

\[
N_U(k) = \eta(k) \left( \left\lceil \frac{N}{L} \right\rceil \right)^N \left( \left\lceil \frac{U_d}{L} \right\rceil \right) \text{floor}(N/L),
\]

where \( \lceil \cdot \rceil \) denotes cardinality,

\[
\eta(k) = \begin{cases} 
1, & \text{if } 0 < k \leq L - \text{rem} (N/L), \\
\lceil U_d \rceil, & \text{if } L - \text{rem} (N/L) < k \leq L,
\end{cases}
\]

and

\[
\eta(k + L) = \eta(k), \quad \forall k \geq 0.
\]

**PROOF.** It follows directly from (5), (6) and (10) that:

\[
N_U(k) = \left( \left\lceil \frac{N}{L} \right\rceil \right)^N \prod_{k \leq \ell < k + N} \left| U_D(\ell) \right|.
\]

On the other hand, careful inspection of Fig. 2 yields:

\[
\prod_{k \leq \ell < k + N} \left| U_D(\ell) \right| = \eta(k) \left( \left\lceil \frac{U_d}{L} \right\rceil \right) \text{floor}(N/L),
\]

from where (11) follows. \( \square \)

As a consequence of this result, computation times can be bounded. Furthermore, computation times have an average value, which is proportional to:

\[
\frac{1}{L} \sum_{k=1}^{L} N_U(k) = \frac{\left( \left\lceil \frac{N}{L} \right\rceil \right)^N \left( \left\lceil \frac{U_d}{L} \right\rceil \right) \text{floor}(N/L)}{L} \cdot (L + \text{rem} (N/L) (|U_d| - 1)).
\]

5. STATE SPACE FORMULATION

Given the prediction model (9), at time \( \ell = k \), \( V_N \) can be minimized based upon previous decisions \( \tilde{u}(\ell) \), where \( \ell < k \). Alternatives can be devised by formulating (9) in state space.

For that purpose, \( G(z) \) is described as:

\[
x'_G(\ell + 1) = A_G x'_G(\ell) + B_G u'(\ell) \\
y'(\ell) = C_G x'_G(\ell),
\]

where \( u'(\ell) = u_D(\ell) + u'_E(\ell) \), while \( H_E(z) \) and \( H_D(z) \) satisfy:

\[
x'_D(\ell + 1) = A_D x'_D(\ell) + B_D \begin{bmatrix} 1 & 0 \end{bmatrix} \tilde{u}(\ell) \\
u'_D(\ell) = C_D x'_D(\ell) \\
x'_E(\ell + 1) = A_E x'_E(\ell) + B_E \begin{bmatrix} 0 & 1 \end{bmatrix} \tilde{u}(\ell) \\
u'_E(\ell) = C_E x'_E(\ell) + D_E \begin{bmatrix} 0 & 1 \end{bmatrix} \tilde{u}(\ell)
\]

The models (12) and (13) can be combined into an overall prediction model:

\[
x'(\ell + 1) = A x'(\ell) + B \tilde{u}(\ell) \\
\begin{bmatrix} y'(\ell) \\
u'(\ell) \end{bmatrix} = C x'(\ell) + D \tilde{u}(\ell), \quad k \leq \ell < k + N,
\]

where:

\[
A \triangleq \begin{bmatrix} A_G & B_G C_D & B_G C_E \\ 0 & A_D & 0 \\ 0 & 0 & A_E \end{bmatrix}, \\
B \triangleq \begin{bmatrix} 0 & B_G D_E \\ B_D & 0 \\ 0 & B_E \end{bmatrix}, \\
C \triangleq \begin{bmatrix} C_G & 0 & 0 \\ 0 & C_D & C_E \end{bmatrix}, \\
D \triangleq \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & D_E \end{bmatrix}.
\]

The initial state for the prediction model (14) is:

\[
x'(k) = x(k) \triangleq \begin{bmatrix} x_G(k) \\ x_D(\ell) \\ x_E(\ell) \end{bmatrix}.
\]

Note that \( x(k) \) not only includes plant states, but also channel and source decoder states.

In practical situations, plant and channels are affected by disturbances and transmission errors. Furthermore, plant outputs need to be coded to be sent through the up-link to the controller, see e.g. (Wong and Brockett, 1999; Goodwin et al., 2004). As a consequence, \( x(k) \) often needs to be replaced by an observed value \( \hat{x}(k) \).

To be more precise, in the NCS architecture of Fig. 1, the erasure channel is affected by random transmission errors. Unless a feedback channel is deployed for the down-link which feeds back plant inputs to the controller, \( x_E(\ell) \) needs to be estimated. The simplest method resides in using an open-loop observer (see (13)):

\[
\hat{x}_E(k + 1) = A_E \hat{x}_E(k) + B_E \tilde{u}_E(k).
\]
For moderate to high error probabilities, a closed loop observer may be preferable. One way to derive such an observer uses an additive erasure-channel noise model:

\[ n_E(t) \triangleq -c(t) \tilde{u}_E(t). \]  

(16)

With this, plant and erasure channel, see (1) and (12), can be re-written as:

\[
\begin{bmatrix}
    x_G(t+1) \\
    x_E(t+1)
\end{bmatrix} =
\begin{bmatrix}
    A_G & B_G C_E \\
    0 & A_E
\end{bmatrix}
\begin{bmatrix}
    x_G(t) \\
    x_E(t)
\end{bmatrix}
+ \begin{bmatrix}
    B_G D_E \\
    B_E
\end{bmatrix} n_E(t) + v(t),
\]

where:

\[ v(t) \triangleq \begin{bmatrix}
    B_G D_E \\
    B_E
\end{bmatrix} \tilde{u}_E(t) + \begin{bmatrix}
    B_G \\
    0
\end{bmatrix} z^{-\Delta} H_D(z) \tilde{u}_D(t) \]

is a known quantity. A linear filter can be used to estimate \( x_G(t) \) and \( x_E(t) \) based upon, possibly noisy, plant output measurements, see e.g. (Goodwin and Sin, 1984). It is worth emphasizing here that \( n_E(t) \) defined in (16) is, strictly speaking, not an independent noise process. Furthermore, \( A_E \) should have all its eigenvalues strictly inside the unit circle to give good performance, see also (Quevedo et al., 2004b).

Note 2. (Implementation). By adapting previous work, see e.g. (Quevedo et al., 2003) it can be shown that the Two-channel Networked Controller can be implemented via linear filters and a vector quantizer, which operate on the reference trajectory and \( \hat{x}(k) \). Due to space limitations, details are omitted. An interesting point resides in the fact that the constraint set \( U(\ell) \) is \( L \)-periodic, see (7). As a consequence, also the vector quantizer, and the networked controller are \( L \)-periodic.

### 6. EXAMPLE

Consider a model of an NCS aimed at making the level of liquid in a tank follow a reference signal. The system is described by (12) with:

\[ A_G = 0.9716, \quad B_G = 0.125, \quad C_G = 0.1979, \]

see (Quevedo et al., 2003). The plant states \( x_G(t) \) are here assumed known to the controller.

The down-link is composed of two virtual channels as in Fig. 1. The delay channel has a sub-sampling ratio of \( L = 4 \), and a time delay of two time steps (\( \Delta = 2 \) in (2)). The data-dropout probability of the erasure channel is \( P = 10\% \), see (1). Both channels can only transmit data, which belongs to the ternary sets \( U_E = U_d = \{-1, 0, 1\} \). Following the results of (Quevedo et al., 2004b), the source decoders are chosen as:

\[ H_D(z) = \frac{0.5z}{z-1}, \quad H_E(z) = \frac{0.5z}{z-0.7} \]

(17)

Fig. 3. The two-channel architecture.

For comparison, three other NCSs, which differ from the NCS architecture proposed in the present work by making use of only one of the two available down-link channels, were investigated, see (Quevedo et al., 2003; Quevedo et al., 2004b). The first NCS utilizes the erasure channel and the decoder \( H_E(z) \) given in (17), while the second system deploys the same channel and the integrating decoder \( H_D(z) \). The third design makes use of the delay channel and \( H_D(z) \), see (17).

Performance of the above architectures is documented in Fig. 4, where also the results corresponding to the two-channel NCS are included. Note that the NCSs which use \( H_D(z) \) exhibit steady state errors in the erasure channel case and excessive overshoot when operating over the delay channel. On the other hand, using the stable source decoder \( H_E(z) \) avoids these phenomena at the expense of a somewhat sluggish response to reference changes. These observations agree with the design trade-offs elucidated in (Quevedo et al., 2004b). All these one-channel schemes are outperformed by the proposed two-channel NCS.

Also in the ideal full-information case where \( x_E(t) \) is known to the controller, the two-channel NCS outperforms all one-channel architectures as can be seen in Fig. 5. It is worth noticing that use of the integrating source decoder over the erasure channel now gives zero steady state error.

### 7. CONCLUSIONS

This paper has described an NCS architecture comprising a down-link communication system which can operate in two modes. The scheme

---

\[ \text{Note 4: } x_D(k) \text{ is error-free and an open-loop observer similar to (15) gives exact results.} \]


