Abstract: An output feedback constrained MPC control scheme for uncertain LFR/Norm-Bounded discrete-time linear systems is discussed. The design procedure consists of an off-line step in which a state-feedback and an asymptotic observer (dynamic primal controller) are designed via BMI optimization and used to robustly stabilize a suitably augmented system. The on-line moving horizon procedure adds $N$ free control moves to the action of the primal controller and its computation consists of solving an online LMI optimization problem whose numerical complexity grows up only linearly with the control horizon $N$. The effectiveness is illustrated by a numerical example.

Keywords: Predictive Control, Uncertain Linear Systems, Output Feedback, Convex Programming

1. INTRODUCTION

Model predictive control (MPC) is a standard control technique based on the on-line solution of a constrained optimization problem (see (Kothare et al., 1996) and references therein). In this paper, we introduce an approach to design output feedback MPC controllers for LFR/Norm Bounded uncertain discrete-time linear systems. The main contribution is an extension of the full-state framework introduced in (Casavola et al., 2004). It is shown that joint state estimation and minmax MPC can be cast into an optimization problem: the off-line step (controller/observer couple) can be reduced to a BMI (solvable using local search optimization algorithms (Kočvara and Stingl, 2003)) whereas the on-line step (receding horizon algorithm) can proved to be an LMI and solvable instead using standard semidefinite programming solvers.

Contributions on output feedback MPC ensuring stability for augmented systems (observer and moving horizon controller) were first established by (Michalska and Mayne, 1995), (Scokaert et al., 1997). More recent contributions have been dealt by (Lee and Kouvaritakis, 2001) and (Wan and Kothare, 2002). In particular, the latter authors left unsolved how to take into account the state estimation error for ensuring MPC solvability, especially in the presence of hard state-related constraints. Such analysis has been considered in the present work which also extends the results of (Wan and Kothare, 2002) in considering control horizons of arbitrary length $N$.

2. PROBLEM FORMULATION

Consider the following discrete-time linear system with uncertainties appearing in the feedback loop
\[
\begin{aligned}
\dot{x}(t+1) &= \Phi x(t) + Gu(t) + B_p p(t) \\
\dot{y}(t) &= Hx(t) + Eu(t) \\
\|p(t)\|_2^2 &\leq \|C_p x(t) + D_{qu}(t)\|_2^2
\end{aligned}
\]

with \( x \in \mathbb{R}^n \) denoting the state, \( u \in \mathbb{R}^m \) the control input, \( y \in \mathbb{R}^p \) the measured output and \( p \in \mathbb{R}^p \) accounting for the uncertainty (\( \| \cdot \|_2 \) denotes the standard euclidean norm). It is further assumed that the plant input is subject to the following ellipsoidal constraint

\[
u(t) \in \Omega_a, \quad \Omega_a = \left\{ u \in \mathbb{R}^m : u^T Q_a u \leq \bar{u} \right\}
\]

The aim is to find a dynamic output-feedback regulation strategy \( \nu(t) = g(y^*, u^{-1}) \) which possibly asymptotically stabilizes (1) subject to (2). In order to reconstruct the state, which is not directly measurable, a full state observer based on the nominal plant realization is proposed

\[
\ddot{x}(t+1) = \Phi \ddot{x}(t) + Gu(t) + L (y(t) - \bar{y}(t))
\]

where \( L \in \mathbb{R}^{n_x \times n_y} \) is the observer gain matrix and \( \ddot{y}(t) \) is the output estimate. By defining the state estimation error as

\[
e(t) \triangleq x(t) - \ddot{x}(t)
\]

we assume that the uncertainty on the initial state satisfies

\[
e^T(0) W e(0) \leq \bar{e}_0^2
\]

Due to the presence of a state dependent signal \( p(t) \) acting both on the state and the output, the separation principle does not hold true and conditions for the quadratic stability must be expressed in terms of the augmented state \( \ddot{x}(t) e^T(t) \)^T. Specifically, the family of systems plant/observer (1),(3) is quadratically stabilizable by a feedback control law based on the state estimate

\[
u(t) = K \ddot{x}(t)
\]

if there exists a controller/observer pair, such that, for all the initial states belonging to (5), all closed-loop augmented state trajectories will converge asymptotically to \( X_0 \). Using the strategy (6), standard arguments (see (Wan and Kothare, 2002) for details) allows one to conclude that quadratic stability conditions can be guaranteed if the following matrix

\[
X_0 \triangleq \begin{bmatrix}
\rho Q & 0 & \Phi^T_{KL} & 0 \\
0 & \mu L_{p} & 0 & \Phi^T_{L} \\
\Phi^T_{KL} & 0 & \mu C^T_{q,K} & 0 \\
0 & \Phi^T_{L} & 0 & \mu C^T_{q,L}
\end{bmatrix}
\]

is positive semidefinite. Notice that (7) is bilinear in \( K \in \mathbb{R}^{n_x \times n_y}, L \in \mathbb{R}^{n_x \times n}, Q = Q^T \in \mathbb{R}^{2n \times 2n} \geq 0, \mu \geq 0, \rho \geq 0, \rho < 1 \) (the latter is a scalar affecting the convergence rate of the augmented state). Moreover, \( \Phi_K \triangleq \Phi + G K, \Phi_L \triangleq \Phi + L H, C_{q,K} \triangleq C_q + D_q K \).

The control performance and the invariance properties need to be defined with respect to the true state, which can be regarded as a linear combination of the augmented state components \( x = \ddot{x} + e \). Then, we need to find conditions under which the control strategy (6) achieves a guaranteed cost

\[
J(x(0), u(\cdot)) \triangleq \max_{p(t) \in S_t} \int_0^{\bar{t}} \sum_{\tau=0}^{\bar{t}-1} \left( \|\dot{x}(t)\|_R^2 + \|u(t)\|_R^2 \right) e^T(0) W e(0) \leq \bar{e}_0^2
\]

where \( R_x = R_x^T \geq 0, R_u = R_u^T \geq 0 \) are state and input weighting matrices (\( \| \cdot \|_R \) and \( \| \cdot \|_{R_u} \) denote matrix weighted euclidean norms) and the sets

\[
S_t \triangleq \left\{ p : \| p \|_2^2 \leq \|(C_{q,K} + D_q K) \ddot{x}(t) + C_q e(t)\|_2^2 \right\}
\]

represent plant uncertainty domains at each time instant \( t \). A bound on (8) is given by

\[
\max_{e^T(0) W e(0) \leq \bar{e}_0^2} \left( \ddot{x}(0) + e(0) \right)^T P (\ddot{x}(0) + e(0)) \leq \bar{e}_0^2
\]

for a suitably chosen matrix \( P = P^T > 0 \in \mathbb{R}^{n_x \times n_x} \). Moreover, if \( K, L \) satisfy (7), the following ellipsoidal set

\[
C(P, \gamma) \triangleq \left\{ x \in \mathbb{R}^n \left| (\ddot{x} + e)^T P (\ddot{x} + e) \leq \gamma, \forall e \right. \right\}
\]

can be proved to be a robust positively invariant region for the state evolutions of the closed-loop system, viz. \( x(0) \subseteq C(P, \gamma) \) implies that

\[
x(t) \subseteq \left\{ \left[ I_n, I_n \right], \left[ \Phi_K - L H \right]^T, \ddot{x}(0), e(0) \right\} \subseteq C(P, \gamma)
\]

for all \( t \). Given the cost (8) and the upper bound (10) the following matrix

\[
X_1 \triangleq \begin{bmatrix}
\rho & 0 & \Phi^T_{KL} & 0 \\
0 & \mu L_{p} & 0 & \Phi^T_{L} \\
\Phi^T_{KL} & 0 & \mu C^T_{q,K} & 0 \\
0 & \Phi^T_{L} & 0 & \mu C^T_{q,L}
\end{bmatrix}
\]

is bilinear in the triplet \( (K, L, P) \) and in the scalar \( \lambda \geq 0 \), must be positive semidefinite where

\[
\Phi_{KL} \triangleq \Phi_K - L H, \quad \Phi_L \triangleq \Phi + L H, \quad C_{q,K} \triangleq C_q + D_q K
\]

The invariance condition \( x(0) \subseteq C(P, \gamma) \) can equivalently be translated, via the S-procedure, into the requirement that the following matrix, linear in \( P, \xi \geq 0 \) and \( \gamma > 0 \)

\[
X_2 \triangleq \begin{bmatrix}
\xi W - P & -P \xi(0) \\
\ast & \gamma - \xi^2 e_0^2 - \xi^2(0) P \xi(0)
\end{bmatrix}
\]

be positive semidefinite. Finally, the input constraint

\[
u^T(t) Q_u u(t) \leq \bar{u}^2
\]

with \( u(t) = K \ddot{x}(t) \) is satisfied iff

\[
\mathcal{U} \triangleq \begin{bmatrix}
\bar{u}^T I & Q^T_u K \\
K^T Q_u^{\frac{1}{2}} & P
\end{bmatrix}
\]
is positive semidefinite as well. All previous discussion can be summarized in the following Theorem 1.

**Theorem 1.** Let the triplet \((P, K, L)\) be a solution of following BMI optimization problem

\[
\min_{K, L, P, Q; \mu, \lambda, \xi} \gamma \\
\text{s.t.} \\
P > 0, \ Q > 0 \\
X_0 \geq 0, \ X_1 \geq 0, \ X_2 \geq 0, \ \forall U \geq 0 \\
\mu \geq 0, \ \lambda \geq 0, \ \xi \geq 0
\]

Then, provided that (5) is satisfied, the control law (6) with the state estimate computed via (3) satisfies the input constraints (2), ensures quadratic stability to the system (1) and achieves a guaranteed cost upper-bound \(\gamma\) to the quadratic index (8) with \(\Phi_{K,L,X}(t) \subseteq C(P, \gamma), \ \forall t\).

**Proof -** By collecting all the above discussion and exploiting standard results. \(\square\)

In order to add predictive capability we consider the following family of virtual commands

\[
u(t) = \begin{cases} 
K \hat{s}_k(t) + c_k(t), & k = 0, 1, \ldots, N - 1, \\
K \hat{s}_k(t) & k \geq N,
\end{cases}
\]

with \(\hat{s}_k(t) \triangleq \hat{x}(t + k | t)\) and \(c_k(t) \triangleq e(t + k | t)\). The latter vectors, over which the optimization takes place, provide \(N\) free perturbations over the action of the stabilizing and admissible primal dynamic controller defined by the \((K, L)\) pair solving (16). Given the strategy (17), it is possible to consider the convex set-valued state predictions \(x_k(t) = \hat{x}_k(t) + e_k(t)\) which are the projections of the augmented state predictions

\[
\begin{bmatrix} \hat{x}_k(t) \\ e_k(t) \end{bmatrix} = \Phi_{K,L} \begin{bmatrix} \bar{x}(t) \\ e(t) \end{bmatrix} + \sum_{i=0}^{k-1} \Phi_{K,L}^{k-1-i} G_{ci}(t) + \sum_{i=0}^{k-1} \Phi_{K,L}^{k-1-i} B_{pi}(t)
\]

along the plant state space, computed under the conditions \(p_i(t) \triangleq p(t + i | t) \in S_i(t), \ e(t)^T We(t) \leq \tilde{c}_i^2\)

\[
S_i(t) \triangleq \left\{ p : \| p \|^2 \leq \max_{x(t), e(t)} \| C_{G_{ki}, G_{ei}}(t) + D_{ci}(t) \|_2^2 \right\}, \quad i = 0, \ldots, k - 1 \text{ with } S_i(t) \text{ characterizing all admissible perturbations along the system trajectories corresponding to the virtual command sequence (17).}
\]

**Remark 1 -** One of the main features of the command family (17) is that it allows the setup of moving horizon strategies based on the notion of “closed-loop” state predictions. The rationale hinges upon the fact that, under (17) and given a couple \((K, L)\) solution of (16), the augmented system state predictions from time \(t\) onwards \(\hat{x}_k(t) \triangleq \bar{x}(t + k | t), \ e_k(t) \triangleq e(t + k | t)\), \(k = 0, \ldots, N - 1\) are linear in terms of the input moves \(c_k(t), k = 0, \ldots, N - 1\). Moreover, the cost (20) can be equivalently rewritten as the following minmax quadratic index

\[
V(\hat{x}(0), P, c_k(t)) \triangleq \gamma \sum_{k=0}^{\infty} \left( \max_{p_i(t) \in S_i(t)} (|e_k(t)| + c_k(t))^2 \right) + \max_{p_{N+2}\in S_{N+2}} (|e_{N+1}| + c_{N+1})^2 \leq \tilde{c}_i^2
\]

(20) to be minimized w.r.t. \(c_k(t), k = 0, \ldots, N - 1\). Then, at each time instant \(t\), our solution will consist of computing

\[
c_k(t) = \arg \min_{c_k(t)} V(\hat{x}(t), P, c_k(t)) \quad \text{s.t.}
\]

\[
K \hat{s}_k(t) + c_k(t) \subseteq \Omega_{v_k}, \quad k = 0, 1, \ldots, N - 1,
\]

\[
\bar{s}_N(t) + e_N(t) \subseteq C(P, \gamma), \quad K z \subseteq \Omega_{v_k}, \quad \forall z \in C(P, \gamma)
\]

where \(C(P, \gamma)\) is a robust invariant set under \(K, (P, K, L, \gamma)\) solution of (16). It will be shown that the above optimization problem is solvable at each time \(t\) provided that it is solvable at time \(t = 0\). The resulting predictive control action, based on the state estimate provided by the observer, satisfies the constraints and stabilizes the plant. At the stage of the estimator design, the speed of the error dynamics is obviously influenced by the triplet \((P, K, L)\), the \(N\) perturbations \(c_k(t), k = 0, \ldots, N - 1\) and the output measurements \(y(t)\). Starting from such a consideration and in order to improve the control performance of the proposed strategy, the error estimation bound

\[
e(t)^T We(t) \leq \tilde{c}_i^2,
\]

(22) can be updated at each time instant. In the next section, along with the moving horizon strategy, sufficient LMI conditions will be derived in order to accomplish this requirement.

### 3. MOVING HORIZON SCHEME

#### 3.1 Upper Bound Conditions

In this section we aim at determining a suitable upper bound to (20) in terms of LMI feasibility conditions. We will suppose the generic time instant \(t\) equal to zero and denote \(c_k = c_k(0), p_k = p(k | 0), e_k = e(k | 0), \bar{s}_k = \bar{s}_k(0), \tilde{x} = \tilde{x}(0)\) and \(S_k = S_k(0)\) for \(k = 0, 1, \ldots, N - 1\) for notational simplicity.

The simplest way to derive an easily computable upper-bound to the cost (20) is that of introducing non-negative reals \(J_0, J_1, \ldots, J_{N-1}\) such that, for arbitrary \(P, K, L\), and \(c_k, k = 0, 1, \ldots, N - 1\), the following inequalities

\[
\max_{p_{k+1} \in S_{k+1}} (i_{k+1} + e_{k+1})^T R_k (i_{k+1} + e_{k+1}) + \tilde{c}_i^2 \leq \tilde{c}_i^2
\]

(23)

\[
\max_{p_{N+1} \in S_{N+1}} (i_{N+1} + e_{N+1})^T R_k (i_{N+1} + e_{N+1}) + \tilde{c}_i^2 \leq \tilde{c}_i^2
\]

(24)

hold true. In such a case, in results that
are positive definite, where \( \Sigma_k \) is the Choleski factor of

\[
el_k = \begin{bmatrix} e_k + \sum_{i=0}^{T} \phi_{i} e_k^i \end{bmatrix},
\]

\[
\begin{bmatrix} \Phi_{k} & \Phi_{k}^T & \vdots & \Phi_{k}^T \\
\Phi_{k} & \Phi_{k}^T & \vdots & \Phi_{k}^T \\
\vdots & \vdots & \ddots & \vdots \\
\Phi_{k} & \Phi_{k}^T & \vdots & \Phi_{k}^T
\end{bmatrix}
\]

are positive definite, where \( \Sigma_k \) is the Choleski factor of

\[
Y_0 \triangleq \left[ \bar{\theta} - \begin{bmatrix} (K \tilde{x} + c_0)^T Q_0 (K \tilde{x} + c_0) \end{bmatrix} \right] \geq 0
\]

whereas the satisfaction of (28) is ensured if the following matrix inequalities

\[
Y_k \triangleq \left[ \bar{\theta} - \begin{bmatrix} (K \tilde{x} + c_0)^T Q_0 (K \tilde{x} + c_0) \end{bmatrix} \right] \geq 0
\]

linear in the triplet \((\tilde{x}, \xi, \bar{\theta})\), is positive semidefinite, where \( T_k \) is the choleski factor of

\[
t_k \triangleq \begin{bmatrix} \tilde{\theta} & \sum_{i=0}^{T} \phi_{i} e_k^i \end{bmatrix},
\]

\[
\begin{bmatrix} \Phi_{k} & \Phi_{k}^T & \vdots & \Phi_{k}^T \\
\Phi_{k} & \Phi_{k}^T & \vdots & \Phi_{k}^T \\
\vdots & \vdots & \ddots & \vdots \\
\Phi_{k} & \Phi_{k}^T & \vdots & \Phi_{k}^T
\end{bmatrix}
\]

are positive definite, where \( \Sigma_k \) is the Choleski factor of

\[
Y_0 \triangleq \left[ \bar{\theta} - \begin{bmatrix} (K \tilde{x} + c_0)^T Q_0 (K \tilde{x} + c_0) \end{bmatrix} \right] \geq 0
\]

whereas the satisfaction of (28) is ensured if the following matrix inequalities

\[
Y_k \triangleq \left[ \bar{\theta} - \begin{bmatrix} (K \tilde{x} + c_0)^T Q_0 (K \tilde{x} + c_0) \end{bmatrix} \right] \geq 0
\]

linear in the triplet \((\tilde{x}, \xi, \bar{\theta})\), is positive semidefinite, where \( T_k \) is the choleski factor of

\[
t_k \triangleq \begin{bmatrix} \tilde{\theta} & \sum_{i=0}^{T} \phi_{i} e_k^i \end{bmatrix},
\]

\[
\begin{bmatrix} \Phi_{k} & \Phi_{k}^T & \vdots & \Phi_{k}^T \\
\Phi_{k} & \Phi_{k}^T & \vdots & \Phi_{k}^T \\
\vdots & \vdots & \ddots & \vdots \\
\Phi_{k} & \Phi_{k}^T & \vdots & \Phi_{k}^T
\end{bmatrix}
\]

are positive definite, where \( \Sigma_k \) is the Choleski factor of

\[
Y_0 \triangleq \left[ \bar{\theta} - \begin{bmatrix} (K \tilde{x} + c_0)^T Q_0 (K \tilde{x} + c_0) \end{bmatrix} \right] \geq 0
\]

whereas the satisfaction of (28) is ensured if the following matrix inequalities

\[
Y_k \triangleq \left[ \bar{\theta} - \begin{bmatrix} (K \tilde{x} + c_0)^T Q_0 (K \tilde{x} + c_0) \end{bmatrix} \right] \geq 0
\]

linear in the triplet \((\tilde{x}, \xi, \bar{\theta})\), is positive semidefinite, where \( T_k \) is the choleski factor of

\[
t_k \triangleq \begin{bmatrix} \tilde{\theta} & \sum_{i=0}^{T} \phi_{i} e_k^i \end{bmatrix},
\]

\[
\begin{bmatrix} \Phi_{k} & \Phi_{k}^T & \vdots & \Phi_{k}^T \\
\Phi_{k} & \Phi_{k}^T & \vdots & \Phi_{k}^T \\
\vdots & \vdots & \ddots & \vdots \\
\Phi_{k} & \Phi_{k}^T & \vdots & \Phi_{k}^T
\end{bmatrix}
\]

are positive definite, where \( \Sigma_k \) is the Choleski factor of

\[
Y_0 \triangleq \left[ \bar{\theta} - \begin{bmatrix} (K \tilde{x} + c_0)^T Q_0 (K \tilde{x} + c_0) \end{bmatrix} \right] \geq 0
\]

whereas the satisfaction of (28) is ensured if the following matrix inequalities

\[
Y_k \triangleq \left[ \bar{\theta} - \begin{bmatrix} (K \tilde{x} + c_0)^T Q_0 (K \tilde{x} + c_0) \end{bmatrix} \right] \geq 0
\]
3.4 State estimation error updating

Let us suppose that the MPC scheme will have a solution $c_{0},c_{1},...,c_{N-1}$ for a generic instant time $t$. Then, the state estimation error is constrained to belong to the following uncertainty set

$$e(t)^T W e(t) \leq \varepsilon_1^2$$

and the problem that we face in this section is how to update such a condition by taking into account the error contraction law. Given, at each time $t$, a $(K,L)$ pair solving the off-line problem (16) and a sequence of free input moves $c_{0},c_{1},...,c_{N-1}$, at the next time instant $t+1$ the state estimation error must satisfy the following equation

$$(\Phi + LH)e(t) + B_p p_0(t))^T W (\Phi + LH)e(t) + B_p p_0(t)) \leq \varepsilon_1^2,$$

with the uncertainty radius $\varepsilon_1^2$ to be computed, $\forall e(t), p_0(t)$, s.t.

$$e(t)^T W e(t) \leq \varepsilon_1^2$$

$$\|p_0(t)\|^2 \leq \|C_q K \bar{x}(t) + D_q e_0 + C_q e(t)\|^2$$

This can be done by means of the S-procedure. In fact, the following statement (34) holds true for all $e(t), p_0(t)$ satisfying (35),(36)

holds true if there exist scalars $\varsigma_e, \varsigma_p \geq 0$ such that the following matrix

$\Delta \equiv \begin{bmatrix} -LH + C_q K \varsigma_e + D_q \varsigma_p e_0 & -LH + C_q K \varsigma_e + D_q \varsigma_p e_0^T \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigma_e \varsigma_p \varsigm
for all \( t \) realization of the plant is equal to the nominal plant over 200 sec. of simulation time, by supposing that the burden (1876 flops per step for \( N = 1 \) and 4387 flops per step for \( N = 2 \))

\[
K = \begin{bmatrix} -0.6144 & -5.9035 \end{bmatrix} \quad L = \begin{bmatrix} -0.9793 & -0.3386 \end{bmatrix}
\]

The on-line moving horizon part has been solved via standard LMI solvers for \( N = 1 \) and \( N = 2 \) over 200 sec. of simulation time, by supposing that the realization of the plant is equal to the nominal plant for all \( t \), the initial state (not directly available and measurable) is equal to \( y(0) = [0.1 \, 0.1]^T \), the initial state estimate is equal to \( \hat{y} = 0 \), and the initial state estimation error belongs to the ball of radius \( \bar{E} = 0 \). As expected, the use of increasingly larger control horizons improves the control performance at expenses of a modest increment of the on-line computational burden (1876 flops per step for \( N = 1 \) and 4387 flops per step for \( N = 2 \); 0.2421 sec. average CPU time per step for \( N = 1 \) and 2.1753 sec. average CPU per step for \( N = 2 \))

5. CONCLUSIONS

In this paper, a novel output feedback MPC strategy has been presented for input-saturated LFR/Norm-Bounded uncertain systems. The novelty in this scheme relies on the simultaneous off-line design of an observer/controller pair, capable to cope with the model mismatch. Even if the joint selection of state-feedback/observer pair is a non-convex problem, a feasible couple w.r.t. a LQ performance measure can usually be found in a finite number of steps by means of a local algorithm. The on-line MPC strategy is based on minimizing, at each time instant, an upper bound on the worst-case cost for an augmented plant (state estimate/state estimation error), under the constraint that all future state prediction are robustly steered within \( N \)-steps into a feasible positively invariant set whose shape is fixed and derived in the off-line phase. The numerical experiences accomplished on a benchmark problem have shown the effectiveness of the approach.

REFERENCES


