FLATNESS-BASED CONTROL OF A PARALLEL ROBOT ACTUATED BY PNEUMATIC MUSCLES

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Abstract: This paper presents a flatness-based control for a two-degree-of-freedom parallel robot driven by two pairs of pneumatic muscle actuators. The robot consists of a light-weight closed-chain structure with four moving links connected by revolute joints. The two base joints are active and driven by pairs of pneumatic muscles by means of a toothed belt and pulley. Exploiting differential flatness with end-effector position and mean muscle pressures as flat outputs, a cascaded trajectory control is designed. Simulation results demonstrate an excellent control performance and point out the potential of this novel robot. Copyright © 2005 IFAC

Keywords: Model-based control, robotics, nonlinearity, mechanisms, pneumatic systems

1. INTRODUCTION

Pneumatic muscles are tension actuators, which consist of a fibre-reinforced vulcanised rubber tubing with connection flanges at both ends. The working principle is based on the specially designed fibre structure that leads to a muscle contraction in longitudinal direction when the pneumatic muscle is filled with compressed air by means of a proportional valve. Pneumatic muscles offer several advantages in comparison to classical cylinders: significantly less weight, no stick-slip effects, insensitivity to dirty working environment, and a larger maximum force. The achievable closed-loop performance using such pneumatic muscle actuators in combination with sophisticated non-linear control has already been investigated thoroughly by experiments at a one-degree-of-freedom test rig (Aschemann and Hofer, 2004). This experimental platform consists of a carriage with pneumatic muscles arranged at opposite sides, which allow for rectilinear movements on two guideways. Current research at the University of Ulm involves the use of pneumatic muscles as actuators for parallel robots, which are known for providing high stiffness, and especially for the capability of performing fast and highly accurate motions of the end-effector.

The parallel robot, which serves as a platform for the development and the investigation of non-linear control approaches, is depicted in fig. 1. It is characterised by a closed-chain kinematic structure formed by four moving links and the robot base offering two degrees of freedom. All joints are revolute joints, two of which - the cranks - are actuated by a pair of pneumatic muscles, respectively. Here, the coordinated contraction of a pair of pneumatic muscles is transformed into a rotation of the according crank by means of a toothed belt and a pulley. The mass flow rate of compressed air into and out of a pneumatic muscle is controlled by means of a proportional valve. The incoming air is available at a maximum pressure of 7 bar, whereas the outlet air is discharged at atmospheric pressure, i.e. 1 bar. To avoid pressure
The chosen multi-body model of the parallel robot part consists of three rigid bodies (fig. 1): the two cranks as actuated links with identical properties (mass $m_A$, reduced mass moment of inertia w.r.t. the actuated axis $J_A$, center of gravity distance $s_A$ to the centre of gravity $C_A$, length of the link $l_A$, pulley radius $r$) and the end-effector $E$ (mass $m_E$), which can be modelled as lumped mass. The inertia properties of the remaining two links with length $l_P$, which are designed as light-weight construction, shall be neglected in comparison to the other links.

For a given end-effector position $r$ the corresponding crank angles follow from the inverse kinematics $q = q(r, k_1, k_2)$, which can be determined in symbolic form. The given ambiguity is taken into account by introducing the configuration parameters $k_1$ and $k_2$ as shown in fig. 2. The relationship between the corresponding velocities is obtained by differentiation $\dot{q} = J(r, k_1, k_2)\dot{r}$, where $J(r, k_1, k_2)$ denotes the Jacobian. Analogously, the acceleration relationship is given by $\ddot{q} = J(r, k_1, k_2)\ddot{r} + J(r, k_1, k_2)\dddot{r}$.

The direct kinematics yields the vector of end-effector coordinates for given crank angles, i.e. $r = r(q, k_3)$. Similar to the inverse kinematics, the configuration parameter $k_3$ is introduced to cope with two possible configurations. The relationships between velocities and between accelerations are derived by taking advantage of the inverse dynamics. This leads directly to $\ddot{r} = J^{-1}(q, k_1, k_2, k_3)\dddot{q}$ and $\dddot{r} = J^{-1}(q, k_1, k_2, k_3)[\dddot{q} - J(q, k_1, k_2, k_3)\dddot{r}]$. At this, singularities in the Jacobian can be avoided by model-based trajectory planning.

The according equations of motion for the actuated links follow directly from the free-body diagramm applying Euler’s Law.
Here, the drive torque \( \tau_i \) of drive \( i \) depends on the corresponding muscle forces, i.e. \( \tau_i = r \cdot [F_{Mij} - F_{Mis}] \). The disturbance torque \( \eta_i \) accounts for friction effects as well as remaining uncertainties in the muscle force characteristics (5) of drive \( i \), respectively. The coupling forces \( F_{1E} \) and \( F_{2E} \) are obtained from Newton’s Second Law applied to the end-effector

\[
\begin{bmatrix}
    m_E \cdot \ddot{x}_E \\
    m_3 \cdot (g + \ddot{z}_E)
\end{bmatrix} = \begin{bmatrix}
    \cos \gamma_1 - \cos \gamma_2 \\
    \sin \gamma_1 & \sin \gamma_2
\end{bmatrix} \begin{bmatrix}
    F_{1E} \\
    F_{2E}
\end{bmatrix}
\]

(2)

The equations of motion in minimal form for the crank angles can be obtained in two steps. First, the last equation has to be solved for the unknown forces \( F_{1E} \), which then can be eliminated in (1a) and (1b). Second, the substitution of the variables \( \gamma_i = \gamma_i(q_i) \), \( \beta_i = \beta_i(q_i) \), and \( \mathbf{f} = \mathbf{f}(q_i, q_i, \dot{q}_i) \) resulting from direct kinematics leads to the envisaged minimal form of the equations of motion

\[
\dot{q} = \dot{q}(q_i, \dot{q}_i, \tau_1, \tau_2).
\]

(3)

### 2.2 Modelling of the Pneumatic Actuators

The parallel robot is equipped with four pneumatic muscle actuators. At this, the indices of all variables describing a particular pneumatic muscle are chosen as follows: the first index \( i = 1, 2 \) denotes the drive under consideration, described by the generalised coordinate \( q_i(t) \), whereas the second index \( j = l, r \) stands for the mounting position, i.e. for the left or the right pneumatic muscle. A mass flow \( \dot{m}_{Mij} \) into the pneumatic muscle leads to an increase in internal pressure \( p_{Mij} \). In the case of a free movable muscle without external forces, the increased internal pressure results in an enlarged tubing diameter associated with a contraction \( \Delta l_{Mij,free} \) of the muscle in longitudinal direction due to specially arranged fibres. This contraction effect can be exploited to generate forces. The resulting force \( F_{Mij} \) of a pneumatic muscle depends non-linearly on the according internal pressure \( p_{Mij} \), as well as the contraction length \( \Delta l_{Mij} \). The contraction lengths of the pneumatic muscles are related to the generalised coordinates, i.e. the crank angles \( q_0 \). The position of the crank angle, where the corresponding right pneumatic muscle is fully contracted, is denoted by \( q_0 \). Consequently, by considering the transmission consisting of toothed belt and pulley, the following constraints hold for the contraction lengths of the muscles

\[
\Delta l_{Mij}(q_i) = r \cdot (q_i - q_0).
\]

(4a)

\[
\Delta l_{Mij}(q_i) = \Delta l_{M,\text{max}} - r \cdot (q_i - q_0). \quad (4b)
\]

Here, \( \Delta l_{M,\text{max}} \) is the maximum contraction given by 25% of the uncontracted length. The force characteristic \( F_{Mij}(p_{Mij}, \Delta l_{Mij}) \) of the pneumatic muscle yields the resulting static tension force for given internal pressure \( p_{Mij} \) as well as given contraction length \( \Delta l_{Mij} \). This non-linear force characteristic has been identified by static measurements and, then, approximated by the following polynomial description

\[
F_{Mij} = \bar{F}_{Mij} (\Delta l_{Mij}) \cdot p_{Mij} - f_{Mij} (\Delta l_{Mij})
\]

\[=
\sum_{n=0}^{3} (a_m \cdot \Delta l_{Mij}^n) \cdot p_{Mij} - \sum_{n=0}^{4} (b_n \cdot \Delta l_{Mij}^n). \quad (5)
\]

The dynamics of the internal muscle pressure follows directly from a mass flow balance in combination with the pressure-density relationship. As the maximum internal muscle pressure is limited by a maximum value of \( p_{M,\text{max}} = 7 \text{ bar} \), the ideal gas equation \( p_{Mij} = p_{Mij} \cdot R \cdot T_{Mij} \) can be utilised as accurate description of the thermodynamic behaviour of the compressed air. Here, the density \( p_{Mij} \), the gas constant \( R \), and the thermodynamic temperature \( T_{Mij} \) are introduced. For the thermodynamic process a polytropic change of state is employed

\[
\frac{p_{Mij}}{\rho_{Mij}^n} = \text{const}. \quad \text{or} \quad p_{Mij} = \frac{p_0}{\rho_0^n} \cdot \rho_{Mij}^n, \quad (6)
\]

where \( n \) denotes the identified polytropic exponent. Thus, the relationship between the time derivative of the pressure and the time derivative of the density results in

\[
\dot{p}_{Mij} = n \cdot R \cdot T_{Mij} \cdot \dot{\rho}_{Mij}. \quad (7)
\]

The mass flow balance for the pneumatic muscle yields

\[
\dot{\rho}_{Mij} \cdot V_{Mij} = \dot{m}_{Mij} - p_{Mij} \cdot \dot{V}_{Mij}. \quad (8)
\]

The volume characteristic of the pneumatic muscle can be approximated with high accuracy by
the following non-linear function of both contraction length and muscle pressure, where the coefficients in this polynomial approximation have been identified by measurements

\[ V_{Mij}(\Delta \ell_{Mij},p_{Mij}) = \sum_{m=0}^{3} a_m \cdot \Delta \ell_{Mij}^m \sum_{n=0}^{1} b_n \cdot p_{Mij}^n. \]

(9)

Finally, by inserting (7) and (9) into (8), the pressure dynamics for the muscle i becomes

\[ \dot{p}_{Mij} = \frac{V_{Mij} + n \cdot \frac{\partial V_{Mij}}{\partial p_{Mij}} \cdot p_{Mij}}{V_{Mij} + n \cdot \frac{\partial V_{Mij}}{\partial p_{Mij}} \cdot p_{Mij} - \frac{\partial V_{Mij}}{\partial \Delta \ell_{Mij}} \cdot \frac{\partial \Delta \ell_{Mij}}{\partial q_i} \cdot p_{Mij} \cdot \dot{q}_i} \cdot (10). \]

3. FLATNESS-BASED FEEDBACK CONTROL DESIGN

3.1 Differential Flatness

Differential flatness is a prerequisite for flatness-based control of non-linear systems, which are usually given in state space representation, i.e. \( \dot{x} = f(x, u) \). A system is denoted as differentially flat (Fliess et al., 1995) if appropriate flat outputs \( y = y(x, u, \dot{u}, \ldots, u^{(l)}) \) exist that

(i) allow for expressing all system states \( x \) and all system inputs \( u \) as a function of these flat outputs \( y \) as well as their time derivatives, i.e. \( x = x(y, \dot{y}, \ldots, y^{(\beta)}) \) and \( u = u(y, \dot{y}, \ldots, y^{(\beta+1)}) \),

(ii) are differentially independent, i.e. they are not connected by differential equations.

If the first condition is fulfilled, the second condition is equivalent to \( \text{dim}(u) = \text{dim}(y) \).

3.2 Flatness-Based Pressure Control

The non-linear state equation (10) for the internal muscle pressure \( p_{Mij} \) represents the basis for the decentralized pressure control. It can be reformulated as

\[ \dot{p}_{Mij} = k_{u_{ij}} (\Delta \ell_{Mij}, p_{Mij}) \cdot \dot{m}_{Mij} - k_{p_{ij}} (\Delta \ell_{Mij}, \Delta \ell_{Mij}, p_{Mij}) \cdot p_{Mij}. \]

(11)

With the internal muscle pressure as flat output candidate \( y_{ijp} = p_{Mij} \), (11) can be solved for the mass flow as control input \( u_{ijp} = \dot{m}_{Mij} \) and leads to the inverse model for the pressure control

\[ \dot{m}_{Mij} = \frac{1}{k_{u_{ij}} (\Delta \ell_{Mij}, p_{Mij})} \cdot (\dot{p}_{Mij}) - k_{p_{ij}} (\Delta \ell_{Mij}, \Delta \ell_{Mij}, p_{Mij}) \cdot p_{Mij}. \]

(12)

Since the internal pressure \( p_{Mij} \) as state variable is identical to the flat output and \( \text{dim}(y_{ijp}) = \text{dim}(u_{ijp}) = 1 \) holds, the differential flatness property is proven. The contraction length \( \Delta \ell_{Mij} \) as well as its time derivative \( \Delta \ell_{Mij} \) can be considered as scheduling parameters in a gain-scheduled adaptation of \( k_{u_{ij}} \) and \( k_{p_{ij}} \).

With the internal pressure as flat output, its first time derivative \( \dot{p}_{Mij} = v_{ijp} \) is introduced as new control input. Consequently, the state variable of the corresponding Brunovsky form has to be provided by means of measurements, i.e. \( z_{ijp} = p_{Mij} \). Each pneumatic muscle is equipped with a pressure transducer mounted at the connection flange that connects the muscle with the toothed belt. The scheduling parameter \( \Delta \ell_{Mij} \) results from the measured crank angle \( q_i \), which is obtained by an encoder providing high resolution. Furthermore, the second scheduling parameter \( \Delta \ell_{Mij} \) is derived from the crank angle \( q_i \) by means of real differentiation using a DT1-System with the transfer function \( G_{DT1}(s) = s/(T_1 \cdot s + 1) \). The error dynamics of each muscle pressure \( p_{Mij} \) can be asymptotically stabilised by the following control law

\[ v_{ijp} = \dot{p}_{Mijd} + \alpha_{ijp} \cdot (p_{Mijd} - p_{Mij}), \]

(13)

where the constant \( \alpha_{ijp} \) is determined by pole placement. Here, the desired value \( \dot{p}_{Mijd} \) can be obtained either by real differentiation of the corresponding control input \( u_{ij} \) in (20) or by model-based calculation using only desired values, i.e. \( \dot{p}_{Mijd} = \dot{p}_{Mijd}(T_d, \dot{T}_d, \dot{T}_d, \dot{p}_{Mijd}) \). The corresponding desired trajectories are obtained from a trajectory planning module that provides synchronous time optimal trajectories according to given kinematic constraints. Defining \( e_{ijp} = p_{Mijd} - p_{Mij} \) as control error w.r.t. the internal muscle pressure, the corresponding error dynamics is governed by the first order differential equation

\[ \dot{e}_{ijp} + \alpha_{ijp} \cdot e_{ijp} = 0. \]

(14)

In each input channel, the non-linear valve characteristic (VC) is compensated by pre-multiplying with its approximated inverse valve characteristic (IVC). This inverse valve characteristic is implemented as look-up-table and depends both on the commanded mass flow and on the measured internal pressure.

3.3 Flatness-Based Decoupling Control

For the outer control loop, the following flat output candidates are chosen: the end-effector position in the \( xz \)-plane, i.e. \( x_E \) and \( z_E \), and the mean pressure for each pair of muscle, i.e. the mean value \( p_{Mij} = (p_{Mij} + p_{Mij'})/2 \) of the left and the right muscle pressure. The trajectory control of the mean pressure allows for increasing
stiffness concerning disturbance forces acting on the carriage (Bindel et al., 1999). As the decentralised pressure controls have been assigned a high bandwidth, these four subsidiary controlled muscle pressures $p_{Mij}$ can be considered as ideal control inputs of the outer control loop. Subsequent differentiation of the first two flat output candidates until one of the control inputs appears leads to

$$
y_1 = x_E, \quad \dot{y}_1 = \ddot{x}_E,
$$

$$
\ddot{y}_1 = x_E (x_E, z_E, \dot{x}_E, \dot{z}_E, p_{M1j}, p_{M2j}),
$$

(15a)

$$
y_2 = z_E, \quad \dot{y}_2 = \ddot{z}_E,
$$

$$
\ddot{y}_2 = z_E (x_E, z_E, \dot{x}_E, \dot{z}_E, p_{M1j}, p_{M2j}),
$$

(15b)

whereas the third and fourth flat output candidates directly depend on the control inputs

$$
y_3 = p_{M1} = 0.5 \cdot (p_{M11} + p_{M1r}),
$$

(16a)

$$
y_4 = p_{M2} = 0.5 \cdot (p_{M21} + p_{M2r}),
$$

(16b)

The differential flatness can be proven as follows: all system states can be directly expressed by the flat outputs and their time derivatives

$$
x_E = y_1, \quad \dot{x}_E = \dot{y}_1, \quad z_E = y_2, \quad \dot{z}_E = \dot{y}_2.
$$

(17)

Analogously, the internal muscle pressures as inputs are given by the following function of the flat outputs and a finite number of their time derivatives

$$
u = \begin{bmatrix}
p_{M11}\left(y_1, \dot{y}_1, \ddot{y}_1, \dddot{y}_1, y_2, y_3, y_4\right) \\
p_{M1r}\left(y_1, \dot{y}_1, \ddot{y}_1, \dddot{y}_1, y_2, y_3, y_4\right) \\
p_{M21}\left(y_1, \dot{y}_1, \ddot{y}_1, \dddot{y}_1, y_2, y_3, y_4\right) \\
p_{M2r}\left(y_1, \dot{y}_1, \ddot{y}_1, \dddot{y}_1, y_2, y_3, y_4\right)
\end{bmatrix}.
$$

(18)

The non-linear state transformation that yields the measured state variables of the corresponding Brunovský form, i.e. the end-effector position and the end-effector velocity in $x$- and $z$-direction, is given by the direct kinematics of the parallel robot

$$
z_x = \begin{bmatrix}
x_E(q) \\
\dot{x}_E(q, \dot{q})
\end{bmatrix}, \quad z_z = \begin{bmatrix}
z_E(q) \\
\dot{z}_E(q, \dot{q})
\end{bmatrix},
$$

(19)

Here, in contrast to the approach presented in (Aschemann and Hofer, 2004), the end-effector acceleration has not to be determined neither by evaluation of the equation of motion (3) nor by a double real differentiation of the measured position signal. By inserting the new defined inputs $v_1 = \ddot{x}_E, v_2 = \ddot{z}_E, v_3 = p_{M1}$, and $v_4 = p_{M2}$ as well as the transformed states into (18), the inverse dynamics becomes

$$
u = u(z_x, z_z, v_1, v_2, v_3, v_4).
$$

(20)

The error dynamics of the end-effector positions $x_E$ and $z_E$ can be asymptotically stabilized with the control laws

$$
y_1 = x_E, \quad \dot{y}_1 = \ddot{x}_E,
$$

$$
\ddot{y}_1 = x_E (x_E, z_E, \dot{x}_E, \dot{z}_E, p_{M1j}, p_{M2j}),
$$

(21a)

$$
y_2 = z_E, \quad \dot{y}_2 = \ddot{z}_E,
$$

$$
\ddot{y}_2 = z_E (x_E, z_E, \dot{x}_E, \dot{z}_E, p_{M1j}, p_{M2j}),
$$

(21b)

Here, the desired trajectories for the end-effector position in $x$-direction $x_{Ed}$ and in $z$-direction $z_{Ed}$ as well as their first two time derivatives have to be provided, whereas the desired trajectories for the mean pressure of drive 1, i.e. $p_{M1d}$, and the mean pressure of drive 2, i.e. $p_{M2d}$, are directly employed in a feedforward manner

$$
v_3 = p_{M1d}, \quad v_4 = p_{M2d}.
$$

Here, the desired trajectories for the end-effector position in $x$-direction $x_{Ed}$ and in $z$-direction $z_{Ed}$ as well as their first two time derivatives have to be provided, whereas the desired trajectories for the mean pressure of drive 1, i.e. $p_{M1d}$, and the mean pressure of drive 2, i.e. $p_{M2d}$, are directly employed in a feedforward manner

$$
v_3 = p_{M1d}, \quad v_4 = p_{M2d}.
$$

Due to the integral control parts, which are meant to counteract the disturbance torque $\eta_i$ acting on drive $i$, respectively, and guarantee a vanishing steady-state control error w.r.t. the end-effector position, the dynamics of the corresponding position errors in $x$-direction $e_x = x_{Ed} - x_E$ as well as in $z$-direction $e_z = z_{Ed} - z_E$ are of third order. The coefficients $\alpha_{2x}, \alpha_{1x}$, and $\alpha_{0x}$ as well as $\alpha_{2z}, \alpha_{1z}$, and $\alpha_{0z}$ are specified by pole placement according to a desired pole configuration. The implementation of the flatness-based control structure is depicted in fig. 4.

4. SIMULATION RESULTS

Tracking performance as well as steady-state accuracy w.r.t. end-effector position and mean pressure have been investigated by simulation studies of a parallel robot with the following dimensions: $a = 0.5 \text{ m}$, $l_A = 0.4 \text{ m}$, $s_A = 0.2 \text{ m}$, $l_p = 0.8 \text{ m}$. For this purpose, the tracking of a triangle-like desired trajectory for the controlled variables as shown in fig. 5 has been considered. At this, the desired value for the mean pressure is held constant at 4.0 bar.

The first part consists of a motion from the starting point $(x = 0 \text{ m}, z = 1 \text{ m})$ to the point $(x = -0.2 \text{ m}, z = 0.6 \text{ m})$. The second part comprises a movement in $x$-direction by
robot with two degrees of freedom driven by pneumatic muscles. The modelling of this mechatronic system leads to a system of non-linear differential equations of eighth order. For the characteristics of the pneumatic muscles polynomials serve as good approximations. The non-linearity of the valve is linearised by means of a pre-multiplication with its approximated inverse characteristic. The inner control loops of the cascade involve a flatness-based control of the internal muscle pressure with high bandwidth. The outer control loop achieves a decoupling of the end-effector position and the mean pressures as controlled variables. Simulation results emphasise the excellent closed-loop performance with maximum position errors of approx. 3 mm during the movements, vanishing steady-state position error and steady-state pressure error of less than 0.01 bar. Future research will address experiments at a prototype system.

REFERENCES

5. CONCLUSIONS

In this paper, a cascaded trajectory control based on differential flatness is presented for a parallel