Abstract: The application of the Sliding Mode Thermal Control System to laser spectroscopic system is proposed. An adaptive controller is designed based on results obtained in the sphere of discontinuous system design. The derived Sliding Mode (SM) control algorithm should improve the control performance of the important characteristic of systems such as the thermal field of the oven. The suggested controller program written in LabView environment consists of the following blocks: the sliding mode indicator determining the occurrence of SM; the observer block, identifying the state vector of the system; the anticipatory compensation block. The obtained control algorithm is verified on the real system. Copyright© 2005 IFAC

Keywords: Temperature control, Sliding Mode controller, indicator of Sliding Mode, observer; anticipatory compensation, Discontinuous Control and Setting Adjustment.

1. INTRODUCTION

Theoretical and experimental investigations on resonant interaction of laser radiation with atomic vapors are important both for fundamental physics and numerous applications.

Numerous basic studies have been performed on systems involving alkali vapors and their mixtures with buffer gases. There are several reasons why these systems are very suitable subjects for experimental as well as theoretical investigations. Alkali resonance lines are in a spectral range attainable by available lasers and alkali vapors can be easily generated in cells. On the other hand, due to their simple hydrogen-like structure, alkali atoms are convenient subjects for theoretical calculations and for modeling. Many applications such as magnetometry (Scully and Fleischhauer, 1992), electro-magnetically induced transparency, quantum computing, are based on the interaction of these vapors with laser radiation.

In present experiment the thermal regimes of selective excitation of light from boundary of cell’s dielectric window and atomic vapor (Chevrollier, et al., 1992) are investigated. As is known, selective reflection of light from the boundary of cell’s dielectric window and atomic vapor is a convenient spectroscopic tool for this kind of measurement. Relevant spectroscopic investigations imply recording and processing of a large number of selective reflection spectra, along with frequency reference spectra, at various thermal conditions of a vapor cell. The heating unit that provides proper temperature regime of a vapor cell has two independent working areas. Temperature of one of the areas (side arm of a cell) determines alkali metal vapor density, and temperature of the second area determines regime of atom-atom and atom-dielectric surface interaction in the cell (Taylor and Langmuir,
In this connection it is important to precisely set and keep proper values of temperature.

The swift development of electronics, computer and software engineering in recent years had led to broad applications of the adaptive control system in various fields. Thus many complex algorithms can be now applied for real-time control, with PCs with AD/DA cards and modern software such as LabView programming. Within the framework of the present work the Sliding Mode Thermal controller based on LabView software under Windows environment is proposed (Mkrttchian and Khachaturova, 2003; Khachaturova and Mkrttchian, 2003).

2. PROCESS DESCRIPTION AND MODELLING

This section includes the experimental setup description and derives the model of the thermal control system.

2.1 Process description.

The block-scheme of an experimental setup is shown in Fig.1 and operates as follows. The parallel radiation beam of the single –frequency laser diode was directed on a beam splitter. One part of divided radiation beam was directed onto the alkali vapor cell at working temperature 50-300°C.

![Block-scheme of the experimental setup](image)

Fig.1. Block-scheme of the experimental setup.

The signal of selective reflection from the “dielectric window-vapor” interface was detected by the photodiode with an operational amplifier. The second radiation beam was directed into a room temperature reference cell with alkali metal vapor, where the saturated absorption scheme has been implemented, allowing to determine frequency positions of atomic hyperfine lines to be determined. Detection of a saturated absorption spectrum was carried out by the second similar photodiode with an operational amplifier. The output signals from the amplifiers of photodiodes were imported to an analog input of the DAQ board, PC-1200AI. And the output (temperature) signals from oven and cell were also imported to analog inputs of the DAQ board (Mkrttchian and Khachaturova, 2004).

2.2 Model of the thermal control system (TCS).

The TCS objective is to control the thermal regime of the oven. The mathematical model of TCS is very useful in determining the sliding surfaces and control functions and is represented by the nonlinear SISO system with partially known parameters. Model equations simulate the dynamics within TCS, which include the response of the oven with a cell inside.

The transfer function of the system is:

\[
\frac{Y(p)}{X(p)} = \frac{k_T \exp(-p \tau_T)}{(T_T p + 1)(T_{TO} p + 1)},
\]

where \( k_T = k_{T1} k_{T2} \) - is the gain value, that is a product of a heat source gain \((k_{T1})\) and plant’s gain \((k_{T2})\); \( \tau_T \) is delay; \( T_T \) – is time constant of a heat source; \( T_{TO} \) - time constant of a cell.

From (1) yields:

\[
\frac{Y(p)}{X(p)} = \frac{k_T \exp(-p \tau_T)}{[T_T T_{TO} p^2 + T_T p + T_{TO} p + 1]}
\]

Applying Laplace transform, yields

\[
\frac{d^2 Y}{dt^2} + T_T T_{TO} \frac{dY}{dt} + (T_T + T_{TO}) Y = k_T (t - \tau) x
\]

Dividing both parts of the equality (3) on \( T_T T_{TO} \), yields

\[
\frac{d^2 Y}{dt^2} + \left[ \frac{T_T + T_{TO}}{T_T T_{TO}} \right] \frac{dY}{dt} + \frac{1}{T_T T_{TO}} Y = \frac{k_T}{T_T T_{TO}} x(t - \tau)
\]

Denote

\[
T_1 = \frac{T_T + T_{TO}}{T_T T_{TO}}; \quad T_2 = \frac{1}{T_T T_{TO}}; \quad T_3 = \frac{k_T}{T_T T_{TO}}.
\]

Then,

\[
\frac{d^2 Y}{dt^2} + T_1 \frac{dY}{dt} + T_2 Y = T_3 x(t - \tau)
\]

The description of the system in normal form is shown as

\[
Y = Y_1;
\]

\[
Y_1 = Y_2;
\]

\[
Y_2 = -Y_3 T_1 - Y_2 T_2 + T_3 x(t - \tau)
\]

A set of differential equations can be presented in matrix form:

\[
\begin{bmatrix}
\dot{Y}_1 \\
\dot{Y}_2 
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & -\frac{T_3}{T_2}
\end{bmatrix}
\begin{bmatrix}
Y_1 \\
Y_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
\frac{T_1}{T_2}
\end{bmatrix} x(t - \tau)
\]
3.1 The Sliding Mode Indicator.

As already noted, the approach used is oriented toward a deliberate introduction of sliding modes over the intersection of surfaces on which the control vector components undergo discontinuity. Realization of such an approach implies the knowledge of the conditions of the occurrence of sliding mode. Designed for this purpose was the indicator of sliding modes. From the point of view of mathematics, the problem may be reduced to that of finding the area of attraction to the manifold of the discontinuity surfaces intersection (Dubrovsky and Kortnev, 1968).

On the indicator input two signals \( x \) and \( g \) from the equivalent circuit are supplied. The indicator compares those two signals recording the moment of changing the signs of function \( x \) and \( g \), see (8).

\[
g(t) = C_1x + C_2 \frac{dx}{dt}, \tag{8}
\]

where \( C_1 \) and \( C_2 \) are constants of object control; \( g \) is function of switching; \( x \) is parameter of system; \( t \) is time.

According to (Dubrovsky and Kortnev, 1968; Merntchian et al., 1999) the existing indicator diagram is shown in Fig.2.

![Fig. 2. The diagram of the Sliding Mode Indicator.](image)

In forming devices 1 and 2 (FD1 and FD2) it indicates the moment of changing of the signs of functions \( x(t) \) and \( g(t) \). Then in logical cells “AND”, INVERTOR- “Inv”, and “OR” the operation of sign coincidence is taking place. When the signs of functions \( x(t) \) and \( g(t) \) are identical, the indicator logic is “1” (Sliding mode is present), when the signs of function \( x(t) \) and \( g(t) \) are not identical, the indicator is “0” (Sliding mode is not present). The device includes the input restructures (IR1 and IR2), amplifiers (A1 and A2), and potential triggers (TR1 and TR2).

3.2 Construction of SM existence region

As it known, the motion of the system operating in the sliding surface is invariant to disturbances, parameter uncertainties, coupling between channels, and nonlinearities seen in the system. The constant coefficients \( C_i \) of the sliding surface boundary described by equation (8) are chosen to provide the output tracking motion in sliding mode. This is accomplished through construction of SM existence region and choosing the coefficients \( C_i \) from the region.
In this section the graphic method of construction of SM existence region is given, according to (Dubrovsky and Kortnev, 1968). The example of construction of the boundary of sliding mode existence region for the control system is shown in Fig. 3.

Construction of boundaries is carried out as follows. Three systems of coordinates are combined on a plane: \( xOx \), \( xOF(M) \), \( pOF(p) \). The points of origin of the two first systems coincide, and the third system is shifted for \(-1\) on axes \( Ox \). The scales on \( OxF \), \( Ox \) and on \( OF(M) \), \( OF(p) \) are identical, and the directions of axes \( Ox \), \( OF(M) \), \( OF(p) \) coincide. A plot of characteristic polynomial of the system (9) is constructed in \( xOF(p) \) system of coordinates \( p \) from 0 to \(-\infty\). A plot of function (10) is constructed in \( xOF(M) \) system of coordinates.

\[
F(p) = p^2 + T_1p + T_2
\]

(9)

\[
F(M) = -T_3Mx^{-1}
\]

(10)

where \( T_3 \) is input constant; \( M \) is amplitude of control action. Instead of \( M \) the value of \( M_1 \) or \( M_2 \) is substituted depending on what boundary (upper or lower) \( B_1 \) or \( B_2 \) is constructed.

Fig. 3. Construction of the boundary of sliding mode existence region.

Then a point \( p^* \in (0, -\infty) \) is taken on axis \( O_p \). The distance of \( O_p p^* \) is marked on an axis \( OF(p) \) and denoted as a point \( N \). Through the point \( N \) the straight line \( NO \) is drawn, which coincides with a straight line \( xOx \) (the boundary equation). Then from a point \( p \) the vertical straight line is conducted up to crossing with polynomial graph (9) in a point \( K \). From point \( K \) the horizontal straight line is conducted up to crossing with hyperbolic curve (10) in a point \( L \). The vertical straight line which has been drawn from a point \( L \), crosses the straight line \( ON \) in point \( R \), belonging to boundary of area of existence of the sliding mode. Taking various points of \( p^* \) and making for them similar constructions, a number of points \( R \) is obtained. The curve obtained by connection of points \( R \) is the boundary of SM existence region, constructed in system of coordinates \( xOx \).

The shaded regions on Fig. (3) represent the sliding mode existence area, and \( B_1, B_2 \) are the boundaries of these regions.

3.3 The SM observer and anticipatory device.

The problem of identifying the state vector is solved based on data provide by the observer. The state vector of the equivalent circuit is not given. The given data include only certain dimensions of the vector. The problem of identifying the state vector is solved based on data providing by the observer (11).

\[
\frac{dy}{dt} = Ay + Bx - L(z - Ky)
\]

(11)

In (11) \( A, B, L, K \) are the known matrixes; \( y \) is exit parameter of system; \( z \) is the vector composed in the form of linear combination of the state vector components of the system and in fact, is its estimation. In (11) the state vector of the system can be directly measured, and the last member of (11) characterizes the gap between the intended and actual parameters of the state vector of the equivalent circuit. The value of the gap will be the solution of the homogeneous differential equation with any desired distribution of roots of the characteristic equation. We can receive components of matrix \( L \) for each case by solving (11) and transforming the differential integrating links. For construction of the block-diagram of the observer the differential equation of object will be transformed to the convenient form. Let us rewrite the equation (5) in the following form

\[
\dot{y} = -a_{2T_1}y - a_{1T_1}y + k_{Gr1}x,
\]

(12)

where

\[
a_{1T_1} = \frac{1}{T_y T_{T0}}; \quad a_{2T_1} = \frac{T_y + T_{T0}}{T_y T_{T0}}; \quad k_{Gr1} = \frac{k_{1T1}k_{2T1}}{T_y T_{T0}}
\]

The block-diagram of the observer is built according to the expressions obtained for the observer diagram.

The technique of anticipatory compensation is based upon the preliminary supply of input signal with regard to the duration of delay. Considering the delay, the problem of controlling a multi-level equivalent circuit can be presented as follows, see (13):

\[
\frac{dy}{dt} = Ay + Bx(t - \tau).
\]

(13)

where \( \tau \) is delay.
The solution on the equation (13) is:

\[ y(t) = \exp(At)y_0(t) + \int_0^t \exp(A(t-\gamma))Bx(\gamma-t)d\gamma \]

(14)

The integral in equation (14) is replaced by the sum and \( y \) is replaced by \( z+\tau \) yielding the expression (15):

\[ y(t+\tau) = \exp(At)y(t) + B\sum_{i=0}^{\infty} \exp(-Az_i)x(z_i + t - \tau) \frac{\tau}{N} \Delta z \]

(15)

The equation (15) yields a diagram for delay compensation.

The designed SM controller algorithm for temperature control of the oven with a cell inside is shown in Fig.4.

The results of applying the designed controller to the thermal control system are shown in Fig.5 and present the graph of the optimally controlled state (temperature).

4. EXPERIMENTAL RESULTS

The TCS includes: oven with heater and thermocouple; an amplifier that has sufficient gain (k=100) so as to provide the range of voltage needed; the PC hardware; voltage-controlled power supply for the oven heater. Software support is provided by the designed controller program written in the LabView programming language environment. The DAQ board measures output from the operational amplifier. The voltage from the PC hardware is the control signal that will turn ON or OFF the heating element to keep temperature set programmatically and maintain the Sliding Mode. The model arrangement including the oven used in real experiment was allowed to achieve stability of ~0.5 °C.

Fig.4. Sliding Mode Controller algorithm for temperature control of oven with a cell inside.

Fig.5. The result of applying the designed controller to the thermal control system.
5. CONCLUSIONS

- This paper formulates the TCS problem in laser using the methods of sliding mode control. The Sliding Mode Controller is designed based on results obtained in the sphere of discontinuous system design. The controller consists of the following blocks: the sliding mode indicator determining the occurrence of SM; the observer block, identifying the state vector of the system, and the anticipatory compensation block.

- The experimental results of applying the designed controller have illustrated the improvement of the achieved performance applied to the spectroscopic system equipped with a thermal system composed of a heating source.

REFERENCES


