ADAPTIVE COORDINATED DECENTRALIZED CONTROL OF LARGE-SCALE SYSTEMS WITH MULTIVARIABLE SUBSYSTEMS

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Abstract: Exact decentralized output-feedback Lyapunov-based designs of direct MRAC for linear interconnected systems with MIMO subsystems are introduced. The design process uses a coordinated decentralized structure of adaptive control with reference model coordination. We develop decentralized MRAC on the base of a priori information about only the local subsystems gain frequency matrices without additional a priori knowledge about the full system gain frequency matrix. To achieve a better adaptation performance we propose proportional, integral time-delayed adaptation laws.

Keywords: Decentralized adaptive control, Coordination, Robust control.

1. INTRODUCTION

An increasing number of control problems for composite interconnected systems requires the use of an adaptive decentralized control structure with physically distributed controllers. These problems are found in various application areas, such as large-scale computer networks, power systems, automotive systems, web handling systems etc. In the case of plants with single-input single-output (SISO) interconnected subsystems, these problems are extensively studied and many important results have been obtained. The design has been based mainly on the following approaches: the traditional certainty equivalence approach or Morse’s “dynamic certainty equivalence approach” (see, e.g., (Ioannou, 1986), (Gavel and Šiljak, 1989), (Ortega, 1996), (Mirkin, 1999), (Narendra and Oleng’, 2002), (Mirkin, 2003)), and the nonlinear design tool based on recursive backstepping (see, e.g., (Jiang, 2000), (Krishmanurthy and Khorrami, 2003)).

However, for the conventional DMRAC scheme, these results do not appear to be easily applied to the case of composite systems with multi-input multi-output (MIMO) subsystems. One reason for this is that the type of a priori information pertaining to the overall MIMO plant structure is not as apparent in the case of MIMO subsystems as it is in the SISO case.

One of main difficulties of this case, even without the requirement of a decentralized control structure, is the generalization of the high frequency gain sign condition, since we deal with matrix gains instead of scalar gains. Current MIMO adaptive control algorithms for systems without a decentralized control structure require some a priori knowledge or constraints on the high-frequency gain matrix $K_p$ of the overall plant (controlled object) $W(s)$. Most available results assume that either the high frequency matrix of the overall plant is known fully or partially, see e.g. (Sastry and Bodson, 1989), or satisfies some positive definiteness condition, see e.g. (Ioannou and Sun, 1996). To reduce the amount of a priori requirements to execute the design, some recent solutions based on Morse’s fac-

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torizations of $K_p$ were suggested. See, e.g. the recent monograph (Tao, 2003) for a summary of the state of the art.

We postulate that the controller is decentralized which creates information constraints. These pose control design difficulties in addition to those caused by the centralized case. The adaptive control problem becomes more complex in the decentralized case, and is yet an unsolved problem. To the authors’ knowledge, results for composite systems with MIMO subsystems have not been reported in the literature.

We propose to design the decentralized controller without a priori knowledge about the full matrix $K_p$. Instead we assume some a priori knowledge about the subsystem matrices $K_{pi}$. The reason is that the transfer function of the overall controller has a block-diagonal form, and is not a full matrix as in the centralized case. Our concept of reference model coordination (Mirkin, 2003; Mirkin and Gutman, 2003) and recent advances in output feedback design for MIMO centralized systems, e.g. (Costa et al., 2003; Tao, 2003) provide tools to overcome the difficulties caused by the lack of a priori centralized information.

We develop an adaptive decentralized control parametrization for the class of composite linear systems with MIMO subsystems which admits output decentralized model reference adaptive designs with zero asymptotical errors. Thus this paper generalizes the results for composite systems with MIMO subsystems which admits output decentralized parameterizations of $K_{pi}$, Motivated by the similarities with the SISO case (Mirkin and Gutman, 2003), we will use the decentralized adaptive control scheme with reference model coordination to achieve the control objective - asymptotic exactly tracking. The control law for the $i$th local MIMO subsystem $u_i$ is chosen to be of the form

$$u_i(t) = u_{i1}(t) + u_{ci}(t)$$

where the part of the control law $u_{i1}(t)$ is based only on the local signals of the $i$th subsystem, and the component $u_{ci}(t)$ is the coordinated component which is based on the reference signals of the all other subsystems. Exchange of the reference signals between subsystems can be easily implemented in real-life control systems.

**Remark 2.** For the case of systems with MIMO interconnected subsystems, the main difficulties and the main difference from the case with SISO interconnected subsystems is the choice of a suitable parameterization of the local and coordinated component of the control law (3), in order to anticipate the effect of the cross-coupling. The component based on local signals, $u_{i1}(t)$, will be modified in comparison with (Mirkin and Gutman, 2003) by adding a term based on local data. As the basic building block for the coordinated component $u_{ci}(t)$, we suggest a dynamical system (pre-filter) with adjustable parameters that describes how the reference signal $r_j(t)$ of the $j$-th reference model acts on the $i$-th control input. We consider here the case of a priori knowledge about $K_p$, based on the SDU decomposition of $K_{pi}$, e.g. (Costa et al., 2003).
\( \omega_{fi}(t) \) and \( \omega_{i}(t) \) are the local feedback and the local feedforward signals respectively and are given by the following equation (i, j = 1, ..., M,)

\[
\omega_{fi}(t) = \begin{bmatrix}
I_{m_i} & 0 & 0 & 0 & \cdots & 0 \\
0 & \Phi_i(s) & 0 & 0 & \cdots & 0 \\
0 & 0 & \Phi_i(s) & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & \cdots & 0 \\
0 & 0 & 0 & \cdots & \cdots & 0 \\
\end{bmatrix} \begin{bmatrix}
e_i(t) \\
x_{1i}(t) \\
x_{2i}(t) \\
\vdots \\
\vdots \\
\end{bmatrix}
\]

\[
\omega_i(t) = \begin{bmatrix}
I_{m_i} & W_{rs}(s) \\
\end{bmatrix} \begin{bmatrix}
[r_i(t)] \\
\end{bmatrix}, \quad \omega_{fi}(t) = [e_i^T, x_{1i}^T, x_{2i}^T]^T
\]

with

\[
\Phi_i(s) = \left[ I_{m_i}, s^{\nu_i-2}, \ldots, I_{m_i}, s, I_{m_i} \right]^T
\]

\[
\Phi_t(s) \in \mathbb{R}^{m_i \times (m_i-1) \times m_i}, I_{m_i} \in \mathbb{R}^{m_i \times m_i} \text{ is an identity matrix and } \Lambda_i(s) = s^{\nu_i-1} + \ldots + \lambda_{m_i} s + \lambda_0 \text{ is a monic Hurwitz polynomial.}
\]

The coefficient matrix \( \theta_{ui}(t) \) in \( \theta_{fi}(t) \) has the specific upper triangular structure with zero diagonal element like in (Costa et al., 2003) for the centralized case, i.e.

\[
\theta_{ui}(t) = \begin{bmatrix}
0 & \theta_{ui}^{12}(t) & \cdots & \theta_{ui}^{1m_i}(t) \\
0 & \theta_{ui}^{23}(t) & \cdots & \theta_{ui}^{2m_i}(t) \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
\end{bmatrix}
\]

This upper triangular matrix structure guarantees that the control component (4) is implementable without singularity, that is,

\[
u_{li}^{m_i}(t) = \theta_{fi}^{m_iT} \omega_{fi}(t) + \theta_{fi}^{m_iT} \omega_{i}(t) + \sum_{k=2}^{m_i} \theta_{ui}^{kL} u_{i}^{(k)}(t)
\]

\[
u_{li}^{2}(t) = \theta_{fi}^{2T} \omega_{fi}(t) + \theta_{fi}^{2T} \omega_{i}(t) + \sum_{k=3}^{m_i} \theta_{ui}^{2k} u_{i}^{(k)}(t)
\]

\cdots

\[
u_{li}^{m_i}(t) = \theta_{fi}^{m_iT} \omega_{fi}(t) + \theta_{fi}^{m_iT} \omega_{i}(t) + 0
\]

where \( \theta_{fi}(t) = [\theta_{ei}^T, \theta_{ui}^{1T}, \theta_{ui}^{2T}, \ldots, \theta_{ui}^{m_iT}] \) and \( \omega_{fi}(t) = [e_i^T, x_{1i}^T, x_{2i}^T]^T \)

If we denote

\[
\Theta_{li}^{1}(t) = [\theta_{fi}^T \theta_{ui}^{1T} \theta_{ui}^{2T} \cdots \theta_{ui}^{m_iT}]^T
\]

\[
\Theta_{li}^{2}(t) = [\theta_{fi}^T \theta_{ui}^{2T} \theta_{ui}^{3T} \cdots \theta_{ui}^{m_iT}]^T
\]

\cdots

\[
\Theta_{li}^{m_i}(t) = [\theta_{fi}^T \theta_{ui}^{m_iT}]^T
\]

we can rewrite the local control component (4) as

\[
u_{li}(t) = \begin{bmatrix}
\Theta_{li}^{1}(t) \Omega_{li}^{1}(t) \\
\vdots \\
\Theta_{li}^{m_i}(t) \Omega_{li}^{m_i}(t)
\end{bmatrix}
\]

**Remark 3.** When comparing the control component based on local signals, \( u_{li}(t) \), of subsystem \( i \) regarded as an isolated system, with the case of MIMO centralized adaptive control (Costa et al., 2003), one notices the following differences: In \( u_{li}(t) \) the additional term \( \theta_{ri}^* W_{ri}(s)[r_i(t)] \) is present, and the tracking error \( e_i(t) = y_i(t) - y_{ri}(t) \) is used in the local feedback signal vector \( \omega_{fi}(t) \) instead of \( y_i(t) \).

The coordinated control component, \( u_{ci}(t) \), which is based on the reference signals of the all other subsystems is chosen as follows

\[
u_{ci}(t) = \sum_{j=1}^{M} \theta_{ij}^T \omega_{ij}(t)
\]

\[
u_{ij}(t) = P_{ij}(s)[x_{rj}(t)] = \left[ I_{m_j} \Phi_i(s) \right][x_{rj}(t)]
\]

where \( \theta_{ij}(t) = [\theta_{ei}^T, \theta_{2ij}^T, \ldots, \theta_{m_i}^T] \in \mathbb{R}^{m_i \times m_i}, \theta_{1ij}(t) \in \mathbb{R}^{m_i \times m_i}, \theta_{2ij}(t) \in \mathbb{R}^{m_i \times (m_i-1) \times m_i} \) are some time-varying parameter matrices, and \( \omega_{ij}(t) \) is the output of the dynamic system with the transfer function \( P_{ij}(s) \)

\[
\Phi_i(s) = \left[ I_{m_j}, s^{\nu_i-2}, \ldots, I_{m_i}, s, I_{m_i} \right]^T
\]

4. ERROR EQUATION AND STABILITY ANALYSIS

To develop an adaptation law for the controller (3)-(12), we need to express the closed-loop system in terms of the tracking error \( e_i(t) = y_i(t) - y_{ri}(t) \).

With the specification of \( \Lambda_i(s), \Phi_i(s) \) and \( W_i(s) \) in the local control component (4) there exist some constant matrices \( \theta_{ei}^* = K_{ei}^*, \theta_{ri}^* = \theta_{ei}^* \) (Sastry and Bodson, 1989; Ioannou and Sun, 1996) such that

\[
I - \theta_{ei}^* W_i(s) = \theta_{ei}^* \theta_{ri}^* W_i(s) - \theta_{ei}^* W_i(s)
\]

\[
= \theta_{ei}^* W_i^{-1}(s) W_i(s)
\]

Then from (1) and (14), for any \( u_i \) we have the following equation for the tracking error \( e_i \), with

\[
e_i(t) = W_{ri}(s) K_{ei} \begin{bmatrix}
u_i - \theta_{ei}^* y_i - \theta_{ei}^* x_{1i} \\
x_{2i} - \theta_{ei}^* x_{2i} + \sum_{j=1}^{M} A_{ij} x_{rj}(t)
\end{bmatrix}
\]

\[
= -\sum_{j=1}^{M} \theta_{ei}^* x_{rj}(t) + \sum_{j=1}^{M} A_{ij} x_{rj}(t)
\]

Denoting \( A_{ij}^* = [\theta_{ei}^{1T} A_{ij}, \theta_{ei}^{2T} A_{ij}, \ldots, \theta_{ei}^{(m_i-1)T} A_{ij}] \), \( A_{ij}^* \in \mathbb{R}^{m_i \times m_i (m_i-1)} \), using (6) and (13) and doing some manipulations with the transfer functions we can write

\[
\theta_{ei}^* \Phi_i(s) A_{ij}^* = A_{ij} \Phi_i(s)
\]

In view of (16), (5) and (12)-(13) after substituting \( x_{2i} = x_{rj} \) in the right part of (15), the equation (15) can be rewritten as
\[
e_i = W_{ri}(s)K_{pi} u_i - \hat{\theta}_i^T \hat{\omega}_f(t) - \theta_i^T \omega_i(t)
+ \sum_{j=1,j\neq i}^M \left( \hat{A}_{ij}^* c_{ej}(t) - A_{ij}^* \Phi_{ij}(s)[e_{xj}(t)] \right)
\]

where \(\hat{\theta}_i^* = [\theta_{i1}^*, \theta_{i2}^*, \ldots, \theta_{in_i}^*]^T\), \(\theta_i^* = [\theta_{i1}^*, \ldots, \theta_{in_i}^*]^T\) and \(\theta_i^* = [-A_{ij}^T, A_{ij}^T]^T\).

By using the high-frequency gain matrix decomposition \(K_{pi} = S_i D_i U_i\) (Morse, 1993), where \(S_i\) is symmetric positive definite, \(D_i\) is diagonal, and \(U_i\) is unity upper triangular, and in view of \(U_i u_i = u_i - (I_{m_i} - U_i u_i)\) we derive an error equation from the equation (17)

\[
e_i = W_{mi}(s)S_i D_i \left[ u_i - \hat{\theta}_i^T \hat{\omega}_f(t) - \theta_i^T \omega_i(t) \right]
- \sum_{j=1,j\neq i}^M \left( \hat{\theta}_j^T \omega_i(t) + \sum_{j=1,j\neq i}^M \hat{\Phi}_{ij}(s)[e_{xj}(t)] \right)
\]

where \(\hat{\theta}_i^* = \theta_i^*\), \(\hat{\theta}_j^* = U_i \theta_j^*\), and matrix \(\hat{\theta}_i^* = I_{m_i} - U_i\) has the same specific upper triangular form as (7) with zero diagonal element but with unknown constant coefficients.

Then using the control law as given by (3), (4) and (11) the tracking error equation (18) can be written as

\[
e_i = W_{mi}(s)S_i D_i \left[ u_i - \hat{\theta}_i^T \hat{\omega}_f(t) - \theta_i^T \omega_i(t) \right]
+ \sum_{j=1,j\neq i}^M \left( \hat{\theta}_j^T \omega_i(t) + \hat{\Phi}_{ij}(s)[e_{xj}(t)] \right)
\]

where \(\hat{\theta}_i^T = \Theta_i^T(t) - \Theta_i^k(t)\) and \(\hat{\theta}_j^T = \Theta_j^T(t) - \Theta_j^k(t)\) are the parameter errors, \((k = 1, 2, \ldots, m_i; i, j = 1, \ldots, M)\).

Let define the augment state \(X_i = [x_i^T, x_i^1, x_i^2]^T\) by combining the \(i\)-th subsystems state \(x_i\) of the plant (1) with the filter states \(x_i^1\) and \(x_i^2\) from (5). With \(X_{mi} = [x_{mi}^T, x_{mi1}, x_{mi2}]^T\) we denote the state of the corresponding nonminimial realization \(C_i(sI - A_i)^{-1} B_i\) of \(W_{mi} S_i\) that is SPR (Costa et al., 2003).

Let \((A_{\phi ij}, B_{\phi ij}, C_{\phi ij}, D_{\phi ij})\) be a minimal stable space realization for the stable transfer matrix \(\Phi_{ij}(s)\) from (18). Then, the augment state error \(\hat{e}_i = X_i(t) - X_{mi}(t)\) and the output error \(e_i(t)\) in (19) satisfy

\[
\dot{\hat{e}}_i = \hat{A}_i \hat{e}_i(t) + \hat{B}_i D_i \left( \hat{\Theta}_i^T \Omega_i \ldots \hat{\Theta}_i^m_i \Omega_i \right)_i
+ \sum_{j=1,j\neq i}^M \hat{B}_j D_i \left( \hat{\Theta}_j^T \omega_i(t) + D_{\phi ij} L^T \hat{e}_j(t) \right)
+ C_{\phi ij} Z_{\phi ij}(t)
\]

\[
Z_{\phi ij}(t) = A_{\phi ij} Z_{\phi ij}(t) + B_{\phi ij} L^T \hat{e}_j(t)
\]

\[
z_{\phi ij}(t) = C_{\phi ij} Z_{\phi ij}(t) + D_{\phi ij} L^T \hat{e}_j(t)
\]

\[
e_i(t) = y_i(t) - y_{mi}(t) = \hat{C}_i \hat{e}_i(t)
\]

where \(L = [I \ 0 \ 0]^T\) and \(0_{n_i \times l_i(n_i-1)}\) is a zero matrix.

Because \(\dot{C}_i^T (sI - A_i)^{-1} \hat{B}_i = W_{mi}(s)S_i\) is SPR, the triple \((\hat{A}_i, \hat{B}_i, \hat{C}_i)\) satisfies the following equations given by the matrix version of the KY Lemma (Narendra and Annaswamy, 1989, page 67)

\[
\hat{A}_i^T P_i + P_i \hat{A}_i + Q_i = 0 \quad P_i \hat{B}_i = \hat{C}_i^T
\]

where \(A_{\phi ij}, P_{\phi ij} + P_{\phi ij} A_{\phi ij} + Q_{\phi ij} = 0\) is stable, it also hold that \((i, j = 1, \ldots, M)\)

\[
\Theta_i^T(t) = -\int_0^t \eta_i^k(s)ds - \eta_i^k(t) \quad \eta_i^k(t) = \gamma_k \text{sign}(d_i(t) \Omega_i^k(t)) (k = 1, \ldots, l_i)
\]

\[
\Theta_j^T(t) = -\int_0^t \eta_j(s)ds - \eta_j(t) \quad \eta_j(t) = \text{Sign}(D_i \Gamma_i^k(t) \omega_j(t))
\]

(23)

where \(\text{Sign}(D_i) = \text{diag} \{\text{sign}(d_i^1), \ldots, \text{sign}(d_i^l_i)\}\), \(\Gamma_i = \Gamma_i^T > 0\) and \(h_i\) and \(h_{ij}\) are some arbitrary design parameters to be chosen.

**Remark 4.** Although only the integral component of the adaptation algorithm is needed for stability and exact asymptotic tracking, the use of the proportional and the proportional delayed terms in the adaptation algorithm (23) makes it possible to achieve better adaptation performance than the traditional I and PI schemes. This adaptation algorithm includes the traditional I and PI schemes as a special case. The design parameters \(h_i\) and \(h_{ij}\) are chosen in the same way as the traditional gains \(\gamma_i^k\) and \(\Gamma_i^k\) in (23).

For the stability analysis we use the following Lyapunov-Krasovskii type functional

\[
V = \sum_{i=1}^M \left( \frac{1}{2} \|\hat{e}_i(t)\|^2 + \sum_{j=1,j\neq i}^M \left( \|v_{ij}(t) + v_{zi}(t)\|^2 \right) \right)
\]

\[
V_{\phi ij} = e_i^T P_i \hat{e}_i, \quad V_{\phi ij} = Z_{\phi ij}^T P_{\phi ij} Z_{\phi ij}
\]

\[
V_{\phi ij} = \sum_{k=1}^l_i (\gamma_i^k)^{-1} \left| d_i^k \right| \eta_i^k(t) \eta_i^k(t) + \int_{t-h_i}^t \eta_i^k(t) \Omega_i^k(t)\eta_i^k(s)ds
\]

\[
V_{\phi ij} = \sum_{j=1,j\neq i}^M \left( \eta_{ij}(t)\Gamma_i^k \eta_{ij}(s)ds + \int_{t-h_{ij}}^t \eta_{ij}(s)\Gamma_i^k \eta_{ij}(s)ds \right)
\]

(24)

where \(\eta_{ij}(t) = \eta_i(t) + \eta_j(t) + \eta_{ij}(t - h_{ij})\) and \(\eta_{ij}(t) = \eta_j(t) + \eta_{ij}(t - h_{ij})\), \(\hat{D}_i = \text{diag} \{d_i^1, \ldots, d_i^l_i, \ldots, d_i^m_i\}\) and \(d_i^k\) are the entries of \(D_i\). The "artificial" vector \(\eta_i^k = \rho_0 (2d_i^k t)^{-1}[1, 0, \ldots, 0]^T\).

(25)

has the same dimension as \(\Theta_i^T\), and \(r_0\) is an as yet unspecified positive constant.
Using (21) and (22) the time derivatives of the components of (24) along (20) can be written

\[
\dot{V}_{ei} = -\dot{e}_i^T(t)Q_i\dot{e}_i(t) + \sum_{j=1, j \neq i}^M 2\dot{e}_i^T(t)P_i\hat{B}_iD_iD_{\phi ij}L_i^T\dot{e}_j(t) \\
+ \sum_{j=1, j \neq i}^M 2\dot{e}_i^T(t)P_i\hat{B}_iD_iC_{\phi ij}Z_{eij}(t) \\

(26)
\]

\[
\dot{V}_{zij} = -\sum_{j=1, j \neq i}^M Z_{eij}^T(t)Q_{zij}Z_{eij}(t) \\
+ \sum_{j=1, j \neq i}^M 2Z_{eij}^T(t)P_{zij}\hat{B}_{\phi ij}L_j^T\dot{e}_j(t) \\

(27)
\]

\[
\dot{V}_{\eta ij} = -\text{tr}(\eta_{ij}(t)\Gamma_i\hat{B}_{ij}^T(t) + \hat{\Theta}_{ij}(t)\Gamma_i\hat{B}_{ij}^T(t)) \\
- \|\eta_{ij}(t) + \eta_{ij}(t - h_{ij})\|_{\Gamma_i, D_i} \\

(29)
\]

Further using (25), (26)–(29) and dropping negative terms we obtain

\[
\dot{V}_{\|20\|} \leq \sum_{i=1}^M \left[-\dot{e}_i^T(t)Q_i\dot{e}_i(t) - r_0\dot{e}_i^T(t)\hat{B}_iD_i^T\hat{B}_i^T\dot{e}_i(t) \\
- \sum_{j=1, j \neq i}^M Z_{eij}^T(t)Q_{zij}Z_{eij}(t) \\
+ \sum_{j=1, j \neq i}^M 2\dot{e}_i^T(t)P_i\hat{B}_iD_iD_{\phi ij}L_i^T\dot{e}_j(t) \\
+ \sum_{j=1, j \neq i}^M 2\dot{e}_i^T(t)P_i\hat{B}_iD_iC_{\phi ij}Z_{eij}(t) \\
+ \sum_{j=1, j \neq i}^M 2Z_{eij}^T(t)P_{zij}\hat{B}_{\phi ij}L_j^T\dot{e}_j(t) \\

(30)
\]

We can estimate the mixed terms of (30) as follows

\[
2\dot{e}_i^T(t)P_i\hat{B}_iD_iD_{\phi ij}L_i^T\dot{e}_j(t) \\
\leq \dot{e}_j^T(t)P_i\hat{B}_i\Sigma_{1ij}^T\dot{e}_j(t) + \dot{e}_j^T(t)G_i\dot{e}_j(t) \\
2\dot{e}_i^T(t)P_i\hat{B}_iD_iC_{\phi ij}Z_{eij}(t) \\
\leq \dot{e}_j^T(t)P_i\hat{B}_i\Sigma_{2ij}^T\dot{e}_j(t) + Z_{eij}^T(t)G_iZ_{eij}(t) \\
2Z_{eij}^T(t)P_{zij}\hat{B}_{\phi ij}L_j^T\dot{e}_j(t) \\
\leq Z_{eij}^T(t)G_iZ_{eij}(t) + e_j^T(t)\Sigma_{3ij}^T\dot{e}_j(t) \\

(31)
\]

where \(G_i = G^T_i > 0\) is a some constant matrix and

\[
\Sigma_{1ij} = D_iD_{\phi ij}L_i^T \Sigma_{1ij}^T \\
\Sigma_{2ij} = D_iC_{\phi ij}G_i^{-1} \Sigma_{2ij}^T \\
\Sigma_{3ij} = LBL_{\phi ij}P_{zij}G_i^{-1} \Sigma_{3ij}^T \\

(32)
\]

Applying (31) to (30) and selecting values of \(r_o, Q_{zij}\) and \(Q_i\) from the inequalities \((i, j = 1, \ldots, M)\)

\[
\frac{\lambda_{\text{max}}(\Sigma_{1ij}) + \lambda_{\text{max}}(\Sigma_{2ij}) + \lambda_{\text{max}}(\Sigma_{3ij})}{\lambda_{\text{min}}(\Sigma_{1ij})} > \lambda_{\text{max}}(G_i + \Sigma_{1ij}G_i^{-1}) \\
\lambda_{\text{min}}(Q_{zij}) > \lambda_{\text{max}}(G_i) \\
\lambda_{\text{min}}(Q_i) > 2\lambda_{\text{max}}(G_i + \Sigma_{1ij}) \\

(33)
\]

where \(\lambda_{\text{min}}(*)\) and \(\lambda_{\text{max}}(*)\) are the minimum and maximum eigenvalues of (*), respectively we obtain

\[
\dot{V}_{\|20\|} \leq -\sum_{i=1}^M \left(\dot{e}_i^T(t)Q_i\dot{e}_i(t) \\
+ \sum_{j=1, j \neq i}^M Z_{eij}^T(t)Q_{zij}Z_{eij}(t) \right) \\

(34)
\]

This implies (Hale and Lunel, 1993), that \(\dot{V}\) and, therefore, \(\dot{e}_i(t), \dot{e}_j(t), Z_{eij}(t), \hat{\Theta}_{ij}, \hat{\Theta}_{ij}, \theta_{ij}, \theta_{ij}\) is \(L_\infty\). The remainder of the stability analysis follows directly using the steps in (Ioannou and Sun, 1996).

Remark 5. We note that \(r_o\) and the matrices \(Q_{zij}\) and \(Q_i\) are used only for analysis and do not influence the control law. Decentralized controller gains adjust automatically to counter the non-desirable effects of interconnections and parameter uncertainties.

5. ROBUSTNESS OF THE DECENTRALIZED CONTROLLER

As in adaptive centralized control theory (Ioannou and Sun, 1996), the coordinated adaptive controller proposed here can be shown to be robust with respect to disturbances and unmodeled dynamics by introducing modifications to the adaptive law. A brief example of this property is mentioned below where the integral adaptation law is discussed for ease of exposition. If disturbance \(f_i(t)\), \(\|f_i\| \leq f_o\) is present in the subsystems, and e.g. a \(\sigma\)-modification as in (Ioannou and Sun, 1996) is used, the error equation (19) and the adaptation algorithms (23) can be rewritten as

\[
e_i = W_{ei}(s)\Sigma_i \left(\hat{\Theta}_{i1}^T \Omega_{1i} \cdots \hat{\Theta}_{im}^T \Omega_{mi} \right) + \hat{f}_i(t) \\
+ \sum_{j=1, j \neq i}^M \left[\hat{\Theta}_{ij}^T \omega_{ij} + \hat{\Phi}_{ij}(s)[e_{ij}(t)]\right] \\

(35)
\]

\[
\hat{\Theta}_{i1}(t) = -\eta_i^k(t) - \sigma_i^k \Theta_{i1}^k \\
\hat{\Theta}_{ij}(t) = -\eta_i^j(t) - \sigma_j^k \Theta_{ij}^k \\

(36)
\]

where \(\hat{f}_i(t) = \hat{\Phi}_i(s) f_i(t)\) is guaranteed to be bounded with the bound \(f_o\) due to the boundedness of \(f_i(t)\) and the stability of \(\Phi_i(s) = I - \Phi_i(s)\).

Instead of \(V_{\eta ij}\) and \(V_{q ij}\) in (24) we use the following functions

\[
V_{\eta ij} = \sum_{k=1}^I (\eta_k^i)^{-1} |d_l| \hat{\eta}_k^i(t) \\
V_{q ij} = \sum_{j=1, j \neq i}^M \text{tr}(\eta_{ij}(t)\Gamma_i \hat{D}_i\hat{\eta}_j^T(t)) \\

(37)
\]
with \( \eta_i^k(t) = \hat{\theta}_i^k(t) + \eta_i^k \) and \( \eta_{ij}(t) = \hat{\theta}_{ij}(t) \)

Using (35) and (36) and in view of (37) after some simplifications, we obtain the time derivative of (24)

\[
\dot{V} \leq \sum_{i=1}^{M} \left[ - \hat{e}_i^T(t) Q_i \hat{e}_i(t) - \sum_{k=1}^{l_i} \sigma_i^k |d_i^k| \hat{\theta}_i^k(t) \eta_i^k(t) + \beta_i^* - \sum_{j=1, j \neq i}^{M} \left( Z_{ej}^T(t) Q_{zij} Z_{ej}(t) - \sigma_{ij} \text{tr}(\eta_{ij}(t) D_i \eta_{ij}^T(t)) \right) \right] 
\]

where

\[
\beta_i^* = \bar{f}_i^2 ||G_i|| + \sum_{k=1}^{l_i} \sigma_i^k |d_i^k| |\theta_i^k - \eta_i^k|^2 + \sum_{j=1, j \neq i}^{M} \sigma_{ij} \text{tr}(\eta_{ij}(t) D_i \eta_{ij}^T(t)) 
\]

Thus, if we select \( Q_{zij} \) and \( Q_i \) from (33) and the values of \( a \) and \( r_a \) from the inequalities

\[
\alpha = \min \left( \frac{1}{\lambda_{max}(Q_i)} \lambda_{min}(Q_{zij}), \frac{\sigma_i^k |d_i^k| \lambda_{min}(\sigma_{ij}^k |d_i^k|) \lambda_{max}(D_i)}{\lambda_{max}(D_i) \lambda_{max}(D_i)} \right) 
\]

and

\[
\frac{r_a}{M-a} > \lambda_{max}(\Gamma_{ij} + \Gamma_{2ij} + \Gamma_{4ij}) 
\]

where \( \Gamma_{4ij} = D_i G_{i}^{-1} D_i \) we obtain after some manipulations from (38)

\[
\dot{V} = - \alpha V + \sum_{i=1}^{M} \beta_i^* 
\]

Furthermore, using standard arguments from the robust adaptive control theory, see, e.g. (Ioannou and Sun, 1996), we conclude that all closed-loop signals are bounded, and \( \epsilon_i(t) \) converges to the residual set bounded by the same constant. We can show that by decreasing the values \( \sigma_i \) and \( \sigma_{ij} \) sufficiently, the upper bound on the steady-state error \( \epsilon_i \) may be made as small as desired. The system designer can e.g. tune the size of the residual set by adjusting these parameters which were introduced in the adaptation laws described in (36).

6. SUMMARY

In this paper an adaptive coordinated decentralized controller is proposed for large-scale linear systems with MIMO subsystems which admits output decentralized model reference adaptive designs with zero asymptotical errors. We develop decentralized MRAC on the base of a priori information about only the local subsystems gain frequency matrices without additional a priori knowledge about the full system gain frequency matrix. A novel Lyapunov-Krasovskii type functional derived to design the adaptive laws and in order to prove stability. The decentralized controller is also shown to be robust to disturbances. This paper represents the first results for decentralized MRAC of large-scale system with MIMO subsystems.

REFERENCES


