STEPWISE DECELERATION FOR PICK AND PLACE APPLICATIONS

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Abstract: In this paper we present a method that has been developed for fast stopping in pick and place robotic applications with high inertia loads and self-blocking transmissions. The latter is the case of worm gears and power transmission screws. Hence a desired trajectory can be executed at maximum speed and near optimal time with a controlled overloading of the torque at the transmission output shaft. The method can also be applied in other high reduction gearing, which resemble closely the behavior of a self-blocking mechanism. Copyright © 2002 IFAC

Keywords: robotics, dynamics, actuators, transient response, torque control

1. INTRODUCTION

Electric motors such as those used in motion control show a tendency to get smaller in time. The power to weight ratio is further increased by higher operating speeds. Thus for motion control applications power transmission gearing with high speed reduction ratio is commonly required.

On the other hand in pick and place applications where heavy inertia loads are moved it is desirable to operate at maximum speed for economical reasons. To move a load from one place to another as fast as possible, an electric motor can work at full load during acceleration, thus making full use of the actuator capacity. However bringing the load to a stop at the end of the trajectory generates problems if it is done abruptly because the speed reduction gear may act as a self-blocking mechanism whenever the load tries to drive the motor. In a sense a speed reducer with a large reduction ratio acts like a mechanical diode preventing the output shaft from driving the input shaft.

A blocking torque is generated when the load (due to its inertia) tries to drive the electric motor passing through the speed reduction unit. The magnitude of this torque can be high enough to damage the speed reduction unit. This is the case of worm gears and power transmission screws but it also includes transmission units with high-speed reduction ratio.

To avoid the occurrence of high blocking torques the high inertia loads must be decelerated gradually. The presence of coulomb friction or viscous damping is helpful to reduce the magnitude of the blocking torque, but nevertheless the best way to control the occurrence of a high magnitude blocking torque is by gradual deceleration.

From a productivity point of view however, if the deceleration is too mild it will increase substantially the total trajectory duration and by extension the overall task time. Hence it is important to produce an appropriate and convenient deceleration of the load at the end of the trajectory.
The finding of an optimal trajectory in this case is a problem of Optimal Control. As such this problem has been addressed by Fotouhi and Szyszkowski (1998 a,b) and by Geering et al.(1986), and the solutions proposed are computationally demanding.

In this article we introduce an alternative method by which the blocking torque magnitude during stopping is controlled stepwise in a way such that the speed reduction unit is not harmed and at the same time the deceleration is not too slow to affect adversely the overall task time.

2. THEORETICAL BACKGROUND

To illustrate the methodology we are proposing let us assume a mechanical system such as that shown in the figure below.

![Figure 1. Schematic diagram of a pick and place robotic joint](image)

Here an electric motor drives a worm gear speed reduction unit, which in turn drives a large inertia load. If we use a DC servomotor controlled by a Pulse Width Modulation (PWM) technique, then we can write the following equations:

\[ V = K \phi \omega_M + i_a R_a \]
\[ T_M = K \phi i_a \]

where

- \( V \): PWM voltage supply [Volt]
- \( K \): Geometric constant of motor
- \( \phi \): Magnetic Flux
- \( \omega_M \): Electric motor angular velocity [rad/s]
- \( i_a \): Armature current [amp]
- \( R_a \): Armature resistance [ohm]
- \( T_M \): Motor torque [Nm]

A complete discussion regarding motors for motion control applications can be found in Kaiser(1998).

**Speed Reducer Input Shaft Torque Equation:** To find the dynamic equations of motion of a mechanism the Newton-Euler or Lagrange approaches can be used. In particular Sciaicco et al. (1996) have discussed this matter before in the case of a robotic manipulator. The kinematical and dynamic characteristics of a worm gear speed reducer can be found in Shigley and Mischke(1996). In this work we have used the conventional Newton-Euler approach, which is extensively discussed by Chiang(1995) among many others.

A schematic diagram of forces in a worm gear speed reducer is shown below

![Figure 2. Velocity and Free Body Diagrams at the engagement zone of a worm gearing speed reduction unit.](image)

The following equation can be written
\[ K \phi i_a = (J_M + J_W) \omega_M + (N_c \sin \beta + f \cos \beta) r_w \]  \hspace{1cm} (2)

where

- \( J_M \) = Motor Moment of Inertia [kgm²]
- \( J_W \) = Worm Moment of Inertia [kgm²]
- \( N_c \) = Normal contact force on worm [kgm²]
- \( \beta \) = friction lead angle
- \( f \) = friction force in engagement zone [N]
- \( r_w \) = worm radius [m]

**Speed Reducer Output Shaft Torque Equation:** The following equation can be written for the output shaft and load:

\[ (N_c \cos \beta - f \sin \beta) R_G = (J_G + J_L) \omega_G + T_f + c \omega_G \]  \hspace{1cm} (3)

where

- \( J_G \) = Moment of Inertia of Wormgear [kgm²]
- \( J_L \) = Moment of Inertia of Load [kgm²]
- \( T_f \) = Coulomb friction Torque [Nm]
- \( c \) = damping coefficient [Ns/m]

The speed reduction ratio for a worm gear is given by the expression

\[ \frac{\omega_G}{\omega_M} = \frac{r_w}{r_G} \tan \beta \]  \hspace{1cm} (4)

Also the friction force \( f \) can be related to normal contact \( N_c \) by the Coulomb friction law

\[ f = \mu N_c \]  \hspace{1cm} (5)

Using equations (1) to (5) we can write the equation of motion for the system during acceleration, which is:

\[ \left( J_M + (J_G + J_L) \right) \left( \sin \beta + \mu \cos \beta \frac{r_w}{r_G} \tan \beta \right) \omega_M = K \phi V \left( \frac{K^2 \phi^2}{R_u} + c \left( \frac{r_w}{r_G} \right)^2 \sin \beta + \mu \cos \beta \tan \beta \right) \omega_M - T_f \left( \frac{r_w}{r_G} \right) \sin \beta + \mu \cos \beta \tan \beta \]  \hspace{1cm} (6)

The driving torque exerted by the worm gear can be written as

\[ T_G = (J_G + J_L) \omega_G + T_f + c \omega_G \]  \hspace{1cm} (7)

If the load attempts to drive the speed reducer from the output end, then the friction force depicted in Figure 2 changes direction and therefore equations (2) and (3) become (notice the sign change):

\[ K \phi i_a = (J_M + J_W) \omega_M + (N_c \sin \beta - f \cos \beta) r_w \]  \hspace{1cm} (8)

\[ (N_c \cos \beta + f \sin \beta) R_G = (J_G + J_L) \omega_G + T_f + c \omega_G \]  \hspace{1cm} (9)

If the voltage source is abruptly disconnected then the above equations apply and equation (6) now becomes

\[ \left( J_M + (J_G + J_L) \right) \left( \frac{r_w}{r_G} \right)^2 \sin \beta - \mu \cos \beta \tan \beta \right) \omega_M = K \phi V \left( \frac{K^2 \phi^2}{R_u} + c \left( \frac{r_w}{r_G} \right)^2 \sin \beta - \mu \cos \beta \tan \beta \right) \omega_M - T_f \left( \frac{r_w}{r_G} \right) \sin \beta - \mu \cos \beta \tan \beta \]  \hspace{1cm} (10)

The above equation can be simplified if we consider that

\[ \mu \leq \tan \beta \]  \hspace{1cm} (11)

Hence we obtain

\[ J_M \omega_M + \left( \frac{K^2 \phi^2}{R_u} \right) \omega_M = 0 \]  \hspace{1cm} (12)

Upon disconnecting the voltage source, the motor behaves according to the above equation, which is independent of the load characteristics. The solution to equation (12) is

\[ \omega_M = \omega_i \exp \left( - \frac{K^2 \phi^2}{J_M R_u} t \right) \]  \hspace{1cm} (13)

The blocking torque given in equation (7) becomes now

\[ T_G = \left( c - (J_G + J_L) \left( \frac{K^2 \phi^2}{J_M R_u} \right) \right) \omega_i \exp \left( - \frac{K^2 \phi^2}{J_M R_u} t \right) + T_f \]  \hspace{1cm} (14)

If instead of abruptly disconnecting the voltage it is decreased stepwise from \( V_0 \) to a lower level \( V_1 \) then we have
\[ \omega_M = \omega_h + (\omega_o - \omega_h) \exp \left( -\frac{K^2 \phi^2}{J_M R_a} t \right) \]  
with 
\[ \omega_h = \frac{V_I}{K \phi} \]

Therefore equation (14) becomes 
\[ T_{GSTEP} = \left[ (c - J_G + J_L) J_M R_a \right] (\omega_{G0} - \omega_{G1}) \exp \left( -\frac{K^2 \phi^2}{J_M R_a} t \right) + c + \omega_{G1} \]

Notice that the time constant in equation (14) is the same as in (16). Thus 
\[ \tau = \frac{J_M R_a}{K^2 \phi^2} \]  

The ratio between both torques in equations (14) and (16) can be written approximately as 
\[ \frac{T_{GSTEP}}{T_{G}} = \frac{\omega_{G0} - \omega_{G1}}{\omega_{G0}} \]  

The fact that the time constant in equations (14) and (16) is the same and the torque relation in equation (18) are the basis of the methodology to limit the magnitude of the blocking torque while putting a high inertia load to a stop.

3. METHOD FORMULATION

The method presented here has been developed principally for low cost pick and place applications where the mechanism (robotic joint) is driven by an electric motor whose speed is regulated by PWM techniques. Furthermore for simplicity the duty cycle is limited to a set of discrete values (typically 1-8 values) that are preprogrammed into the motor driver. A given preprogrammed duty cycle value can be activated digitally through a parallel input port in the motor driver. It must be noticed that this feature is common to most commercial electric motor drivers and it is convenient to use because a discrete duty cycle profile can be easily generated by feeding the corresponding input port with the appropriate values at predetermined times.

Our method takes advantage of this feature present in most commercial motor drivers, so that the current active duty cycle/voltage is switched in a stepwise form to bring a heavy inertia load to a stop. The duty cycle at each step is calculated so that the blocking torque does not exceed a previously determined admissible value.

We begin by defining an admissible value of the blocking torque \( T_{GAdm} \) which depends on the structural rigidity of the mechanical system.

Next we compute the maximum blocking torque \( T_{GMax} \) if the load were stopped in just one step. We assume that the load, and for that matter the worm gear is moving initially at an angular velocity \( \omega_{G0} \). Then the magnitude of this torque is obtained from equation (14) and we have the following expression:

\[ T_{GMax} = \left[ (c - J_G + J_L) J_M R_a \right] \omega_{G0} + T_{f} \]  

It can be shown that if the voltage source polarity were inverted instead, then the value obtained above would double.

The number of steps \( N_{steps} \) to use in the stopping process is computed next:

\[ N_{steps} = \frac{T_{GMax}}{T_{GAdm}} \]  

The switching of the voltage feeding the motor is done once the angular velocity reaches the step final reference velocity which according to equations (15) and (17) could be estimated in a time \( \Delta t \approx \tau \).

Finally the variation of Voltage \( \Delta V \) at each step can be computed. Combining the equations so far given at each step also the variation of motor speed \( \Delta \omega \) can be computed. Thus we have the following equations

\[ \Delta V = \frac{V}{N_{steps}} \]  
\[ \Delta \omega = \frac{V}{K \phi N_{steps}} \]  

Since it is customary to use an incremental encoder to monitor the angular position of the electric motor, it may be more practical to produce the voltage or duty cycle switching based on the reading of the encoder. In this case the angular variation at each step would be:

\[ \Delta \omega = \omega - \omega - \omega \]  

Numerical Example: Let us assume the following values for a system such as shown in Fig. 1:
\[ J_M = 8.65e^{-3} \text{kgm}^2 \]
\[ J_G + J_sc = 1.54 \text{kgm}^2 \]
\[ V = 24[V] \]
\[ \beta = 5^\circ \]
\[ K = 0.17 \text{[ohm]} \]
\[ c = 0.01 \text{[Ns/m]} \]
\[ T_f = 10 \text{[Nm]} \]
\[ T_{Gdms} = 1000 \text{[Nm]} \]

Then we obtain
\[ T_{GMax} = 3959 \text{[Nm]} \] according to (19)
\[ N_{step} = 4 \] according to (20)
\[ \tau = 0.057 \text{[s]} \] according to (17)
\[ \Delta t_s = 0.231 \text{[s]} \]
\[ \omega_Go = 3.26 \text{[rad/s]} \]

Thus the following table can be generated

<table>
<thead>
<tr>
<th>Worm gear Angle Variation</th>
<th>Speed Reference</th>
<th>Voltage Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.46</td>
<td>18</td>
</tr>
<tr>
<td>0.755915985</td>
<td>1.64</td>
<td>12</td>
</tr>
<tr>
<td>0.566720412</td>
<td>0.82</td>
<td>6</td>
</tr>
<tr>
<td>0.377524838</td>
<td>Jog</td>
<td>-</td>
</tr>
<tr>
<td>0.188329264</td>
<td>Stop</td>
<td>0</td>
</tr>
</tbody>
</table>

4. EXPERIMENTAL EVALUATION

The methodology previously described has been implemented in two pick and place applications that we describe next

a) Pick and Place Scara Type Robot

The robot is shown in the picture below

A PIC microprocessor is used and it is programmed to receive by serial communication from a PC, a table similar to the one shown in Table 1, for each joint. The PIC microprocessor also reads the encoder signal coming out from each DC servomotor. For a detailed explanation of PIC programming aspects see Angulo(1999).

Each of the two tables received for every trajectory contains two columns; the first one contains the encoder count at which the corresponding speed/voltage should be set. The second column contains the respective output number that will activate the corresponding discrete speed/voltage in the motor driver through the parallel input port.

While completing the trajectory the PIC microprocessor reads the encoder count for each motor and switches the speed reference according to the previously loaded table. The graphic shown next depicts the recorded angular motion of the first joint during completion of a trajectory. The slope magnitude decreases stepwise until reaching the final position.

b) Linear Positioning Device

A similar scheme was applied in the linear positioning device shown below which is used to position steel beams for drilling a hole in each end. The system consists in an electric motor that drives a rail by means of speed reducer with a high reduction ratio. Thus, the beam end is transported to the drilling station center point.
To prevent the occurrence of a high blocking torque when the load is stopped, a PIC microprocessor is used that previously receives the trajectory table from a PC by serial communication. As the trajectory is completed, the PIC activates the corresponding preset reference value in the motor driver. Thus, the trajectory is completed at maximum speed without surpassing the allowed torque at the output shaft of the speed reducer in any time.

The graph below depicts the behavior of the system during a working cycle, that is to say a round trip travel. Notice the slope changes that denote the switching of reference velocity.

**Fig. 6. Recorded position time-history during a working cycle in a linear positioning system**

5. CONCLUSIONS

A method to produce the stopping of a heavy inertia load in pick and place applications is presented. It prevents the occurrence of a blocking (backdrive) torque of large magnitude that can damage the system when stopping the load at the target end point.

The stopping of the load is done in stepwise fashion in such a way that the blocking torque in the output shaft never exceeds an admissible value. For this purpose the number of steps required to stop the load and the time for each step is determined using an appropriate mathematical model such as the one described in the previous sections.

In the simplest form of implementation of this methodology, a table is computed for each degree of freedom in the system. This table contains the reference speed for every step. A microprocessor uses this table to switch discretely the reference velocity (or voltage/duty cycle) as the trajectory is completed according to the instantaneous encoder count reading.

The implementation is of low cost and it has been tested both experimentally as well as in field applications with good results.

**REFERENCES**


