ROBUSTNESS ANALYSIS OF DMC FOR FIRST ORDER PLUS DEAD-TIME PROCESSES

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Abstract: This paper presents a robustness analysis of the Dynamic Matrix Controller (DMC) for dead-time processes. The study considers a wide class of industrial processes, those that can be modeled by a first order transfer function with a dead-time. The analysis shows that DMC can be described using a predictor plus a primary control structure. This description allows to analyze the effect of the dead-time in the robustness of DMC. The results are illustrated with some simulations that compare DMC to the Generalized Predictive Controller (GPC).

Keywords: Predictive control, Delay compensation, Robustness, Robust Stabilizer Process control.

1. INTRODUCTION

Dead-times are present in many processes in industry. In most of the cases, these dead-times are usually associated with mass or energy transport, or due to the accumulation of a great number of low-order systems. It is well known that these types of processes are specially difficult to control because of the phase lag introduced by the dead-time in the closed loop. Also, the control problems caused by dead-time represent, in addition to the problems caused by the disturbances and the interactions between variables, more of 60% of the important problems in the process industry (Takatsu et al., 1998).

In general, when the process exhibits a small dead-time, it is possible to obtain an acceptable solution using classical PID controllers. But if the dead-time is dominant, it is necessary to use some dead-time compensating system to improve the closed-loop performance. Two different control structures can be used for this purpose: (i) a dead-time compensator like a Smith predictor (Smith, 1958) or modified versions (Normey-Rico, 1999); (ii) a model predictive controller (MPC). Several papers have been recently written analyzing the performance and the robustness of these two types of controllers. Particularly, the GPC (Generalized Predictive Controller) was used in (Normey-Rico and Camacho, 1999) to demonstrate that this algorithm is equivalent to a system composed by a primary controller and an optimal predictor (both, for the SISO and MIMO cases). Also, using this structure, a simple analysis of performance and robustness of the GPC has been performed when controlling dead-time processes.

In this paper, a similar study is developed for the Dynamic Matrix Controller (DMC) for processes modeled by a first order transfer function plus a dead-time. It is shown that a primary controller plus a predictor structure can be obtained for the DMC and that, with this new formulation, it is possible to make a more complete analysis of the effect of the dead-time on the robustness.

The paper is organized as follows: in the next section, an overview of the GPC and DMC algorithms is presented. Section 3 shows that DMC can be rep-
resented as a primary controller plus a predictor. The robust stability analysis of the gpc and the DMC is shown in section 4, where the effect of the dead-time in the robustness of the algorithms is specially considered. Some simulation results are described in section 5 and finally the conclusions of the work are presented in section 6.

2. REVIEW OF PREDICTIVE CONTROL

Model Predictive Control (MPC) is not a specific strategy of control, but a set of control methods including common ideas. These ideas, included in all MPC algorithms, are: the use of an explicit prediction model to predict the process outputs in a defined horizon; the minimization of a cost function in order to compute the control sequence and the use of a receding strategy (Camacho and Bordons, 1999).

Two MPC algorithms have been widely studied and used in the industry: the DMC developed by Cutler and Ramaker of Shell Oil Co. in 1980 (Cutler and Ramaker, 1980) and the gpc presented by Clarke and co-authors (Clarke et al., 1987). DMC is mostly used in the industrial world, mainly in the chemical processes. It uses a step response model to describe the behavior of the process and also to compute the predicted values of the process output y(t + j).

The DMC strategy consists in applying a control sequence that minimizes a cost function given by:

$$J = \sum_{j=N_1}^{N_2} [\tilde{y}(t + j | t) - w(t + j)]^2 + \sum_{j=1}^{N_2} \lambda(j)[\Delta u(t + j - 1)]^2 \quad (1)$$

where \( \tilde{y}(t + j | t) \) is the prediction at \( t + j \) of the system output based on the data up the instant \( t \); \( N_1 \) and \( N_2 \) are the minimal and maximum cost horizons; \( \lambda(j) \) is the weighting control sequence, and \( w(t + j) \) is the future reference trajectory.

The objective of the DMC is to calculate the future control sequence \( u(t), u(t + 1), \ldots \) in order to drive the future plant output \( y(t + j) \) to the reference \( w(t + j) \).

The gpc uses the same cost function as DMC and has the same objectives but the predictions of the output of the plant are computed using a CARIMA model given by:

$$A(z^{-1})y(t) = z^{-\delta}B(z^{-1})u(t - 1) + T(z^{-1})\frac{e(t)}{\Delta} \quad (2)$$

where \( y(t) \) and \( u(t) \) are the output and control sequence of the plant; \( e(t) \) is a zero mean white noise; \( A, B, T \) are polynomials in the backward shift operator \( z^{-1} \); \( \Delta = 1 - z^{-1} \) and \( T(z^{-1}) \) is a filter normally used to obtain a good compromise between performance and robustness. The advantage of the model used in the gpc strategy is that it can deal with all types of plants while DMC can be used only with stable or integrative processes.

3. DMC FOR FIRST ORDER PLUS DEAD-TIME PROCESSES

Simple models are very important in process industry because they generate simple controllers and easier tuning rules. There are several claims that in practice low order models coupled with dead-times are sufficient for most purposes. Particularly, the first order transfer function plus a dead-time model is the most used representation in commercial controllers. In general, the three parameters of this model are obtained using the well-known identification step test (Bi et al., 1999). Using this idea, the system can be described with the following transfer function:

$$P_n(s) = \frac{K}{1 + e^{-\tau_d s}} \quad (3)$$

where \( K \) is the static gain, \( \tau \) is the time constant and \( \tau_d \) is the dead-time of the system. In the following the dead-time \( \tau_d \) is considered as a multiple of the sampling time \( T \), it means, \( \tau_d = dT \). Thus, the discrete transfer function is given by:

$$P_n(z^{-1}) = \frac{b z^{-1}}{1 - a z^{-1}} \quad (4)$$

where \( a, b \) and \( d \) are computed as:

$$a = e^{-\tau / \tau_d}, \quad b = K (1 - a), \quad d = \frac{\tau_d}{T}$$

For the processes modeled by equations (3) or (4), it is interesting to formulate the DMC in order to explicitly see the effect of the dead-time in the controller and to evaluate the effect of the dead-time in the closed-loop behaviour. To do it, the prediction at \( t + k \) is written as a function of the predictions up to the dead-time.

Considering the step response model, the predicted values during the horizon are:

$$\hat{y}(t + k | t) = \sum_{i=1}^{k} g_i \Delta u(t + k - i) + y(t) + \sum_{i=1}^{N} (g_{i+k} - g_i) \Delta u(t - i) \quad (5)$$

where the elements \( g_i \) can be described using the systems parameters (4) as:

$$g_i = b \sum_{j=0}^{i} a^j, \quad i = 1 \ldots N,$$
and \( N \) is the control horizon.

Notice that the minimum output horizon \( N_1 \) must be chosen as \( N_1 > d + 1 \), because the system output is not affected by the input \( u(t) \) until \( t = d \). So, in this paper \( N_1 \) and \( N_2 \) will be considered as \( N_1 = d + 1 \) and \( N_2 = d + N \). Now, if the prediction equation (5) is rewritten as a function of \( y(t + d \mid t) \), it is possible to obtain:

\[
\hat{y} = G u + H u_1 + S y_1, \tag{6}
\]

where:

\[
\hat{y} = [\hat{y}(t + d + 1 \mid t) \ldots \hat{y}(t + d + N \mid t)]^T, \\
u = [\Delta u(t) \ldots \Delta u(t + N - 1)]^T, \\
u_1 = [\Delta u(t - 1) \ldots \Delta u(t - 2) \ldots \Delta u(t - N)]^T, \\
y_1 = [\hat{y}(t + d \mid t)]^T
\]

and \( G \) and \( H \) are \( N \times N \) matrices and \( S \) is an unitary vector of length \( N \).

It is important to notice that \( f = H u_1 + S y_1 \) is computed using the past control signals and corresponds to the free response of the system. Note that the matrix \( G \) in (6) is equivalent to the step response matrix used in the gpc algorithm (Clarke and Mohd-Mahdi, 1989).

Substituting \( \hat{y} \) in the cost function equation (1), \( J \) can be expressed as a function of the reference sequence \( w = [w(t + d + 1) \ldots w(t + d + N)]^T \) and the vectors \( u, u_1, y_1 \).

Thus, the minimum of \( J \) using \( \lambda(j) = \lambda \) constant, is obtained by:

\[
M u = P_0 [y(t + d \mid t)] + P_1 u_1 + P_2 w \tag{7}
\]

where \( M, P_1 \) and \( P_2 \) are \( N \times N \) matrices and \( P_0 \) is a \( N \times 1 \) vector.

Because of the receding strategy, only \( \Delta u(t) \) is applied to the system. Therefore, if \( m \) is the first line of \( M^{-1} \), \( \Delta u(t) \) is computed as:

\[
\Delta u(t) = mP_0 \hat{y}(t + d \mid t) + mP_1 u_1 + mP_2 w
\]

Thereby, \( \Delta u(t) \) is:

\[
\Delta u(t) = ly \hat{y}(t + d \mid t) + lu_1 \Delta u(t - 1) + \ldots + l u_n \Delta u(t - N) + \sum_{i=1}^{N} f_i (w(t + d + i))
\]

where \( mP_0 = [ly], mP_1 = [lu_1 lu_2 \ldots lu_n] \) and \( f_i = \sum_{j=0}^{N} m_{ij} z^{-j} \), \( m_{ij} \) are the elements of \( P_2 \) and \( m_j \) the elements of \( m \). The coefficients \( ly, lu_1 \) and \( f_i \) are functions of \( a, b, \lambda \) and \( N \). When the future reference is unknown, \( w(t + d + i) \) is chosen equal to the actual reference \( r(t) \):

\[
w = [1 \ldots 1] r(t)
\]

and the control increment \( \Delta u(t) \) results:

\[
\Delta u(t) = l y \hat{y}(t + d \mid t) + lu_1 \Delta u(t - 1) + \ldots + l u_n \Delta u(t - N) + l r \ r(t) \tag{8}
\]

Figure 1 shows the equivalent control scheme for the dmc where the values of \( \hat{y}(t + d \mid t) \) are obtained using the analyzed prediction equations.

Fig. 1. Control scheme of dmc.

To complete the analysis, the relationship between the input and output of the process and the prediction in \( t + d \) will be computed. To do it, consider the equation:

\[
\hat{y}(t + j \mid t) = b \sum_{i=0}^{j-1} a^{j-i-1} u(t - d + i) + a^j y(t) + \sum_{i=0}^{j-1} a^i e(t) \tag{9}
\]

where \( e(t) \) represents the error between the model and the process output, that is given by:

\[
e(t) = g(t) - P_m(z) u(t).
\]

If the equation (9) is applied to \( j = d \):

\[
\hat{y}(t + d \mid t) = R(z) y(t) + Q(z) u(t) \tag{10}
\]

where:

\[
Q(z) = \frac{b}{z - a} z^{-d} \left[ z^d + \left( \frac{a^{d+1}-1}{1-a} \right) \right]
\]

and \( R(z) \) is a filter given by:

\[
R(z) = \left( \frac{1-a^{d+1}}{1-a} \right) \tag{11}
\]

In addition, it is possible to compute \( Q(z) \) as:

\[
Q(z) = G_n(z) \left[ 1 - z^{-d} R(z) \right] \tag{12}
\]

where \( G_n(z) \) represents the process model without the dead-time \( d \).

Using these expressions and a block diagram transformation, it is possible to describe the dmc with the scheme of figure 2, where the pair \( C(z), W(z) \) represents the primary controller given by:

\[
W(z) = \frac{r \ e(z)}{C(z)} \quad C(z) = \frac{\Delta t_y \ y}{\Delta (1 - l u_1 z^{-1} - \ldots - l u_N z^{-N})}
\]
As the gpc can be also represented using the block diagram of figure 2, this robustness index is also valid for the gpc, using the appropriate \( C(e^{j\omega}) \) and \( R(e^{j\omega}) \) (Nornery-Rico and Camacho, 1999).

In the comparative analysis, it is assumed that the primary controller is tuned to obtain the same nominal performance in the gpc and DMC. This can be obtained using a defined complementary sensitive function (to simplify the expressions, from now on, the dependence with \( \omega \) is omitted):

\[
I_p = \frac{1}{1 + CG_n}
\]

This implies:

\[
I_c = \left| \frac{1 + CG_n}{1 + G_n} \right| = \left| \frac{G_n}{I_p R} \right|
\]

Thereby, the robustness of the gpc and the DMC can be compared, for the same nominal performance, using:

\[
I_{r_{DMC}} = \left| \frac{R_{GPC}}{R_{DMC}} \right| I_{r_{GPC}}
\]

For the DMC, the filter \( R(z) \) is given by equation (11), thus:

\[
|R_{DMC}| = \left| \frac{1 - a^{\omega+1}}{1 - a} \right|, \quad \forall \omega
\]

which shows that \( |R_{DMC}| \) is a constant value greater than one.

For the same first order model with dead-time \( R(z) \) in the gpc is given by (Nornery-Rico, 1999):

\[
|R_{GPC}| = \left| \frac{1 + a}{1 + a e^{-j\omega}} \right|^2, \quad \forall \omega
\]

Also in this case \( |R_{GPC}| \geq 1 \) for all frequencies (note that the sum of the vectors \( 1 + a \) and \(-ae^{-j\omega}\) is always bigger than one).

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**Fig. 2. Equivalent structure of DMC**

The obtained control structure is equivalent to a dead-time compensator, similar to the Smith predictor sp. Note that if \( R(z) = 1 \) and \( W(z) = 1 \) the classical sp is achieved. If \( R(z) \neq 1 \), a modified version of sp is obtained, where \( R(z) \) is used to improve the robustness or the disturbance rejection of the system (Nornery-Rico et al., 1997).

It is important to notice that in the DMC the filter \( R(z) \) is not a tuning parameter, but a function of some of the process model parameters \( \{A(z) \text{ and } d\} \). It is important to emphasize that: (i) if the dead-time is zero, the final control law is a classical control law; (ii) the tuning of the controller, given by the parameters \( N \) and \( \lambda \), only affects the coefficients of the primary controller.

The results of this analysis are interesting basically for two reasons: (i) they clearly show the effect of the dead-time in the structure of the DMC and (ii) it contribute to the best understanding of the qualities and drawbacks of the DMC when controlling dead-time processes.

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**4. ROBUSTNESS ANALYSIS**

In this section, an analysis of the robustness of the DMC and a comparative study with the GPC is performed. Special attention is given to the errors in the dead-time estimation, because they are the main source of uncertainty at high frequencies that are found in industrial processes.

To consider the effect of the modeling errors, an unstructured additive model of the uncertainties \( \{P(z) = P_n(z) + DP(z)\} \) will be used. From now on, \( d_n \) is the nominal dead-time and \( P_n = G_n z^{-d_n} \) is the nominal model of the process and \( DP(z) \) is the modeling error. Under this hypothesis, it is possible to compute (both, for the DMC and the GPC) a measure of the control system robustness considering, for each frequency, the module of the maximum error that keeps the closed-loop stability (Morari and Zafiriou, 1989). This robustness index \( I_r \) can be computed as follows:

\[
I_r(\omega) = \left| \frac{1 + C(e^{j\omega})G_n(e^{j\omega})}{C(e^{j\omega})R(e^{j\omega})} \right|, \quad \forall \omega > 0
\]

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**Fig. 3. \( |R| \) for the GPC and DMC for different values of the dead-time**

To compare the results figure 3 shows the values of \( |1/R| \) for the GPC and DMC for a typical
The heat exchanger can be considered to be an independent process within the plant. The method of controlling the output temperature \( TT_4 \) is by varying the flow of the recirculation water with valve \( V_5 \); thus, the desired temperature is obtained by variations in the flow. In brief, the \( TT_4-V_5 \) system can be be approached by a first order transfer function with pure dead-time. By performing identification experiments it was possible to note an uncertainty in the values of the plant parameters: the gain \( K \) can vary between 0.08 and 0.15, the time constant \( \tau \) can vary between 5.7s and 6.3s, and the dead-time between 12s and 16s. The sampling time was chosen as \( T = 1s \).

Fig. 4. Diagram of the Pilot Plant

5. APPLICATION TO A PILOT PLANT

To illustrate the robustness and performance of the DMC, some simulation results, comparing the DMC with a simple version of the GPC (Bordons and Camacho, 1998) are presented. The simulation has been developed using the model of a pilot plant existing at the Departamento de Ingeniería de Sistemas y Automática of the University of Seville (Camacho and Bordons, 1999).

5.1 Plant Description

A diagram of the plant showing its main elements is given in figure 4. It has a feed circuit with two input pipes, a cold water one and a hot water one, with motorized valves for regulating the input flows, and a thermally insulated tank with a 15 kW electric resistance for heating. The hot water in the tank can be cooled by sending cold water through the cooling circuit, composed of a centrifugal pump that circulates the hot water from the bottom of the tank through a tube bundle heat exchanger, returning at a lower temperature at the top.

The process model, obtained by the reaction curve method, is given by \( P_u(s) = \frac{21.3e^{-2.4s}}{3.6s} \). Initially, no model uncertainties are considered. In order to obtain a fast closed loop response, the control weight was chosen as \( \lambda = 0.8 \) and the horizons in the GPC and DMC were chosen to achieve a similar nominal response.

The closed loop behaviour for GPC and DMC in the nominal case can be seen in the figure 5. In all cases, the solid lines correspond to the GPC and the dashed lines to the DMC. For the simulation, at \( t = 0 \) a step change is performed and a 10% step load disturbance has been applied to the system at \( t = 150s \). The noise is generated with an ARIMA model with uniform distribution in \( \pm 0.005 \). Notice that both systems have similar setpoint tracking and that GPC rejects a little bit faster the disturbances.

In the following simulations, a 14% dead-time uncertainty is considered without changing the tuning of the controllers. The behaviour of both
control systems is compared in the figure 6. As can be observed, only the DMC is stable.

To stabilize the GPC response, a filter \( T \) is used in the next simulation. Figure 7 shows the closed loop behaviour of the system controlled by the DMC and the filtered GPC. As can be noticed, now the GPC has a stable behaviour but its disturbances rejection response is slower than DMC.

![System response - nominal case](image1)

**Fig. 5. System response - nominal case**

![System response - uncertainty case](image2)

**Fig. 6. System response - uncertainty case**

![System response - uncertainty case (filt. T)](image3)

**Fig. 7. System response - uncertainty case (filt. T)**

This analysis provides a better understanding of some aspects of the DMC operation when it is used to control dead-time processes. It is possible to say that DMC is more robust than GPC, specially in the case of dead-time estimation errors.

6. CONCLUSIONS

This work has presented an analysis of the DMC when controlling dominant dead-time systems. A novel study of the DMC considering mainly the effect of the dead-time in the behaviour of the closed loop system has been performed, showing that it can be represented as a simple dead-time compensator. The obtained results allow a simple analysis of the robustness of the DMC. Some comparative studies with the GPC have been presented showing that, in the control of dominant dead-time processes, the DMC is more robust than GPC, specially in the case of dead-time estimation errors.

7. REFERENCES


