A SUPERVISORY ROBUST ADAPTIVE FUZZY CONTROLLER

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Abstract: A fuzzy controller equipped with an adaptive algorithm and two supervisors is
developed in this work to achieve tracking performances for a class of uncertain non-
linear single input single output (SISO) systems with external disturbances. The
convergence of the training algorithm is guarantied by a gradient projection law. The
effect of both the approximation errors and the external disturbances is attenuated to a
prescribed level thanks to $H_{\infty}$ control. The convergence of the tracking error toward zero
is guarantied by a supervisor where linguistic rules are used to accelerate the convergence
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Keywords: Adaptive fuzzy control, $H_{\infty}$ tracking, gradient projection, Riccati equation,
uncertain systems, nonlinear systems.

1. INTRODUCTION
Fuzzy logic control, as one of the most useful
approaches for utilising expert knowledge, has been
a subject of intense research in recent years (King
and Mamdani, 1977; Mamdani and Assilian, 1975;
Sugeno, 1985; Tong et al., 1980). Fuzzy logic
control is generally applicable to plants that are
mathematically poorly modelled and where
experienced operators are available for providing
qualitative guiding. Although achieving many
practical success, fuzzy control has not been viewed
as a rigorous science, because most of the fuzzy logic
algorithms are proposed without analytical tools to
guarantee basic performance criteria.

According to the universal approximation theorem
(Wang, 1996), many important adaptive fuzzy-based
control schemes have been developed to incorporate
the expert information directly and systematically,
and various stable performance criteria are
guaranteed by theoretical analysis (Marino and
Tomei, 1995; Spooner and Passino, 1996; Wang,
1996). The major advantages in all these fuzzy-based
control schemes are that the developed controllers
can be implemented without any precise knowledge
about the structure of the entire dynamic model.

However, the influence of both fuzzy logic
approximation errors and the external disturbances
can not be eliminated with these approaches (Chang,
2001). In this sense, Hamzaoui, et al. (2000) have
proposed a fuzzy logic controller equipped with a
training algorithm to approximate the system and a
$H_{\infty}$ control to attenuate the effect of both fuzzy
approximation errors and external disturbances.
However, only a good choice of the initial
parameters of the fuzzy approximator can guarantee
the convergence of the algorithm. Chen, et al. (1996)
proposed a similar approach with a gradient
projection law to assure the convergence of the
adaptive fuzzy logic system. But, the attenuation
level can not be systematically determined because it
depends on the control signal (Kang, et al., 1998).
Furthermore, no constraints are imposed to keep the
system in a forced region ($g(x)\neq 0$).

In order to alleviate these problems, we propose in
this work a new fuzzy adaptive algorithm equipped
with a gradient projection law and two supervisors.
The first supervisor, $u_{s}$, forces the system to remain
in a given controllability zone. Thus, the controller’s
parameters are bounded and the quadratic integral of
both the minimal approximation error and the
tracking error is bounded, i.e. the tracking error converges to zero. The second supervisor, $u_a$, attenuates the effect of both the approximation errors and the external disturbances to a prescribed level, $\rho$, using H∞ approach. The performances of the resulting controller can be improved by incorporating some linguistic rules describing the dynamic behaviour of the plant. The classical example of inverted pendulum, as presented in (Wang, 1996), is used to illustrate this approach. We show that the proposed algorithm is robust and the control signal is smooth compared to (Hamzaoui, et al., 2000) and (Wang, et al., 2001).

Section 2 presents the problem statement. Section 3 gives, in a constructive manner, the steps for constructing the robust adaptive fuzzy controller, and how to use the two supervisors to meet the control objectives. A pendulum tracking control example is given in section 4 for illustration.

### 2. PROBLEM STATEMENT

We consider the following nth order non-linear dynamic single input single output (SISO) system in the canonical form:

$$
\begin{align*}
\dot{x}(n) &= f(x, \dot{x}, ..., x^{(n-1)}) + g(x, \dot{x}, ..., x^{(n-1)}) u + d \\
y &= x
\end{align*}
$$

where $f$ and $g$ are unknown (uncertain) but bounded continuous functions; $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the input and output of the system, respectively, $d$ denotes the external disturbances (due to system load, external noise, etc) which is assumed to be unknown but bounded. It should be noted that more general classes of non-linear control problem can be transformed into this structure (Slotine and Li, 1991; Chen, et al., 1996). Let $X = \{x, \dot{x}, ..., x^{(n-1)}\}^T \in \mathbb{R}^n$ be the state vector of the system which is assumed to be available for measurement. We require the system (1) to be controllable, thus the condition $g(X) \neq 0$ must be satisfied in a given controllability region $U_c \subset \mathbb{R}^n$.

Without loss of generality we assume that $g(X) > 0$, but the analysis throughout this paper can easily be tailored to systems with $g(X) < 0$.

The control objective is to force $y$ to follow a given bounded reference signal, $y_r$, under the constraint that all the parameters $(u, y, X)$ are bounded and the closed loop system is globally stable and robust. If the system is well-known and free of external disturbances, feedback linearization (Isidori, 1989) can be used to synthesis a control law of the form:

$$
u = \frac{1}{g(X)} \left[ -f(X) + \frac{y(n)}{g(X)} + e \right]
$$

where $e = [e, \dot{e}, ..., e^{(n-1)}]^T$ is the error vector, $e = y_r - y$, and $K = [k_n, k_{n-1}, ..., k_1]^T$ is the dynamic error coefficient vector such that all the roots of the polynomial $H(s) = s^n + k_n s^{n-1} + ... + k_1$ are located in the open left half plane.

The control signal (2) gives the following dynamic error:

$$
e(n) + k_1 e^{(n-1)} + ... + k_n e = 0
$$

which implies that $\lim_{t \to \infty} e(t) = 0$. However, it is impossible to obtain such a control algorithm if $f$ and $g$ are unknown and the system is perturbed. A fuzzy logic approximation, as described in section 3, is therefore employed to treat this tracking control design problem. The following control law, proposed by (Hamzaoui, et al., 2000), can be applied to the system:

$$
u = \frac{1}{g(X)} \left[ -f(X) + \frac{y(n)}{g(X)} + K^T e_a \right]
$$

where $\hat{f}$ and $\hat{g}$ are the approximations of $f$ and $g$, respectively, and $e_a$ is the control signal which attenuates the effect of both the approximation errors and the external disturbances.

### 3. ROBUST TRACKING PERFORMANCE DESIGN IN ADAPTIVE FUZZY SYSTEM

The objective of this work is to guarantee the convergence of both the estimation algorithm and the tracking error. For the first requirement, we impose the convergence of the adjustable parameters using the projection technique. For the second, we add a supervisor control signal, $u_a$, to guarantee the stability, in the sense of Lyapunov, of the system.

#### 3.1 Adaptive fuzzy algorithm

The approximation $\hat{f}$ and $\hat{g}$ (in (4)), can be given by the universal fuzzy systems $\hat{f}(\chi_{\theta_f})$ and $\hat{g}(\chi_{\theta_g})$ (Wang, 1996):

$$
\hat{f}(\chi_{\theta_f}) = \theta_f^T \zeta(X), \quad \hat{g}(\chi_{\theta_g}) = \theta_g^T \zeta(X)
$$

where $\theta_f$ and $\theta_g$ are the vectors of the tuneable parameters and $\zeta(X) = [\zeta_1(X), ..., \zeta_n(X)]$ is a regressor vector as given in (Wang, 1996). The tracking error dynamic equation resulting from (4) can be written as:

$$
\dot{E} = AE + b(\hat{f} - f) + (\hat{g} - g) u_a + d
$$

where:

$$
A = 
\begin{bmatrix}
0 & 1 & 0 & 0 & ... & 0 \\
0 & 0 & 1 & 0 & ... & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & ... & 1 \\
-k_n & -k_{n-1} & -... & -... & -k_1
\end{bmatrix},
\ b = \begin{bmatrix} 0 \ 0 \ \cdots \ 0 \ \end{bmatrix}^T
$$

$A$ is a stable matrix, thus it can be associated with the following algebraic Riccati equation which has a unique positive definite solution, $P = P^T$, if and only if $2 \rho^2 - \frac{1}{\rho^2} \geq 0$, i.e., if $2 \rho^2 \geq r$:

$$
AP + PA + Q - 2PB \left( \begin{bmatrix} 1 \\
\frac{1}{2\rho^2}
\end{bmatrix} \right) B^T P = 0
$$

where $Q$ is a positive definite matrix given by the designer. According to the universal approximation theorem (Wang, 1996), there exists optimal approximation parameters $\theta_{\hat{f}}$ and $\theta_{\hat{g}}$ such that $\hat{f}(\chi_{\theta_{\hat{f}}})$ and...
\[ \dot{\xi} \left( \chi_{g}^{*} \right) \] can, respectively, approximate \( f(X) \) and \( g(X) \) as closely as possible. The minimum approximation error is defined as:
\[ w_{e} = f \left( \chi_{g}^{*} \right) - f(X) + g \left( \chi_{g}^{*} \right) - g(X) \]  
(8)
and the tracking error dynamic equation (6) can be rewritten as:
\[ \dot{E} = AE + B \left( \frac{\dot{f}}{\dot{\chi}_{g}^{*}} \right) \] 
\[ + B \left( \frac{\dot{g}}{\dot{\chi}_{g}^{*}} \right) + u_{a} + w_{e} - d \]  
(9)
From (9), (5) can be rewritten as:
\[ \dot{E} = AE + B \left( \Phi_{f}^{T}(X) + \Phi_{g}^{T}(X) + u_{a} + w_{e} - d \right) \]  
(10)
where \( \Phi_{f} = \theta_{f} - \theta_{f}^{*} \), \( \Phi_{g} = \theta_{g} - \theta_{g}^{*} \).

Let’s choose the Lyapunov function as:
\[ V = \frac{1}{2} E^{T} P E + \frac{1}{2 \gamma_{f}} \Phi_{f}^{T} \Phi_{f} + \frac{1}{2 \gamma_{g}} \Phi_{g}^{T} \Phi_{g} \]  
(11)
where \( \gamma_{f} \) and \( \gamma_{g} \) are positive constants.
The time derivative of \( V \) along the error trajectory (10) is given by:
\[ \dot{V} = -\frac{1}{2} E^{T} Q E + \frac{1}{2} E^{T} P B B^{T} P E \] 
\[ + E^{T} P B \left( \frac{1}{T} B^{T} P E + u_{a} - d + w_{e} \right) \] 
\[ + \frac{1}{\gamma_{f}} \Phi_{f}^{T} \dot{\phi}_{f} + \gamma_{f}^{2} E^{T} P B C(X) \] 
\[ + \frac{1}{\gamma_{g}} \Phi_{g}^{T} \dot{\phi}_{g} + \gamma_{g}^{2} E^{T} P B C(X) \]  
(12)
Then by using the following control and adaptation laws proposed in (Hamzaoui, et al., 2000):
\[ \begin{align*}
\dot{u}_{a} & = \frac{1}{\gamma} E^{T} P B \\
\dot{\theta}_{f} & = -\gamma_{f} E^{T} P B C(X) \\
\dot{\theta}_{g} & = -\gamma_{g} E^{T} P B C(X) \\
\end{align*} \]  
(13)
we obtain:
\[ \dot{V} \leq -\frac{1}{2} E^{T} Q E + \rho \frac{3}{2} \left( w_{e} - d \right)^{2} \]  
(14)
In this case, the parameters \( \theta_{f} \) and \( \theta_{g} \) are not guaranteed to be bounded, which means that \( w_{e} \) is not bounded. The modified algorithm, proposed in the following subsection, has therefore been developed to obtain a stable system.

### 3.2 Modified adaptive fuzzy algorithm

Let the constraint sets \( \Omega_{f} \) and \( \Omega_{g} \) be defined as:
\[ \Omega_{f} = \left\{ \theta_{f} \mid \theta_{f} \leq M_{f} \right\} \]  
\[ \Omega_{g} = \left\{ \theta_{g} \mid 0 < \theta_{g} \leq M_{g} \right\} \]  
(15)
where \( M_{f}, \epsilon, \) and \( M_{g} \) are constants. Since \( \hat{g} \neq 0, \mid \theta_{g} \mid \) must be bounded from below by \( \epsilon > 0 \).

We therefore propose the modified adaptation law where \( u_{a} \) is the same as in (13), but \( \theta_{f} \) and \( \theta_{g} \) are calculated, using the projection technique (Goodwin and Mayne, 1987), as follows:
\[ \dot{\theta}_{f} = \begin{cases} -\gamma_{f} E^{T} P B C(X) & \text{if } \left\| \dot{\theta}_{f} \right\| < M_{f} \\
\dot{\theta}_{f} & \text{or } \left\| \dot{\theta}_{f} \right\| = M_{f} \text{ and } E^{T} P B C(X) > 0 \end{cases} \]  
(16)
\[ \dot{\theta}_{g} = \begin{cases} -\gamma_{g} E^{T} P B C(X) & \text{if } \left\| \dot{\theta}_{g} \right\| < M_{g} \\
\dot{\theta}_{g} & \text{or } \left\| \dot{\theta}_{g} \right\| = M_{g} \text{ and } E^{T} P B C(X) < 0 \end{cases} \]  
(17)
where the projection operator \( Pr_{[\cdot]} \) is defined as:
\[ Pr_{[\cdot]} = -E^{T} P B C(X) + E^{T} P B \frac{\theta}{\left\| \theta \right\|} \]  
If an element \( \theta_{j} \) of \( \theta_{g} \) is equal to \( \epsilon \), then:
\[ \dot{\theta}_{g} = \begin{cases} -\gamma_{g} E^{T} P B C(X) & \text{if } E^{T} P B C(X) < 0 \\
0 & \text{if } E^{T} P B C(X) > 0 \end{cases} \]  
(18)
3.3 Stability and robustness analysis

Since the convergence of the parameters \( \theta_{f} \) and \( \theta_{g} \) is guaranteed by this modified adaptation law, the next step is to guarantee the convergence of the tracking error toward zero.

From (12), the control law (13) gives:
\[ \int_{0}^{t} E^{T} Q E d\tau \leq \frac{2}{\lambda_{\text{min}}(Q)^{-1}} \left( \left\| v_{0} \right\| - \sup \text{ess} \left\| v(\cdot) \right\| \right) \] 
\[ + \frac{1}{\lambda_{\text{min}}(Q)^{-1}} E^{T} P B \int_{0}^{t} \left( \left\| v(\cdot) \right\| - d \right) d\tau \]  
(19)
where \( \lambda_{\text{min}}(Q) \) is the minimum eigenvalue of \( Q \). Let’s choose \( Q \) such that \( \lambda_{\text{min}}(Q) > 1 \). The existence of the integral \( \int_{0}^{t} E^{T} Q E d\tau \) implies that \( \lim_{\tau \to t} E^{T} Q E = 0 \). So, the convergence of the tracking error toward zero depends only on the term \( \left\| v_{0} \right\| - \sup \text{ess} \left\| v(\cdot) \right\| \). We require that \( E^{T} Q E \leq \bar{V}, \) where \( \bar{V} \) is a constant specified by the designer. Then, after some straightforward manipulations, we obtain:
\[ \dot{V} = -\frac{1}{2} E^{T} Q E - \frac{1}{2} \left( \frac{E^{T} P B + \rho d}{\rho} \right)^{2} + \frac{\rho^{2} \left\| d \right\|}{2} \] 
\[ + E^{T} P B \left( \dot{f} - f \right) + \dot{g} - g \]  
\[ \geq \frac{\rho^{2} \left\| d \right\|}{2} + E^{T} P B \left( \dot{f} - f \right) + \dot{g} - g \]  
(20)
Using the control law (13), (20) can be rewritten as:
\[ \dot{V} = -\frac{1}{2} E^{T} Q E - \frac{1}{2} \left( \frac{E^{T} P B + \rho d}{\rho} \right)^{2} \] 
\[ + \frac{\rho^{2} \left\| d \right\|}{2} + E^{T} P B \left( \dot{f} - f \right) + \dot{g} - g \]  
(21)
Since the first term is negative, a good choice of the attenuation factor, \( \rho \), results in a small value for the term \( \left( \rho \left\| d \right\| \right)^{2} /2 \). Since the sign of
\[
E^T P B \left( f - \dot{f} \right) + \left( g - \dot{g} \right) u \] is unknown, we append a supervisor, \( u_s \), to obtain the overall control signal:
\[
u = u + u_s \tag{22}
\]
We now show how to determine \( u_s \) such that \( \dot{V} \leq 0 \) when \( \dot{V} > 0 \).

Substituting (22) into (1) and after some manipulations, the new error equation becomes:
\[
\dot{E} = AE + Bf \left( f + \dot{f} \right) + g_u + u_s - d
\]
(23)

using (23) and (7), we obtain:
\[
\dot{V} = -\frac{1}{2} E^T Q E - \left( \frac{1}{2} E^T P B \rho + \rho d \right)^2 + \rho^2 \left| p \right|^2 / 2 + E^T P B \left( f - \dot{f} \right) + \left( g - \dot{g} \right) u - g_u
\]
and therefore,
\[
\dot{V} \leq -\frac{1}{2} E^T Q E - \left( \frac{1}{2} E^T P B \rho + \rho d \right)^2 + \rho^2 \left| p \right|^2 / 2 + E^T P B \left( f - \dot{f} \right) + \left( g - \dot{g} \right) u
\]
(24)

In order to design \( u_s \) such that the last term of (24) is nonpositive, we need to know the bounds of \( f \) and \( g \), i.e., we have to determine the functions \( f^M(X) \) and \( g^M(X) \), and \( g_u(X) \) such that \( |f(X)| \leq f^M(X) \) and \( |g(X)| \leq g^M(X) \) for \( X \in U_c \).

Consequently, the supervisory control, \( u_s \), is chosen as follows:
\[
u_s = 1 \text{sgn}\left(E^T P B \rho \right) + \left| f^M + \left| g^M \right| u \right|
\]
(25)

where \( I = 1 \) if \( \dot{V} > 0 \), \( I = 0 \) if \( \dot{V} \leq 0 \), and \( \text{sgn}(y) = 1 \) (respectively, -1) if \( y > 0 \) (respectively, \( y < 0 \)).

Substituting (25) into (24) and considering the case \( \dot{V} \leq 0 \), we have:
\[
\dot{V} \leq -\frac{1}{2} E^T Q E - \left( \frac{1}{2} E^T P B \rho + \rho d \right)^2 + \rho^2 \left| p \right|^2 / 2 + E^T P B \left( f - \dot{f} \right) + \left( g - \dot{g} \right) u
\]
(26)

which guarantees that \( \dot{V} \leq 0 \).

From (12), (16)-(18) and (7), we obtain the same inequality given in (14).

Integrating (14) from \( t=0 \) to \( T \) yields:
\[
V(T) - V(0) \leq -\frac{1}{2} E^T Q E + \int_0^T \rho^2 \left( w_e - d \right)^2 / 2 dt
\]
(27)

Since \( V(T) \geq 0 \), the above inequality implies that:
\[
\frac{1}{2} \int_0^T \rho^2 \left( w_e - d \right)^2 / 2 dt \leq \int_0^T E^T Q E dt \]
(28)

From (14), the (28) is equivalent to:
\[
\frac{1}{2} \int_0^T \rho^2 \left( w_e - d \right)^2 / 2 dt + \frac{1}{2} \int_0^T \rho \phi \psi \phi \psi / 2 dt
\]
(29)

This is our \( H_\infty \) criterion.

3.4 Design procedure

In order to minimise the on-line computing time of our algorithm, the design of the robust adaptive fuzzy controller implies an off-line processing step, and an on-line during control execution as shown below:

- On-line processing
  - Apply \( u_s = u + u_u \), where \( u \) is given by (4) and \( u_u \) by (25).
  - Use the adaptation law, given by (16)-(18), to adjust the parameters.

4. SIMULATION EXAMPLE

To validate our approach, we consider the inverted pendulum depicted in Fig. 1.

\[
\dot{x}_1 = \frac{\sin \theta_1 - \frac{\pi}{2} \cos \theta_1}{\frac{1}{m_c} + \frac{1}{m} + \frac{1}{m_c + m}} \]
\[
\dot{x}_2 = \frac{\cos \theta_1 - \frac{\pi}{2} \sin \theta_1}{\frac{1}{m_c} + \frac{1}{m} + \frac{1}{m_c + m}} \]
\[
y = \frac{g - \sin \theta_1}{\frac{1}{m_c} + \frac{1}{m} + \frac{1}{m_c + m}} + \frac{d}{m_c + m}
\]

Fig. 1. The inverted pendulum system.

Let \( x_1 = \theta \) and \( x_2 = \dot{\theta} \). The dynamic equation of the inverted pendulum as is given by (Wang, 1996):
\[
x_1 = x_2
\]
\[
x_2 = \frac{\sin \theta_1 - \frac{\pi}{2} \cos \theta_1}{\frac{1}{m_c} + \frac{1}{m} + \frac{1}{m_c + m}} + y
\]
\[
y = \frac{g - \sin \theta_1}{\frac{1}{m_c} + \frac{1}{m} + \frac{1}{m_c + m}} + \frac{d}{m_c + m}
\]

where \( g \) is the acceleration due to gravity, \( m_c \) is the mass of the cart, \( m \) is the mass of the pole, \( l \) is the half-length of the pole, the force \( u \) represents the control signal, and \( d \) is the external disturbance. We choose \( m_c = 1\text{Kg}, m = 0.1\text{Kg} \) and \( l = 0.5\text{m} \) in the following simulations. The reference signal is assumed here to be \( y(t) = (\pi/30)\sin(t) \), and the system is subject to two disturbances:

- A structural disturbance on the mass of both the cart and the pole, in the form: \( dm = 0.01\text{m} \sin(t) \)

- An external disturbance: \( d(t) = 0.1\sin(t) \)

If we require:
\[
|y| \leq \frac{\pi}{6}, |x| = 180 \tag{31}
\]

and substituting the functions \( \sin(.) \), and \( \cos(.) \) by their limited development we can determine the bounds:
\[
f^M \left( x_1, x_2 \right) = 15.78 + 0.366x_2^2
\]
\[
g^M \left( x_1, x_2 \right) = 1.46, \quad g_{\text{ub}} \left( x_1, x_2 \right) = 1.12 \tag{32}
\]

To satisfy (3) and (19), we choose, for example, \( k_1 = 2, k_2 = 1 \) and \( Q = \text{diag}(10, 10) \). Furthermore to simplify the calculation, we choose \( r = 2\rho^2 \). So, the solution of the algebraic Ricatti equation is:
\[
P = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}
\]
and \( \lambda_{\text{max}}(P) = 2.93 \). To satisfy the
We select 5 Gaussian membership functions for both $x_1$ and $x_2$ (i=1,2) to cover the whole universe of discourse:

$$
\mu^1_i(x_i) = \exp\left( -\frac{(x_i - \pi/6)}{\pi/24} \right)^2
$$

$$
\mu^2_i(x_i) = \exp\left( -\frac{(x_i - \pi/12)}{\pi/24} \right)^2
$$

$$
\mu^3_i(x_i) = \exp\left( -\frac{(x_i + \pi/6)}{\pi/24} \right)^2
$$

(33)

After trial and errors, $\gamma_f=50$ and $\gamma_c=1$ are chosen. The MATLAB command “ode23s” is used to simulate the overall control system with step size 0.01.

The initial position of the pendulum is chosen as far as possible (\(\theta(0)=\pi/12\)) to improve the efficiency of our algorithm.

Two cases are considered to show the influence of the incorporation of the linguistic rules in the control law:

Case one: the initial values of $\theta_1$ and $\theta_2$ are chosen arbitrarily.

Case two: the initial values of $\theta_1$ and $\theta_2$ are deduced from the fuzzy rules describing the dynamic behaviour of the system. For example, if we consider the unforced system, i.e., $u(t)=0$, the acceleration is equal to $f(x_1,x_2)$. So, intuitively, we state that:

“The bigger is $x_1$, the larger is $f(x_1,x_2)$”

Our task now is to transform this fuzzy information into a fuzzy rule. We obtain the rule:

$$
R^f_1: \text{IF } x_1 \text{ is } F^3_1 \text{ and } x_2 \text{ is } F^5_2
$$

THEN $f(x_1,x_2)$ is Positive Big

where “Positive Big” is a fuzzy set whose membership function is $\mu^f_1(x_1)$ given in (33). The acceleration is proportional to the gravity, i.e. $f(x_1,x_2)=\alpha \cdot \sin(x_1)$, where $\alpha$ is a constant. Since $f(x_1,x_2)$ achieves its maximum at $x_1=\pi/2$; thus based on (32), we have $\alpha=16$. Therefore, we the final fuzzy rules characterizing $f(x_1,x_2)$ as shown in fig. 2, which comprises 25 rules.

Now, to determine the fuzzy rules for $g(x_1,x_2)$, we use the following observation:

“The smaller is $x_1$, the larger is $g(x_1,x_2)$”

Similarly to the case of $f(x_1,x_2)$ and based on the bounds (32), this observation can be quantified into the 25 fuzzy rules given in fig. 3.

<table>
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<th>$g(x_1,x_2)$</th>
<th>$x_1$</th>
<th>$x_2$</th>
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<td>-4</td>
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<tr>
<td>$F^5_1$ $\pi/6$</td>
<td>-8</td>
<td>-4</td>
</tr>
<tr>
<td>$F^5_2$ $\pi/12$</td>
<td>-8</td>
<td>-4</td>
</tr>
</tbody>
</table>

Fig. 3. Linguistic rules for $g(x_1,x_2)$

To obtain the same tracking performances, the attenuation level, $\rho$, was equal to 0.2, in the first case and to 0.8 in the second.

For both the two cases fig. 4 illustrates the tracking performance for a sinusoidal trajectory; the pendulum reaches the reference trajectory in 3.14s. The quadratic error is given by the fig. 5.

Fig. 4. The state $x_1$ (solid line) and its desired value $y_i(t)$ (dashed line) for $X(0)=\{\pi/12,0\}^T$

Fig. 5. The quadratic error

Figures 6 and 7 show the difference between the control signal $u_1$ and $u_2$ in the two cases, respectively. As shown in fig.7, when we incorporate the linguistic rules in the controller, and with a high level of attenuation, the initial global control is much smaller than the control signals proposed in (Hamzaoui, et al., 2000) and (Chen, et al., 1996).
5. CONCLUSION

An adaptive fuzzy controller with two supervisors is proposed for the control of a class of nonlinear systems subject to large uncertainties or to unknown variations in the parameters and the structure of the plant. The projection theorem is used to guarantee the convergence of the adaptation laws corresponding to fuzzy approximators. The first supervisor, \(u_s\), ensures the global stability of the system, in the sense of lyapunov. The second supervisor, \(u_a\), uses \(H_\infty\)-technique to attenuate the effect of both external disturbances and approximation errors to a prescribed level. The stability and the robustness are demonstrated analytically, and an illustrative example has been used to show the efficiency of the proposed method. The performances of the approach can be improved by incorporating some linguistic rules. However, the design of the control algorithm needs a good knowledge of the dynamic behaviour of the system in order to determine both the bounds and the linguistic rules of the functions \(f\) and \(g\). Further work is under investigation to apply the proposed robust adaptive algorithm to multi-input multi-output systems.

REFERENCES


