GLOBAL OBSERVABILITY ANALYSIS OF INDUCTION MOTORS UNDER SENSORLESS CONDITIONS

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Abstract: The problem of achieving high performance from induction motor based drives - without sensing mechanical variables, i.e. high performance sensorless control - is a widely studied topic, specially in the last decade. Several works point out some operating regimes where reliable control and speed estimation fail. However, a complete understanding of the nature of these limitations does not exist due to the lack of a formal analysis of the observability properties of this kind of machines. In this paper a method is proposed to make a global analysis of the observability properties of the induction motor under sensorless conditions. The work aims at providing a formal answer to the fundamental question about the existence of indistinguishable internal trajectories for that electric machine. It is shown that there exist diverging trajectories corresponding to the same input/output behavior of the system, i.e. they are indistinguishable from the external signals. This implies the impossibility of constructing an observer that converges for every trajectory of the motor. This indistinguishable trajectories are input dependent, and the system cannot be decomposed into an observable and a non observable part. We give a complete characterization of the set of all indistinguishable trajectories, called indistinguishable dynamics. This description is valid for all values of the parameters.

Keywords: Induction motors, Nonlinear control, Observability, Observers, Motor control.

1. INTRODUCTION

Induction motor control is a topic that in the last decades attracted the interest of the engineering community (both applied and theoretical) due, on the one hand, to the simple and cheaper structure of this kind of machines and, on the other hand, to the availability of low-cost high-performance digital signal processors, that make feasible the implementation of complex nonlinear control laws in a (relatively) simple way. The maturity reached in the field is such that currently it is possible to find industrial applications based on induction motors where the performance required is quite stringent (Bose, 1994; Taylor, 1994). Moreover, the insight gained in understanding some of the usual controllers currently used in practice (like

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1 This work has been done with the financial aid of Conacyt under project 34934A, and DGAPA-UNAM under projects PAPIIT IN115600 and IN106901.
2 Also with PIMAyC – Instituto Mexicano del Petróleo. Apartado Postal 14-805, 07730, México D.F., MEXICO.
field oriented control), makes possible to carry out tuning and commissioning also in a very simple way (Chang et al., 2000).

Despite of the important aforementioned advances and as a result of a natural evolution in industrial applications, in the last decade (mainly) some efforts appeared with the aim of reducing the complexity of induction motor based drives. As a result the technique called (shaft) sensorless control was proposed. Its two main features are: The control must be carried out without sensing mechanical variables and the performances achieved by the proposed controllers must be similar to the obtained with sensored techniques. This objective was motivated by the industrial concern of dropping costs in the production process, although from a technical point of view several advantages can also be found, e.g. minimization of the sensor failure probability due to the reduced number of these devices.

Since several high performance sensored controllers are at disposition, the main research effort in sensorless control has focused on searching for reliable speed estimation methods. In this area a lot of work has been reported and currently it is possible to (roughly) classify the contributions into four approaches: rotor slot ripple method, high frequency current injection method, extended Kalman filter technique and model reference adaptive control (Rajashekara et al., 1996).

Each of the proposed sensorless controllers has particular properties regarding the performance they can exhibit, and a very interesting discussion can be established about which of them is better, as in fact has been done in several publications (see for example (Lorenz, 1999)). However, a topic that, although mentioned in all the reported results, has not been fairly clarified is the one related with the applicability conditions of the algorithms, i.e. conditions when speed estimation fails. This complex problem, evidently related to the observability properties of the induction motor, has been only approached from a heuristic or practical perspective. It has been pointed out, for example, that operation of the control system at low speed can lead to instability (or at least to very poor performances (Asher, 2000)) and also that constant (or low frequency) flux operation (Montanari et al., 2000) leads to control difficulties. Operation at nominal speed with a field oriented control can also exhibit undesirable behavior (Harnefors, 2000). Several remedies have been proposed to alleviate these disadvantages (Asher, 2000), but no formal analysis has been yet done to clearly understand the nature of such undesirable phenomena, except the local observability analysis reported in (Canudas et al., 2000), where the problems of working at low frequencies is studied.

Considering the conditions described above, it is important to study the observability properties of the induction motor in order to understand the theoretical possibilities and limitations of sensorless control strategies. Moreover, since practical difficulties have been reported involving several operation conditions, it is desirable to develop as general as possible the required observability analysis. In this paper some initial steps towards this objective are given by presenting a method to carry out a global analysis of the observability properties of the induction motor under sensorless conditions. This analysis exploits at a fundamental level the indistinguishable trajectories concept, i.e. internal trajectories of a system that are different under the same input/output behavior. The main contribution is the statement of a dynamical system whose behavior, called indistinguishable dynamics, completely characterize all the trajectories that make non observable the induction electric machine. Unfortunately, the analysis of such a system, as usual with dynamical nonlinear systems, is highly complex and (at the current status of this research work) it is not possible to explicitly present all the aforementioned trajectories. However, it is worth pointed out that even under these conditions, several important results can be obtained regarding the indistinguishable dynamics. Namely:

- A formal answer to the fundamental question about the existence of indistinguishable trajectories for the induction machine is provided. It is shown that for every set of physically meaningful parameters there are indistinguishable trajectories.
- Since there exist always divergent indistinguishable trajectories, it follows that there are always operating regimes for which any observer fails to converge, i.e. it is impossible to construct an observer for every trajectory of the motor.
- These dynamics are input dependent, hence the system cannot be decomposed in an observable and a non observable part.
- Once the structure of the dynamical system that generates these dynamics is at hand, standard analysis tools can be applied in order to obtain a deeper understanding on the observability properties of the induction motor.

To illustrate the usefulness and validity of the results, several explicit trajectories are calculated from the indistinguishable dynamics. Some of these results recover well-known results in the field, but, more importantly, there are other that shed more light to the device operation.
The paper is organized as follows: Section 2 is devoted to the presentation of the induction motor model considered in the paper, the rationale behind the construction of the dynamical nonlinear system that generates the indistinguishable dynamics and the explicit form of these dynamics. In section 3, several special cases of indistinguishability are presented, from some trivial cases (zero flux operation) to others more difficult to analyze (fixed input voltage). Finally some concluding remarks are presented in section 4.

2. INDISTINGUISHABLE DYNAMICS

Observability is the lack of indistinguishability: Two (state) trajectories of a (nonlinear) system are called indistinguishable if they are different, although the input and output signals are identical. In this paper the possible indistinguishable trajectories of the induction machine under sensorless conditions will be studied. First, the mathematical model will be presented and then a method to find the indistinguishable trajectories will be introduced and used for the induction motor.

2.1 Mathematical model of the induction motor

A detailed model of the unsaturated induction motor is given by (Meisel, 1966; Marino and Tomei, 1995)

\[
\Sigma: \begin{cases} 
\dot{\psi} = -f \psi + a \psi_T T_L i - \frac{T_L}{J} \\
\dot{i} = -a \psi - n_p \omega \psi + M a \psi \\
\frac{di}{dt} = \beta [a \psi + n_p \omega \psi - (Ma + b) i + cu] 
\end{cases}
\]

(1)

where: rotor speed \( \omega \), rotor fluxes \( \psi \), and stator currents \( i \) are the states; load torque \( T_L \) and stator voltages \( u = (u_a, u_b) \) are the input variables; rotor inertia \( J > 0 \), stator and rotor inductances \( (L_s, L_r) > 0 \), mutual inductance \( M > 0 \), stator and rotor resistances \( (R_s, R_r) > 0 \), the rotor friction \( f \geq 0 \) and the number of pole pairs \( n_p > 0 \) are the parameters. Furthermore,

\[
\psi = \begin{bmatrix} \psi_a \\ \psi_b \end{bmatrix}, \quad i = \begin{bmatrix} i_a \\ i_b \end{bmatrix}, \quad J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad I\!I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

and \( \alpha = \frac{n_p M}{J L_r}, \quad a = \frac{R_r}{L_r}, \quad b = \frac{L_s R_r}{M}, \quad c = \frac{L_r}{M}, \quad \beta = \frac{M}{L_r L_r - M \tau} \).

2.2 The indistinguishable dynamics

Consider that \( w \) is not measured (sensorless), that the parameters of the system are known, and that \((i, u, T_L)\) are measured. Are the trajectories of the system distinguishable? i.e. do different state trajectories \( (\omega, \psi) \) correspond to different external trajectories \((i, u, T_L)\)? To answer this question let us consider two identical inductor motor models with the same external signals \( (T_{L1} = T_{L2}, u_1 = u_2, i_1 = i_2) \), i.e.

\[
\begin{align*}
\Sigma_1: & \quad \dot{\psi}_1 = -a \psi_1 - n_p \omega_1 \psi_1 + M a i \\
& \quad \frac{di}{dt} = \beta [a \psi_1 + n_p \omega_1 \psi_1 - (Ma + b) i + cu] \\
\Sigma_2: & \quad \dot{\psi}_2 = -a \psi_2 - n_p \omega_2 \psi_2 + M a i \\
& \quad \frac{di}{dt} = \beta [a \psi_2 + n_p \omega_2 \psi_2 - (Ma + b) i + cu]
\end{align*}
\]

These equations single out the indistinguishable trajectories of the motor, since two of them satisfy \( \Sigma_1 \) and \( \Sigma_2 \), and viceversa, two trajectories that satisfy \( \Sigma_1 \) and \( \Sigma_2 \) are indistinguishable trajectories of the machine. Introducing as new variables \( \epsilon \equiv \omega_1 - \omega_2 \) and \( \Delta \equiv \psi_1 - \psi_2 \) the equations describing the indistinguishable trajectories can be written as

\[
\begin{align*}
\dot{\omega} &= -f \omega + a \psi_T T_L i - \frac{T_L}{J} \\
\dot{\psi} &= -a \psi - n_p \omega \psi + M a \psi \\
\frac{di}{dt} &= \beta [a \psi + n_p \omega \psi - (Ma + b) i + cu] \\
\dot{\epsilon} &= -f \epsilon + \alpha \Delta^T T_L i \\
\dot{\Delta} &= 0 \\
0 &= (a \epsilon + n_p (\omega - \epsilon) \Delta + n_p \epsilon \psi)
\end{align*}
\]

This system of Differential-Algebraic Equations (DAE) represent a clamped dynamics, i.e. a dynamic system with algebraic restrictions. Their solutions represent the indistinguishable trajectories of the induction motor, called in the sequel Indistinguishable Dynamics.

2.3 Explicit form of the Indistinguishable Dynamics

From the DAE (2) an equivalent differential equation can be derived, that gives an explicit representation of the indistinguishable dynamics, and that is more amenable for analysis than the implicit system described by (2). From the algebraic constraint of (2) and its first time derivative the flux \( \psi \), and the stator current \( i \) for an indistinguishable trajectory can be expressed (explicitly) in terms of the other state variables of (2) \( \Delta, \epsilon \), and \( \omega \) as

\[
\begin{align*}
\psi &= \frac{1}{n_p} (a \epsilon - n_p (\omega - \epsilon) \Delta) \\
i &= (\phi_1 \epsilon + \phi_2) \Delta + \frac{n_p}{J \psi} T_L \Delta
\end{align*}
\]

where
\[
\phi_1 \triangleq \frac{(\epsilon - 2\omega)n_p}{\Psi_2} \left( \frac{\alpha}{\epsilon} \Delta^T \Delta \phi_2 + \alpha \right), \\
\phi_2 \triangleq \frac{1}{\Psi_1} \left[ a (a + f) - n_p^2 \omega (\omega - \epsilon) \right], \\
\Psi_1 \triangleq \frac{Ma_n\epsilon}{\epsilon} - \frac{aa}{\epsilon} \Delta^T \Delta, \\
\Psi_2 \triangleq \frac{Ma_n\epsilon}{\epsilon} + \frac{aa}{\epsilon} \Delta^T \Delta.
\]

Notice that this is only valid on the manifold defined by

\[
M = \{ (\omega, \epsilon, \Delta) \in \mathbb{R}^4 \mid \epsilon \neq \pm \sqrt{\frac{\alpha}{Mn_p}} \Delta^T \Delta, \epsilon \neq 0 \}
\]

Since \( \epsilon = 0 \) does not represent any indistinguishable trajectory (as can be seen by replacing in (2)), the only true restriction is the assumption that \( \epsilon \neq \pm \sqrt{\frac{\alpha}{Mn_p}} \Delta^T \Delta \). Using (3) and (4) the DAE (2) can be re-written on \( M \) as

\[
\dot{\omega} = -f \omega + \rho (\omega, \epsilon, \Delta^T \Delta) - \frac{Mn_p\epsilon^2 T_L}{f (Mn_p^2 + \alpha \Delta^T \Delta)} \\
\dot{\epsilon} = - \left[ f + \frac{\alpha \Delta^T \Delta \{ a (a + f) - n_p^2 \omega (\omega - \epsilon) \}}{Ma_n \epsilon^2 - aa \Delta^T \Delta} \right] \epsilon \\
\dot{\Delta} = 0
\]

(5)

with

\[
\rho (\omega, \epsilon, \Delta^T \Delta) \triangleq \frac{\alpha \Delta^T \Delta p (\omega, \epsilon, \Delta^T \Delta)}{a \left( \frac{Mn_p\epsilon^2}{(Mn_p^2 + \alpha \Delta^T \Delta)^2} \right)} + \{ a (\epsilon - 2\omega) \Delta^T \Delta + 2 \} \left( \frac{Mn_p \epsilon^2}{\alpha \Delta^T \Delta} \right)
\]

The DE (5) is equivalent to the DAE (2) on the manifold \( M \). Any solution of (5) represents a couple of indistinguishable trajectories of the motor. Notice that the initial conditions for \( (\Delta, \omega, \epsilon) \), and the time function \( T_L \), the load torque, can be chosen arbitrarily as long as the solutions of (5) stay on \( M \). Moreover, all other variables of the motor, i.e. \( \psi, i, u \) are functions of the variables of (5), \( (\omega, \epsilon, \Delta, T_L) \). The advantage of working with (5) instead of (2), is that the former is a reduced order (equivalent) representation of the latter. Moreover, (5) is a differential equation, which is more amenable to be analyzed than the DAE (2). The main result of the paper is

**Theorem 1.** Consider that for the induction motor (1) all parameters are known, and the variables \( (i, u, T_L) \) are measured. The solutions of system (5) are the indistinguishable trajectories which live on the manifold \( M \)

**Proof.** Since the calculations are long and involved the detailed proof has not been given here. All the details can be found in the report (Moreno et al., 2001).

**Remark 2.** Notice that for any given set of trajectories of the indistinguishable dynamics, the input that generates them is given as

\[
u_1 = \frac{g_1 (\pi, \Delta, T_L) [\omega, \epsilon]}{g_3 (\pi, \Delta, T_L) [\omega, \epsilon]} \quad \text{and} \quad \nu_2 = \frac{g_2 (\pi, \Delta, T_L) [\omega, \epsilon]}{g_3 (\pi, \Delta, T_L) [\omega, \epsilon]}
\]

where \( g_i (\pi, \Delta, T_L) [\omega, \epsilon], i = 1, 2, 3 \), are polynomials in \( \omega \) and \( \epsilon \), with coefficients that are functions of the parameters \( \pi \) of the system, the load torque \( T_L \) and \( \Delta \).

**Remark 3.** Note that the existence of indistinguishable trajectories is somehow ubiquitous: for every initial rotor velocity \( \omega (0) \) and every torque load function \( T_L (t) \) there exist a lot of indistinguishable trajectories, i.e. the ones obtained by selecting \( \Delta (0) \), and \( \epsilon (0) \) in the indistinguishable dynamics.

**Remark 4.** From the second equation of (5) it follows that if \( \epsilon (0) \gtrless 0 \Rightarrow \epsilon (t) \gtrless 0 \) for all \( t > 0 \). This is so because for \( \epsilon = 0 \) then \( f + \frac{\alpha \Delta^T \Delta \{ a (a + f) - n_p^2 \omega (\omega - \epsilon) \}}{Ma_n \epsilon^2 - aa \Delta^T \Delta} \epsilon = 0 \).

**Remark 5.** It is interesting to note that for every pair of indistinguishable trajectories of the induction machine the difference of the fluxes \( \Delta \) is constant. This allows to consider \( \Delta \) in (5) as a parameter and to look at the fourth order indistinguishable dynamics as a second order one, parameterized by \( \Delta \), and with an input \( T_L \). In fact, since (5) only depends on \( \Delta^T \Delta = \| \Delta \|_2^2 \), the euclidean norm of \( \Delta \), the parameter is the norm of the difference of the indistinguishable fluxes.

3. SOME SPECIAL CASES OF INDISTINGUISHABILITY

It is clear that all possible indistinguishable trajectories of the induction machine can be obtained from (5), except the ones living outside the manifold \( M \). A complete study of its properties would give us all possible forms of unobservability of the system. This is, however, not an easy task and we will give some special cases in this section.

3.1 Zero Flux operation

Note that when \( \epsilon = 0 \) there is no indistinguishable dynamics, as discussed before. However, if \( \Delta = 0 \) (and \( \epsilon \neq 0 \)) then \( \psi = 0 \), and
\[
\dot{\omega} = -f_\omega - \frac{1}{J} T_L
\]
\[
\dot{\epsilon} = -f_\epsilon
\]
\[
\dot{\Delta} = 0
\]

This means that any velocity \( \omega \) is indistinguishable. If \( f > 0 \), then the indistinguishable velocities converge, i.e. there is detectability, and the velocity can be asymptotically reconstructed. If, however \( f = 0 \), then the indistinguishable velocities have a constant difference.

### 3.2 Constant velocity operation

For the manifold \( \mathcal{M} \), if in (5) the load torque is selected as
\[
T_L = \frac{J (Mn_p e^2 + \alpha \Delta^T \Delta)}{Mn_p e^2} (-f_\omega + \rho (\omega, \epsilon, \Delta^T \Delta))
\]
then the velocity will be constant and the indistinguishable dynamics is given by
\[
\dot{\omega} = 0
\]
\[
\dot{\epsilon} = -\left[ f + \frac{\alpha \Delta^T \Delta \left( a (a + f) - n_p^2 \omega (\omega - \epsilon) \right)}{Mn_p e^2 - \alpha \Delta^T \Delta} \right] \epsilon
\]
\[
\dot{\Delta} = 0 .
\]

This is a one dimensional dynamics in \( \epsilon \) parameterized by the constants \( \omega \) and \( \Delta^T \Delta \). Its equilibrium points are given by the (real) solutions of
\[
fMn_p e^2 + \alpha \Delta^T \Delta n_p^2 \omega \epsilon + \alpha \Delta^T \Delta (a^2 - n_p^2 \omega^2) = 0 .
\]

Remember that \( \epsilon = 0 \) does not belong to the indistinguishable dynamics.

For the equilibrium points on the manifold \( \mathcal{M} \) the values of \( \omega, \epsilon, \) and \( \Delta \) are constant. The values of the other variables of the indistinguishable dynamics are
\[
\psi = \frac{1}{n_p \epsilon} (a \| - n_p (\omega - \epsilon) \|) \Delta ,
\]
\[
i = (\phi_1 \| + \phi_2 \|) \Delta + \frac{n_p}{\Psi_2} T_L \Delta ,
\]
\[
u = g (\pi, \Delta, T_L, \omega, \epsilon) .
\]

Note that at the equilibrium points all these values are constant.

We will consider the cases when \( f = 0 \), and \( f \neq 0 \).

#### 3.2.1. Case \( f = 0 \): In this case equation (7) is satisfied by
\[
\epsilon = \frac{(n_p^2 \omega^2 - a^2)}{n_p^2 \omega^2} \quad \text{if} \quad \omega \neq 0 .
\]
If \( \epsilon \neq 0 \) then the corresponding equilibrium point of the indistinguishable dynamics represents an indistinguishable trajectories of the machine.

If \( \omega = 0 \) there is no solution to (7), and no equilibrium points exist (for indistinguishable dynamics). The indistinguishable dynamics in this particular case is given by
\[
\dot{\omega} = 0
\]
\[
\dot{\epsilon} = -\left[ \frac{\alpha \Delta^T \Delta}{Mn_p e^2 - \alpha \Delta^T \Delta} \right] \epsilon
\]
\[
\dot{\Delta} = 0 .
\]

#### 3.2.2. Case \( f > 0 \): In this case equation (7) is satisfied by
\[
\epsilon = -\frac{\alpha \Delta^T \Delta n_p^2 \omega \pm \sqrt{(\alpha \Delta^T \Delta n_p^2 \omega)^2 - \chi}}{2f M n_p} \]
where
\[
\chi = 4fMn_p \alpha \Delta^T \Delta (a^2 - n_p^2 \omega^2)
\]
These solutions correspond to indistinguishable trajectories if they are real and different from zero.

### 3.3 Constant load torque

In this case the indistinguishable dynamics is an autonomous system, and its solutions can be plotted in the phase plane \( (\omega, \epsilon) \), parameterized by \( \Delta \), and \( T_L \).

#### 3.4 Constant load torque and velocity operation

In this case, \( \omega, \epsilon \) and \( T_L \) are constant, for the manifold \( \mathcal{M} \), the values of all the variables are constant for the indistinguishable dynamics, i.e. \( u, i, \psi \) are constant. This can be easily obtained from the indistinguishable dynamics (5). This is an important observation, since it implies that there is (local and global) observability under steady state conditions iff the applied voltage is not constant.

### 3.5 Fixed voltage function

Usually the machine is excited with a fixed voltage function, i.e. an (sinus) armonic signal. Notice that in the indistinguishable dynamics the voltage function is determined by the trajectory of the differential equations (5). If, however, it is fixed as a time function, then the equation for \( u \) becomes a restriction, leading to an indistinguishable dynamics given by
\[
\dot{\omega} = -f_\omega + \rho (\omega, \epsilon, \Delta^T \Delta) - \frac{Mn_p e^2 T_L}{J (Mn_p e^2 + \alpha \Delta^T \Delta)}
\]
\[
\dot{\epsilon} = -\left[ f + \frac{\alpha \Delta^T \Delta \left( a (a + f) - n_p^2 \omega (\omega - \epsilon) \right)}{Mn_p e^2 - \alpha \Delta^T \Delta} \right] \epsilon
\]
\[
\dot{\Delta} = 0 .
\]
\[
g_1 (\pi, \Delta, T_L) [\omega, \epsilon] - u_a (t) g_2 (\pi, \Delta, T_L) [\omega, \epsilon] = 0
\]
\[
g_2 (\pi, \Delta, T_L) [\omega, \epsilon] - u_b (t) g_3 (\pi, \Delta, T_L) [\omega, \epsilon] = 0
\]
i.e. a DAE, where the last two equations are algebraic ones that depend on the time. If $\pi$, $\Delta$, and $T_L$ are fixed, then the algebraic equations generically have for each time a finite number of solutions, which define a finite number of curves on the plane $(\omega, \epsilon)$. If any of these curves satisfy simultaneously the two first differential equations, then such curves correspond to indistinguishable trajectories. The important to be noticed is that for every $(\pi, \Delta, T_L)$ there is at most a finite number of indistinguishable trajectories of the motor, if the voltage is fixed. A complete analysis for fixed voltage operation requires the solution of the DAE presented above, what is difficult in general. However, if in addition to fixing the input voltage and the load torque the latter is constant, it is clear that the machine behaviour will converge (in steady state operation) to a constant velocity. From the analysis presented in section 3.4, it can be concluded that if there exist indistinguishable trajectories under these conditions, they must exist only in the transient dynamics, a fact that has been recognized in several experimental studies.

4. CONCLUSIONS

In this paper the problem of carrying out an observability analysis of induction motors under sensorless conditions is approached from a global perspective. In this sense, it is proposed a method to find a dynamical system whose behavior, the indistinguishable dynamics, completely characterize all trajectories that render the electric machine non observable. The lack of observability is stated by considering at a fundamental level the indistinguishability concept. Thus, it is proved that the induction electric machine is not (globally) observable, in the sense that there are different internal trajectories under the same input/output behavior. The main drawback of the presented result – derived from the nonlinear nature of the indistinguishable dynamics – is that, at the current status of the research, it is not possible to explicitly present all the indistinguishable trajectories for the sensorless induction motor. However, it is the authors belief that the fact of having the structure of the dynamical system that completely generates it, establishes an important step towards the complete understanding of this problem.

It is important to note that the conditions used in this paper to find the indistinguishable trajectories are the less stringent for a sensorless operation: perfect model and parameter knowledge, and measurability of all inputs (load torque included). This is in practice not always met, and two comments are in order: i) The method used in this paper is fully applicable to the more general cases, i.e. $T_L$ and parameters not known. Of course, the calculations and the results will be more complicated; ii) The set of indistinguishable trajectories for these more general situations is bigger than for the case analyzed in the present work, i.e. the indistinguishable dynamics (5) describes a subset of the indistinguishable trajectories in the more general situations. This implies that all our results are still valid in those cases.

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