ADAPTIVE SLIDING MODE CONTROL OF VEHICLE TRACTION

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Abstract: In this paper we present a sliding mode based vehicle control strategy, which includes anti-lock braking and anti-spin acceleration. This strategy uses the slip velocity as controlled variable instead of wheel slip. The sliding mode control is integrated with an adaptation process for the unknown and varying adhesion coefficient of the tire/road interface. The proposed control method is verified through one-wheel simulation model with a “Magic formula” tire model. Simulations results show the effectiveness of this controller scheme.

Keywords: Sliding mode control, Adaptive control, Tyre forces, Antilock Braking Systems

1. INTRODUCTION

Vehicle traction control, which is composed of an anti-lock braking and an anti-spin acceleration, has been widely studied and many controllers are proposed in literature (Tan and Chin, 1991)(Unsal and Kachroo, 1999)(Canudas de Wit and Tsiotras, 1999)(Hedrick and Yip, 2000)(El Hadri et al., 2001). The objective of this advanced vehicle traction control system is to obtain desired vehicle motion (of a single vehicle or a platoon of closely spaced vehicles), to maintain adequate vehicle stability and steerability.

Generally, the major difficulty involved in the design of a vehicle control system is that the performance depends strongly on the knowledge of the tire/road characteristic. This characteristic depends on the wheel slip as well as road conditions. The wheel slip is defined as the difference between the vehicle speed and the wheel speed normalized by the vehicle speed for braking phase and the wheel speed for acceleration phase. This difference defines the so called slip velocity. For most of the traction control design the wheel slip is chosen as the controlled variable because of its direct influence on the vehicle traction force. In the strategy of the control proposed here, we use the relationship between the wheel slip and the slip velocity and so, we choose the slip velocity as the controlled variable. The design of traction controller (anti-lock braking and anti-spin acceleration) is based on the assumption that vehicle and wheel speeds are available. The tire force can be described by Bakker-Pacejka’s formula (Bakker et al., 1989). However, the longitudinal adhesion (or friction) coefficient defined as the ratio between the longitudinal tire force and normal load, is not explicit in this formulation. We use then the so called similarity technique (Pacejka, 1989)(Pasterkamp and Pacejka, 1994). This form allows the unknown friction coefficient adaptation. Due to the high nonlinearity of the vehicle traction system, with possible time-varying parameters and uncertainties, we choose the sliding mode approach to design a controller known for its advantages in this cases (Utkin, 1977)(Slotine et al., 1986).

This paper is organized as follows: in the next section, we describe the model of the vehicle traction system and deduce the dynamical equation of the controlled variable. The strategy of control is described in section III. An adaptive sliding mode controller is proposed.
and discussed in section IV and the application of this controller to the vehicle traction control is given in section V. Simulation results are given in section V. Conclusion and extension of this work are presented in section VI.

2. SYSTEM DYNAMICS

In this section, we describe the model for vehicle traction system. This model will then be used for system analysis and computer simulations. The model described in this study retains the main characteristics of the vehicle traction system. The application of Newton’s law to wheel and vehicle dynamics gives the equations of motion. The input signal considered here is the torque applied to the wheel.

The dynamic equation for the angular motion of the wheel is:

\[ I \ddot{\omega} = T - f_w \omega - r F_{rr} - r F_z \]  \hspace{1cm} (1)

where \( I \) is the moment of inertia of the wheel, \( f_w \) is the viscous rotational friction and \( r \) is the effective radius of the wheel. The applied torque \( T \) comes from the difference between the shaft torque from engine and the brake torque. \( F_r \) is the tire tractive/braking force which result from the deformation of the tire at the tire/ground contact patch. \( F_{rr} \) represents the rolling resistance.

The vehicle motion is governed by the following equation

\[ M_v \ddot{v} = F_z - F_{rr} - F_{air} \]  \hspace{1cm} (2)

where \( F_{air} \) is the aerodynamic resistance and \( M_v \) is the vehicle mass. The aerodynamic drag is proportional to square of vehicle velocity and is expressed as:

\[ F_{air} = c_{air} v^2 \]  \hspace{1cm} (3)

where \( c_{air} \) represent the aerodynamic drag coefficient.

The rolling resistance is defined as

\[ F_{rr} = \eta F_z \]  \hspace{1cm} (4)

where \( F_z \) is the normal force and \( \eta \) is the rolling resistance coefficient witch is a nonlinear function of the vehicle velocity. It is usually written as function of the squared vehicle velocity:

\[ \eta(v) = c_{1r} + c_{2r} v^2 \]  \hspace{1cm} (5)

where \( c_{1r} \) and \( c_{2r} \) are the rolling resistance parameters.

The tractive (braking) force, produced at the tire/road interface when a driving (braking) torque is applied to the pneumatic tire, oppose the direction of relative motion between the tire and road surface. The relative motion determines the tire slip properties. The longitudinal wheelslip is defined by the following kinematic relationship (Gim and Nikravesh, 1991):

\[ s = \frac{v_s}{\max(v, r \omega)} \]  \hspace{1cm} (6)

where \( v_s = v - r \omega \), represent the slip velocity in the contact patch.

Then by use of equation (1) and (2), the dynamic equation of slip velocity can be written as:

\[ \dot{v}_s = \frac{r F_w}{T} \omega - \left( \frac{1}{M_v} \frac{r^2}{T} F_{rr} - \frac{1}{M_v} F_{air} \right) \]  \hspace{1cm} (7)

\[ + \left( \frac{1}{M_v} \frac{r^2}{T} \right) F_x - \frac{1}{T} T \]

The longitudinal tire force is generally described as function of wheel slip \( s \), adhesion coefficient \( \mu \) and normal load \( F_z \). We can write:

\[ F_x = f(s, \mu, F_z) \]  \hspace{1cm} (8)

To calculate tire force we use the Magic Formula tire model (Bakker et al., 1989) which is an empirically defined model, made to fit measurements as well as possible. The model is described by:

\[ F_x = D \sin(C \arctan(B \varphi)) + S_0 \]  \hspace{1cm} (9)

where

\[ \varphi = (1 - E)(s + S_{hx}) + \frac{E}{B} \arctan(B(s + S_h)) \]

\( B, C, D, E, S_h \) and \( S_o \) are the tire coefficients adjusted to fit the test data. \( B = stiffness factor, C = shape factor, D = peak factor, E = curvature factor, S_h = horizontal shift, S_o = vertical shift \). The coefficient \( B, C, D \) depend on the tire/road adhesion and normal load on the tire, and their product \( BCD \) represents the slip stiffness. Unfortunately, no explicit parameter of the friction coefficient is present in this formulation. We use then instead the similarity technique (Pacejka, 1989)(Pasterkamp and Pacejka, 1994). This technique allows to simulate the effects of various friction values and, if desired, various normal loads. In this technique, the longitudinal tire force can be written as:

\[ F_x = \mu F_s(s, \mu_0, F_z) \]  \hspace{1cm} (10)

where \( \mu \) and \( \mu_0 \) are respectively the actual and the nominal value of the tire/road friction coefficient.

3. CONTROL FORMULATION

By controlling the wheel slip, we control the tractive force to obtain a desired relative motion of vehicle. Let \( s^d \) be the desired wheel slip. Then by use the relationship (6), the desired slip velocity can be obtained by the following relationship as:

\[ v_s^d = v^* s^d \]  \hspace{1cm} (11)

with \( v^* = \max(v, r \omega) \). As \( v^* \) is known, then the desired \( v_s^d \) is also known.

Let \( e_s \) and \( e_{os} \) denote respectively the control errors of wheelslip and slip velocity, \( (e_s = s - s^d, e_{os} = v_s - v_s^d) \). Then we can write:

\[ e_{os} = v^* e_s \]  \hspace{1cm} (12)
We can see that for all \( v^* \neq 0 \) if \( \dot{e}_{os} \rightarrow 0 \) when \( t \rightarrow \infty \) then \( \dot{e}_v \rightarrow 0 \) when \( t \rightarrow \infty \). This justify the choice of the variable to be controlled.

The wheel slip error dynamic equation is given by:

\[
\dot{e}_{os} = \dot{b}_s - s^d \dot{v}^* \tag{13}
\]

3.1 Braking phase

In this case \( v^* = \dot{v} \) and the equation (13) can be rewritten as:

\[
\dot{e}_{os} = \frac{rf_{so}}{I} \dot{\omega} - \left( \frac{1 - s^d}{M_o} \right) F_{rr} - \frac{1 - s^d}{M_o} F_{air} + \left( \frac{1 - s^d}{M_o} \right) F_x - \frac{r}{I} T
\]

By use of (3), (4) and (10), we obtain:

\[
\dot{e}_{os} = \frac{rf_{so}}{I} \dot{\omega} - \left( \frac{1 - s^d}{M_o} \right) F_{rr} - \frac{s^d}{M_o} c_{air} v^2 + \left( \frac{1 - s^d}{M_o} \right) F_x - \frac{r}{I} T
\]

The applied torque can be decomposed as:

\[ T = T_b + u_b \]

where \( T_b \) is the braking torque activated by driver’s system and \( u_b \) is the control input.

Finally, we can write \( \dot{e}_{os} \) as:

\[
\dot{e}_{os} = \pi_b + \theta_\mu \varphi_b(s) - \frac{r}{I} u_b \tag{15}
\]

with

\[
\pi_b = \frac{rf_{so}}{I} \dot{\omega} - \left( \frac{1 - s^d}{M_o} \right) \eta(v) F_z - \frac{1 - s^d}{M_o} c_{air} v^2
\]

\[
\varphi_b = \left( \frac{1 - s^d}{M_o} \right) F_x + \left( r^2 \frac{1 - s^d}{I} \right) F_{so}(s, \mu_0, F_z)
\]

and \( \theta_\mu = \mu \) represents the unknown parameter.

3.2 Acceleration phase

In acceleration phase \( (13) \) can be written with \( v^* = r \dot{\omega} \) as:

\[
\dot{e}_{os} = \frac{r(1 + s^d) f_{so}}{I} \dot{\omega} - \left( \frac{1}{M_o} \right) \eta(v) F_z - \frac{r(1 + s^d)}{I} F_{rr} - \frac{1}{M_o} F_{air} + \left( \frac{1}{M_o} \right) r^2 \left( 1 - s^d \right) F_x - \frac{r(1 - s^d)}{I} T
\]

Similarly, by use of the equation (10), (4) and (3) as previously, we obtain:

\[
\dot{e}_{os} = \frac{r(1 + s^d) f_{so}}{I} \dot{\omega} - \left( \frac{1}{M_o} \right) \eta(v) F_z - \frac{r^2 (1 + s^d)}{I} F_{rr} - \frac{1}{M_o} c_{air} v^2 + \left( \frac{1}{M_o} \right) r^2 \left( 1 - s^d \right) \mu F_{so}(s, \mu_0, F_z) - \frac{r(1 - s^d)}{I} T
\]

In acceleration phase we consider that the traction torque \( T \) is the desired torque applied to the wheel to achieve the control. We note then:

\[ T = u_a \]

Thus the dynamic equation of the slip velocity can be written as:

\[
\dot{e}_{os} = \pi_a + \varphi_a(s) - \frac{r(1 - s^d)}{I} u_a \tag{17}
\]

with

\[
\pi_a = \frac{r(1 + s^d) f_{so}}{I} \dot{\omega} - \left( \frac{1}{M_o} \right) \eta(v) F_z - \frac{r^2 (1 + s^d)}{I} F_{rr} - \frac{1}{M_o} c_{air} v^2 + \left( \frac{1}{M_o} \right) r^2 \left( 1 - s^d \right) \mu F_{so}(s, \mu_0, F_z)
\]

4. ADAPTIVE SLIDING MODE CONTROL

Let us consider the following system:

\[
\begin{aligned}
\dot{x}(t) &= \pi + \theta \varphi(t, x, y) - bu \\
\dot{\theta} &= 0
\end{aligned}
\tag{18}
\]

where \( \pi \) and \( \varphi(t, x, y) \) are analytical functions and \( b \in R \). \( \theta \) is the unknown parameter. \( y \) represent the measurements.

Consider the following assumptions:

- The nonlinear function \( \pi \) is not well known but estimated as \( \hat{\pi} \) and the extent of the imprecision on \( \pi \) is upper bounded by a known positive function \( II \) such as \( \pi - \hat{\pi} \leq II \).
- The nonlinear function \( \varphi \) is unknown but estimated as \( \hat{\varphi} \) and the estimation error on \( \varphi \) is upper bounded by known positive function \( \Phi \) such as \( \varphi - \hat{\varphi} \leq \Phi \).
- The control gain is bounded such as \( 0 \leq b_{\min} \leq b \leq b_{\max} \). Since the control input is multiplied by the control gain in the dynamics. It is recommended to choose as estimate \( \hat{b} \) of the gain \( b \) the geometric mean of the lower and upper bounds as \( \hat{b} = \sqrt{b_{\min} b_{\max}} \) (Slotine and Li, 1991). The bounds can be written in the form \( \beta_{b_{\min}}^{-1} \leq \hat{b} \leq \beta_{b_{\max}} \).

For system (18), the sliding mode controller can be designed as:

\[
u = \frac{1}{b} \left( \hat{\pi} + \hat{\varphi} + K \text{sign}(x) \right) \tag{19}\]
Choosing as Lyapunov function candidate:

\[ V = \frac{1}{2} \dot{x}^2 + \frac{1}{2} \dot{\theta}^2 \]

with \( \dot{\theta} = \theta - \dot{\theta} \), we have then:

\[ \ddot{V} = x (\pi - \dot{x} + (1 - b \dot{\theta}) \dot{x} + \theta \dot{x} + (1 - b \dot{\theta}) \dot{\theta}) \]

\[ - \frac{b}{K} \text{sign}(x) + \dot{\theta} \dot{x} - \dot{x} \dot{\theta} \]

By choosing the adaptive law as:

\[ \dot{\theta} = \frac{1}{2} \dot{\phi} x \]

Then \( \ddot{V} \) can be made negative definite if we choose the gain \( K \) as:

\[ K \geq \beta_b \Pi + (\beta_b - 1)(\dot{\theta} \dot{\phi} + \dot{x}) + \beta_b \theta_{\text{max}} \Phi \quad (20) \]

5. APPLICATION TO THE VEHICLE TRACTION CONTROL

5.1 Acceleration case

In acceleration case, the dynamic error (17) can be rewritten in the form given by (18) with:

\[ \pi = \frac{r (1 + s^d) f_{\text{sw}} |o|}{I} - \left( \frac{1}{M_o} - \frac{r^2 (1 + s^d)}{I} \right) \eta |v| F_z \]

\[ \varphi = \left( \frac{1}{M_o} + \frac{r^2 (1 - s^d)}{I} \right) F_x \]

\[ b = \frac{r (1 - s^d)}{I} \]

The uncertainty in the function \( \pi \) is due to the following set of parameters \( p = \{ r, c_{1_{c_{1_{r}}}}, c_{2_{c_{2_r}}}, c_{\text{air}} \} \). Then the estimation of the nonlinear function \( \pi \) is given by:

\[ \tilde{\pi} = \tilde{r} (1 + s^d) f_{\text{sw}} |o| - \left( \frac{1}{M_o} - \frac{\tilde{r}^2 (1 + s^d)}{I} \right) \tilde{\eta} |v| F_z \]

\[ - \frac{1}{M_o} \tilde{c}_{\text{air}} v^2 \]

with

\[ \tilde{\eta}(v) = \tilde{c}_{1_{c_{1_{r}}}} + \tilde{c}_{2_{c_{2_r}}} v^2 \]

we can write estimation error \( \tilde{\pi} = \pi - \tilde{\pi} \) as:

\[ \tilde{\pi} = \tilde{r} (1 + s^d) f_{\text{sw}} |o| - \frac{1}{M_o} \tilde{\eta} F_z \]

\[ + \left( \frac{1}{I} - \frac{1}{I} \right) F_z (r^2 \tilde{\eta} - r^2 \tilde{\eta}) - \frac{1}{M_o} \tilde{c}_{\text{air}} v^2 \quad (21) \]

with \( \tilde{r} = r - \tilde{r}, \tilde{c}_{\text{air}} = c_{\text{air}} - \tilde{c}_{\text{air}} \) and \( \tilde{\eta} = \eta - \tilde{\eta} \)

We define a priori estimated values of the \( p \) parameters as the geometric mean of the bounds \( (p_{\text{min}} < p_i < p_{\text{max}}) \) for \( p_i \in p \), as:

\[ \tilde{\pi} = \sqrt[\pi]{p_{\text{min}} p_{\text{max}}} \quad (22) \]

As previously, the bounds can be written in the form \( \beta_i^{-1} < \frac{\tilde{p}}{p_{\text{sw}}} < \beta_i \) and \( \beta_i^{-1} < \frac{\tilde{p}}{p_{\text{sw}}} < \beta_i \) with \( \beta_i = \sqrt[\pi]{p_{\text{max}}/p_{\text{min}}} \). So, we can found that:

\[ \beta_i^{-1} \leq \frac{\pi}{\beta_i} \leq \beta_i \]

with \( \beta_i = \max(\beta_{c_{1_{c_{1_{r}}}}}, \beta_{c_{2_{c_{2_r}}}}) \).

Then the error estimation (22) can be bounded as:

\[ |\tilde{\pi}| \leq \beta |\tilde{r}| (1 + s^d) |F_{\text{sw}}| |v| + \frac{1}{M_o} |\beta^{-1}_\pi| \]

\[ \left( \frac{1}{\beta_{\phi}} \right) |\dot{\varphi}| F_{\text{max}} + \frac{1}{M_o} |\beta^{-1}_{c_{\text{air}}}| c_{\text{air}} |v|^2 \]

Similarly, the function \( \varphi \) is estimated as:

\[ \tilde{\varphi} = \frac{\tilde{\varphi} (1 - s^d)}{I} F_x \]

The extent of the imprecision on \( \varphi \) is given by:

\[ |\tilde{\varphi}| \leq \frac{\beta_{\varphi} (r_{\text{max}} + \tilde{r}) (1 + s^d)}{I} |F_x| \]

Finally, to control the wheel slip in acceleration phase we use the control law given in (19) with the sliding gain \( K \) chosen as in (20) where:

\[ \Pi = \beta_{\phi} \tilde{r} (1 + s^d) |F_{\text{sw}}| |v| + \frac{1}{M_o} |\beta^{-1}_\pi| \]

\[ \left( \frac{1}{\beta_{\phi}} \right) |\dot{\varphi}| F_{\text{max}} + \frac{1}{M_o} |\beta^{-1}_{c_{\text{air}}}| c_{\text{air}} |v|^2 \]

\[ \Phi = \beta |\tilde{r}| (r_{\text{max}} + \tilde{r}) (1 + s^d) |F_x| \]

5.2 Braking phase

Similarly to the previous case, the dynamic error (15) can be rewritten in the form given by (18) with:

\[ \pi = \frac{r f_{\text{sw}} |o|}{I} - \left( \frac{1}{M_o} - \frac{r^2}{I} \right) \eta |v| F_z \]

\[ - \frac{1 - s^d}{M_o} c_{\text{air}} v^2 - \frac{r}{I} T_b \]

\[ \varphi = \frac{1 - s^d}{I} + \frac{r^2}{I} F_x \]

\[ b = \frac{r}{I} \]
As in the previous case, the estimation of the function \( \pi \) is
\[
\hat{\pi} = \frac{\hat{f}_m}{I} - \frac{1 - s^d}{M_o} \hat{\eta}(\nu) F_z - \frac{1 - s^d}{M_o} \hat{\zeta}_{air} \omega^2 - \frac{\hat{r}}{I} \hat{T}_b.
\]
Then, the estimation error \( \tilde{\pi} = \pi - \hat{\pi} \) is:
\[
\tilde{\pi} = \frac{\hat{f}_m}{I} - \frac{1 - s^d}{M_o} \hat{\eta}(\nu) F_z + \left( \frac{\hat{r}^2}{I} - \frac{\hat{\eta}}{I} \right) F_z - \frac{1 - s^d}{M_o} \hat{\zeta}_{air} \omega^2 - \frac{1}{I} (r \hat{T}_b - \hat{r} \hat{T}_b).
\]
The previous expression can be bounded as:
\[
|\tilde{\pi}| \leq \beta \left( \frac{r_{max} + \hat{r}}{I} \right)^2 |F_{xy}|
\]
Then, the control law (19) can be used in braking phase if we choose the sliding gain \( K \) as in (20) with:
\[
\Pi = \beta \frac{\hat{f}_m}{I} - \frac{1 - s^d}{M_o} \frac{\hat{\eta}_i}{I} + \frac{1}{I} (\beta \hat{\eta}^2 - 1) \hat{\eta} |F_z|
\]
\[
+ \frac{1 - s^d}{M_o} \beta \frac{\hat{\zeta}_{air} \omega^2}{I} - \frac{1}{I} (r_{min} Y + \beta \hat{r} \hat{T}_b)
\]
\[
\Phi = \beta \frac{(r_{max} + \hat{r})^2}{I} |F_{xy}|
\]

6. SIMULATION RESULTS

To illustrate the performance of the proposed control strategy we consider a one-wheel model (equations (1) and (2)) with “Magic formula” tire model. The model parameters are listed in table (1).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>400</td>
<td>Kg</td>
</tr>
<tr>
<td>( J )</td>
<td>1.3</td>
<td>Kg m^2</td>
</tr>
<tr>
<td>( r )</td>
<td>0.3</td>
<td>m</td>
</tr>
<tr>
<td>( \zeta_{air} )</td>
<td>0.3</td>
<td>N/m2/s^2</td>
</tr>
</tbody>
</table>

Table 1. The parameters of the tyre model.

The equation (6) is used to calculate the value of the slip ratio. The sliding gains are chosen as (20) in braking phase. We consider an error of 25% on the parameters listed in table (1). In simulation, the actual tire force is generated by “Magic formula” tire model.

In braking phase the applied torque shown in Figure (1) without ABS cause the wheel locking (see figure 2). The use of the control law (19) with \( K = 200 \) and \( \lambda = 100 \) when the desired wheel-slip is chosen at 15% allows to avoid the locking of the wheel as shown in figure (3). The chattering in the sliding mode control signal is reduced by using a low-pass filter. The desired and actual wheel-slip are shown in figure (4). The control law is activated only during braking phase (1.1s to 3.6s). To test the robustness of this control we change during braking phase the value of the adhesion coefficient (see figure 5).

The performance of the sliding mode based control is satisfactory. The simulation results show that the adaptive control is robust with respect to the parameters uncertainties and the changes on the road conditions. The corresponding control signal is shown in figure (6).
In this paper, we have presented an adaptive sliding mode control for vehicle traction. In the strategy of control we choose as controlled variable the slip velocity based on desired wheel slip. We have studied this control strategy in both cases of acceleration and braking. This strategy is based on adaptation of the unknown tire/road coefficient. To realize this, we use the similarity technic to represent the tire model. The simulation results illustrate the ability of this approach to give good results. In braking phase, the regulation of wheel-slip allows to achieve quiet and safe braking. The disadvantage of this strategy is the necessity of both vehicle and wheel speeds. The perspective of this work is to use of observers based methods to reduce the needed sensors.

REFERENCES