COORDINATED FEEDBACK AND SWITCHING FOR WAVE SUPPRESSION

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Abstract: This work focuses on the problem of coordinating feedback and switching for the stabilization of the zero solution of the one-dimensional Kuramoto-Sivashinsky equation (KSE) with periodic boundary conditions and input constraints. Galerkin’s method is initially used to derive a finite-dimensional system that captures the dominant dynamics of the KSE for a given value of the instability parameter. This ODE system is then used as the basis for the integrated synthesis, via Lyapunov techniques, of stabilizing nonlinear feedback controllers together with switching laws that orchestrate the switching between the admissible control actuator configurations, in a way that respects input constraints, accommodates inherently conflicting control objectives, and guarantees closed-loop stability. The theoretical results are successfully illustrated through computer simulations of the closed-loop system using a high-order discretization of the KSE.

Keywords: Feedback control, Switching times, Actuators, Constraints, Waves.

1. INTRODUCTION

The KSE is a nonlinear dissipative partial differential equation (PDE) of the form:

$$\frac{\partial U}{\partial t} = -v \frac{\partial^4 U}{\partial z^4} - \frac{\partial^2 U}{\partial z^2} - U \frac{\partial U}{\partial z}$$

where \(v > 0\) is the so-called instability parameter, which describes incipient instabilities in a variety of physical and chemical systems, including falling liquid films (Chen and Chang, 1986), unstable flame fronts (Sivashinsky, 1980), Belousov-Zabotinskii reaction patterns (Kuramoto and Tsuzuki, 1976) and interfacial instabilities between two viscous fluids (Hooper and Grimshaw, 1985). Both the dynamics and control of the KSE with periodic boundary conditions have been the subject of significant research work. Dynamical studies have led to the discoveries of steady and periodic wave solutions, chaotic behavior for very small values of \(v\), and the fact that the dominant dynamics of the KSE can be adequately characterized by a small number of degrees of freedom (see, e.g., (Temam, 1988; Chen and Chang, 1986; Kevrekidis et al., 1990)). Control studies of the KSE have focused on a variety of problems, including the design of finite-dimensional output feedback controllers for stabilization of the zero solution of the KSE based on ordinary differential equation (ODE) approximations (Armaou and Christofides, 2000), the global stabilization of the KSE via distributed static output feedback control (Christofides and Armaou, 2000), control of the KSE with input constraints (El-Farra et al., 2001), and stabilization enhancement via boundary control (Liu and Krstic, 2001).

While the above research efforts have led to the development of systematic approaches for control of the KSE, an underlying theme of these approaches is the use of a fixed (with respect to spatial location) control actuator/measurement sensor configuration in order to accomplish the desired control objectives. There are many practical situations, however, where it may be desirable, and sometimes even necessary, to consider multiple actuator/sensor configurations and switch between them in a specific manner, in order to achieve

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the specified control objectives. One example is the problem of actuator failure where, upon detecting a fault in an actuator, it may be necessary to switch to an alternative actuator, that is already placed at a different spatial location, in order to preserve stability of the closed-loop system. Switching between the actuators in this case provides a means for fault-tolerant control. In other cases, switching between actuator configurations may be motivated by some additional performance objective, such as the desire to optimize a given performance criterion or accommodate inherently competing control objectives that cannot be reconciled using a single actuator configuration.

Motivated by the above considerations, we focus in this work on the problem of coupling feedback and switching in the stabilization of the zero solution, \( U(z, t) = 0 \), of the KSE with periodic boundary conditions and input constraints. The problem is addressed on the basis of finite-dimensional Galerkin approximations of the KSE and entails the integrated synthesis, via Lyapunov techniques, of stabilizing nonlinear switching laws that orchestrate the switching between the admissible control actuator configurations, in a way that respects actuator constraints, accommodates inherently conflicting control objectives, and guarantees closed-loop stability. Precise conditions that guarantee stability of the constrained closed-loop KSE under switching are provided, and an application of the theoretical results to the problem of actuator failure in control of the KSE is demonstrated through computer simulations.

2. PRELIMINARIES

2.1 Mathematical description

Consider the one-dimensional Kuramoto-Sivashinsky equation with distributed control:

\[
\frac{\partial U}{\partial t} = -v \frac{\partial^3 U}{\partial z^3} - \frac{\partial^2 U}{\partial z^2} - U \frac{\partial U}{\partial z} + \sum_{i=1}^{l} b_i u_i(t) \\
y^k_m = \int_{-\pi}^{\pi} s_k(z) U dz, \; k = 1, \ldots, p \\
y^c_i = \int_{-\pi}^{\pi} c_i(z) U dz, \; i = 1, \ldots, l
\]

subject to the periodic boundary conditions and initial condition:

\[
\frac{\partial^j U}{\partial z^j}(-\pi, t) = \frac{\partial^j U}{\partial z^j}(+\pi, t), \; j = 0, \ldots, 3 \\
U(z, 0) = U_0(z)
\]

where \( U(z, t) \) is the state of the system, \( z \in [-\pi, \pi] \) is the spatial coordinate, \( t \) is the time, \( v \) is the instability parameter, \( u_i \in [u_{\text{min}}, u_{\text{max}}] \subset \mathbb{R} \) is the \( i \)-th constrained manipulated input, \( l \) is the total number of manipulated inputs, \( b_i (z) \) is the \( i \)-th actuator distribution function (i.e., \( b_i(z) \) determines how the control action computed by the \( i \)-th control actuator, \( u_i(t) \), is distributed (e.g., point or distributed actuation) in the spatial interval \([-\pi, \pi]\)), \( y^k_m \) denotes a controlled output, \( c_i(z) \) is a known smooth function of \( z \) which is determined by the desired stability/performance specifications, \( y^k_m \in \mathbb{R} \) denotes a measured output, and \( \delta_k(z) \) is a known smooth function of \( z \) which is determined by the location and type of the measurement sensors (e.g., point/distributed sensing). Whenever the control action enters the system at a single point \( z_0 \), with \( z_0 \in [-\pi, \pi] \) (i.e. point actuation), the function \( b_i(z) \) is taken to be nonzero in a finite spatial interval of the form \([z_0 - \varepsilon, z_0 + \varepsilon] \), where \( \varepsilon \) is a small positive real number, and zero elsewhere in \([-\pi, \pi]\). We will use the order of magnitude notation \( O(\varepsilon) \). In particular, \( \delta(\varepsilon) = O(\varepsilon) \) if there exist positive real numbers \( k_1 \) and \( k_2 \) such that: \( \delta(\varepsilon) \leq k_1 |\varepsilon| \), \( \forall |\varepsilon| < k_2 \).

To present our theoretical results, we cast the system of Eqs.2 as an infinite dimensional system in the Hilbert space \( \mathcal{H}([-\pi, \pi]; \mathbb{R}) \), with \( \mathcal{H} \) being the space of sufficiently smooth vector functions defined on \([-\pi, \pi]\) that satisfy the boundary condition of Eq.3, with inner product and norm:

\[
\langle \omega_1, \omega_2 \rangle = \int_{-\pi}^{\pi} (\omega_1(z), \omega_2(z)) \text{d}z \\
\| \omega_1 \|_2 = (\omega_1, \omega_1)^{\frac{1}{2}}
\]

where \( \omega_1, \omega_2 \) are two elements of \( \mathcal{H}([-\pi, \pi]; \mathbb{R}) \) and the notation \( (\cdot, \cdot)_\mathcal{H} \) denotes the standard inner product in \( \mathbb{R} \). Defining the state function \( x \) on \( \mathcal{H}([-\pi, \pi]; \mathbb{R}) \) as:

\[
x(t) = U(z, t), \; t > 0, \; z \in [-\pi, \pi].
\]

the operator \( \mathcal{A} \) in \( \mathcal{H}([-\pi, \pi]; \mathbb{R}) \) as:

\[
\mathcal{A}x = -v \frac{\partial^3 U}{\partial z^3} - \frac{\partial^2 U}{\partial z^2}
\]

\[
x \in D(\mathcal{A}) = \{ x \in \mathcal{H}([-\pi, \pi]; \mathbb{R}) : \} \\
\frac{\partial^j U}{\partial z^j}(-\pi, t) = \frac{\partial^j U}{\partial z^j}(+\pi, t), \; j = 0, \ldots, 3 \}
\]

and the input, controlled output, and measured output operators as:

\[
\mathcal{B}u = \sum_{i=1}^{m} b_i u_i, \; \mathcal{C}x = (c, x), \; \mathcal{F}x = (s, x)
\]

the system of Eqs.2-3 takes the form:

\[
\dot{x} = \mathcal{A}x + \mathcal{B}u + f(x), \; x(0) = x_0 \\
y_c = \mathcal{C}x, \; \; \; \; \; \; \; \; \; \; \; \; y_m = \mathcal{F}x
\]

where \( f(x(t)) = -U \frac{\partial U}{\partial z} \) and \( x_0 = U_0(z) \).

For \( \mathcal{A} \), we can formulate the following eigenvalue problem:
\[ \mathcal{A} \phi_n = -\frac{\partial^4 \phi_n}{\partial z^4} - \frac{\partial^2 \phi_n}{\partial z^2} = \lambda_n \phi_n \]  

(9)

where \( n = 1, \ldots, \infty \), subject to:

\[ \frac{\partial^4 \phi_n}{\partial z^4} (-\pi) = \frac{\partial^4 \phi_n}{\partial z^4} (+\pi), \quad j = 0, \ldots, 3 \]  

(10)

where \( \lambda_n \) denotes an eigenvalue and \( \phi_n \) denotes an eigenfunction. A direct computation of the solution of the above eigenvalue problem yields \( \lambda_0 = 0 \) with \( \psi_0(z) = \frac{1}{\sqrt{2\pi}} \), and \( \lambda_n = -n^4 + n^2 \) (\( \lambda_n \) is an eigenvalue of multiplicity two) with eigenfunctions \( \phi_n(z) = \frac{1}{\sqrt{2\pi}} \sin(nz) \) and \( \psi_n(z) = \frac{1}{\sqrt{2\pi}} \cos(nz) \) for \( n = 1, \ldots, \infty \).

We also define the eigenspectrum of \( \mathcal{A} \), \( \sigma(\mathcal{A}) \), as the set of all eigenvalues of \( \mathcal{A} \), i.e. \( \sigma(\mathcal{A}) = \{ \lambda_1, \lambda_2, \ldots \} \). From the expression for the eigenvalues, it follows that for a fixed value of \( \nu > 0 \) the number of unstable eigenvalues of \( \mathcal{A} \) is finite and the distance between two consecutive eigenvalues (i.e., \( \lambda_k \) and \( \lambda_{k+1} \)) increases as \( n \) increases. Furthermore, for a fixed value of \( \nu > 0 \), \( \sigma(\mathcal{A}) \) can be partitioned as \( \sigma(\mathcal{A}) = \sigma_1(\mathcal{A}) \cup \sigma_2(\mathcal{A}) \), where \( \sigma_1(\mathcal{A}) \) contains the first \( m \) (with \( m \) finite) “slow” eigenvalues (i.e., \( \sigma_1(\mathcal{A}) = \{ \lambda_1, \ldots, \lambda_m \} \)) and \( \sigma_2(\mathcal{A}) \) contains the remaining “fast” eigenvalues (i.e., \( \sigma_2(\mathcal{A}) = \{ \lambda_{m+1}, \ldots \} \)) where \( \lambda_{m+1} < 0 \). This implies that the dynamics of the KSE can be described by a finite-dimensional system. Finally, we define \( \varepsilon = \frac{\lambda_1}{\lambda_{m+1}} \).

2.2 Problem formulation

Consider the system of Eq.2, where the manipulated inputs \( u \) are constrained in the interval \([-u_{\text{max}}, u_{\text{max}}]\), and assume that a family of \( N \)-spatially-distributed control actuator configurations \( \bar{z}_k \), \( k = 1, \ldots, N \) (with \( N \) finite) are available for control purposes. Only one configuration can be engaged for control at any time instance. The index \( k \) in \( \bar{z}_k \) denotes the active actuator configuration, while \( \bar{z}_k \) is a vector whose components represent the corresponding spatial locations of the actuators associated with the \( k \)-th configuration. To ensure controllability of the system, we allow only a finite number of switches between configurations over finite time. The problem is how to coordinate switching between the different control actuator configurations in a way that respects actuator constraints and guarantees closed-loop stability. To address this problem, we formulate the following objectives. Initially, Galerkin’s method is used to derive a nonlinear finite-dimensional ODE system that captures the dominant dynamics of the KSE. Next, the ODE approximation is used as the basis for the synthesis, of bounded nonlinear output feedback controllers of the general form

\[ u = p(y_m, \bar{v}, u_{\text{max}}, \bar{z}_k) \]  

(11)

that enforce asymptotic stability and reference-input tracking in the constrained closed-loop system and provide an explicit characterization of the stability region, associated with each control actuator configuration. Referring to Eq.11, \( p(\bar{x}, \bar{v}, u_{\text{max}}, \bar{z}_k) \) is a bounded nonlinear vector function (i.e. \( |u| \leq u_{\text{max}} \)), and \( \bar{v} \) is a generalized reference input (which is assumed to be a smooth function of time). The controller synthesis is carried out via Lyapunov-based control techniques and is inspired by the results on bounded control in (Lin and Song, 1991). Finally, a set of switching rules is derived to determine which of the \( N \) control actuator configurations can be engaged at any given time, and an upper bound on the separation between the slow and fast eigenvalues, which guarantees stability of the closed-loop finite-dimensional system, is computed.

3. INTEGRATING FEEDBACK AND SWITCHING

3.1 Galerkin’s method

Let \( \mathcal{H}_i \), \( \mathcal{H}_f \) be modal subspaces of \( \mathcal{A} \), defined in the following manner: \( \mathcal{H}_i = \text{span}\{\phi_1, \phi_2, \ldots, \phi_m\} \) and \( \mathcal{H}_f = \text{span}\{\phi_{m+1}, \phi_{m+2}, \cdots\} \) (the existence of \( \mathcal{H}_i \), \( \mathcal{H}_f \) follows from assumption 1). Defining the orthogonal projection operators \( P_i \) and \( P_f \) such that \( x = P_ix + P_fx \), the state \( x \) of the system of Eq.8 can be decomposed as:

\[ x = x_i + x_f = P_ix + P_fx \]  

(12)

Applying \( P_i \) and \( P_f \) to the system of Eq.8 and using the above decomposition for \( x \), the system of Eq.8 can be equivalently written in the following form:

\[ \frac{dx_i}{dt} = \mathcal{A}_i x_i + \mathcal{B}_i u + f_i(x_i, x_f) \]  

\[ \frac{dx_f}{dt} = \mathcal{A}_f x_f + \mathcal{B}_f u + f_i(x_i, x_f) \]  

(13)

where \( \mathcal{A}_i = P_i \mathcal{A}, \mathcal{B}_i = P_i \mathcal{B}, f_i = P_i f, \mathcal{A}_f = P_f \mathcal{A}, \mathcal{B}_f = P_f \mathcal{B} \) and \( f_f = P_f f \) and the notation \( \frac{dx_f}{dt} \) is used to denote that the state \( x_f \) belongs in an infinite-dimensional space. In the above system, \( \mathcal{A}_i \) is a diagonal matrix of dimension \( m \times m \) of the form \( \mathcal{A}_i = \text{diag}\{\lambda_1, \lambda_2, \cdots\} \) and \( f_i(x_i, x_f) \) are Lipschitz vector functions, and \( \mathcal{A}_f \) is an unbounded differential operator which is exponentially stable (following from part 3 of assumption 1 and the selection of \( \mathcal{H}_i, \mathcal{H}_f \)). Neglecting the fast and stable infinite-dimensional \( x_f \)-subsystem in the system of Eq.13, the following \( m \)-dimensional slow system is obtained:

\[ \frac{d\bar{x}_i}{dt} = \mathcal{A}_i \bar{x}_i + \mathcal{B}_i u + f_i(\bar{x}_i, 0) \]  

\[ y_c = \mathcal{C}_s \bar{x}_i, \quad y_m = \mathcal{F} x_i \]  

(14)

where the bar symbol in \( \bar{x}_i, \bar{y}_i \) and \( \bar{y}_m \) denotes that these variables are associated with a finite-dimensional system.
3.2 Main result

In this section we will present the main result of this paper. To this end, we initially transform the system of Eq.14, by means of a standard coordinate change, into an interconnection of two subsystems: one describing the input-output dynamics of the system of Eq.14 and another describing its inverse dynamics:

\[
\begin{align*}
\dot{e} &= F e + K (e, \eta) + C (e, \eta) u \\
\eta &= \Psi (e, \eta) \\
\tilde{y}_i &= e^{(i)}_1 + v_i, \quad i = 1, \ldots, m
\end{align*}
\]  

(15)

where \(e\) is an \(\sum_i r_i \times 1\) vector of the tracking errors, \(e^{(i)}_1 = \tilde{y}_i - v_i\), and their respective time-derivatives, up to order \(r_i\) which is the relative order of the \(i\)-th output with respect to the vector of manipulated inputs, \(v_i\) is \(i\)-th the reference input, \(F, K, C\) are constant matrices, \(\Psi (\cdot)\) is a smooth vector function, and \(G (\cdot)\) is the characteristic matrix of the system of Eq.14 which, for simplicity, is assumed to be nonsingular, uniformly in \(\bar{x}_e\). Finally, we define \(f (e, \eta) = F e + K (e, \eta)\) and denote by \(\tilde{g}_i\) the \(i\)-th column of the matrix \(G (e, \eta) = KC (e, \eta)\).

Assumption 2: The \(\eta\)-subsystem of Eq.15 is input-to-state stable with respect to \(e\) and locally exponentially stable when \(e = 0\).

Theorem 1 below provides both the state feedback control law (see remark 4 and the simulation study in section 4 for output feedback controller design and implementation) as well as the necessary switching laws and states precise conditions that guarantee closed-loop stability and asymptotic reference-input tracking in the closed-loop system. The proof of this Theorem can be found in (El-Farra and Christofides, 2001).

Theorem 1: Consider the system of Eq.14, for which assumption 2 holds, under the feedback control law

\[
u(x, \bar{z}_k) = -k(x, \bar{z}_k) (L_k V)^T (\bar{z}_k)
\]

(16)

where \(k(x, \bar{z}_k) =\)

\[
\frac{L_j V + \sqrt{(L_j V)^2 + (u_{\text{max}} |(L_j V)|)^4}}{|(L_j V)^T|^2 [1 + \sqrt{1 + (u_{\text{max}} |(L_j V)|)^2}]^2}
\]

(17)

\[
\bar{z}_k = [\bar{z}_{k_1}, \bar{z}_{k_2}, \ldots, \bar{z}_{k_N}]^T, \quad k = 1, \ldots, N, \quad L_j V = L_j V + \rho |e|^2, \quad \rho > 0, \quad L_j V is a row vector of the form [L_{j_1} V \cdots L_{j_m} V]. V = e^T F e, \quad F is a positive definite matrix that satisfies the Riccati inequality \(T^T \Phi + \Phi T - \Phi F K^T \Phi F < 0\). Let \(\delta_\phi\) be a positive real number such that the compact set \(\Omega(\bar{z}_k) = \{\bar{z}_k \in \mathcal{H}_r \mid |\bar{z}_k| \leq \delta_\phi\}\) is the largest invariant set embedded within the unbounded region described by the following inequality

\[
L_j V \leq u_{\text{max}} |(L_j V)|^2 (\bar{z}_k)
\]

(18)

Without loss of generality, assume that \(\bar{x}_i (0) \in \Omega (\bar{z}_k)\) and \(\bar{z}_{k(0)} = \bar{z}_k\). Then if, at any given time \(t^*\), the condition

\[
\bar{x}_i (t^*) \in \Omega (\bar{z}_k)
\]

holds, for some \(j \in \{1, \ldots, N\}\), then setting \(\bar{z}_j (t^*) = \bar{z}_j\) guarantees that the closed-loop system is asymptotically stable in the sense that there exists a function \(\beta\) of class KL such that \(\bar{x}_i (t) \leq \beta (|\bar{x}_i (0)|, t)\), \(\forall t \geq 0\). Now, consider the system of Eq.2, for which assumption 1 holds, under the nonlinear state feedback controller of Eq.16. Then given any pair of positive real numbers \((\delta_\phi, \delta_{\eta})\) such that \(\beta (\delta_\phi, 0) + d \leq \delta_{\eta}\), and given any positive real number \(\delta_{\eta}\), there exists \(\bar{e} > 0\) such that if \(e \in (0, \bar{e})\), \(|x (0)| \leq \delta_\phi\), \(|y (0)| \leq \delta_{\eta}\): (1) The infinite-dimensional closed-loop system is asymptotically (and locally exponentially) stable.

(2) The outputs of the closed-loop system satisfy:

\[
\limsup_{t \to \infty} |y_i (t) - v_i| = |O (e)|, \quad i = 1, \ldots, m.
\]

Remark 1: For a given actuator configuration (fixed \(\bar{z}_k\)), the inequality of Eq.18 describes the region where the control action satisfies the constraints and \(V\) is negative-definite, along the trajectories of the finite-dimensional closed-loop system. By computing the largest invariant set \(\Omega (\bar{z}_k)\) within this region, we obtain an estimate of the stability region associated with each configuration. Finally, the large separation between the slow and fast eigenmodes of the spatial differential operator of the KSE allows preservation of this estimate for the infinite-dimensional system.

Remark 2: The switching law of Eq.19 orchestrates the transition between the control actuator configurations in a way that respects the constraints and guards against any potential instability that may arise due to the switching. The basic problem here is that, depending on the state’s location, at a given time, with respect to the \(N\) stability regions associated with the \(N\) actuator configurations, a switch from one configuration to another may land the state outside the stability region of the intended configuration. To guard against this possibility, the switching law of Eq.19 tracks the slow state in time and allows switching to another actuator configuration only when the state resides within the corresponding stability region. This condition therefore determines, implicitly, the earliest safe switching time.

Remark 3: Under the assumption that the number of measurements is equal to the number of slow modes and that the inverse of the operator \(\mathcal{F}\) exists, an output feedback controller can be designed by combining the state feedback controller of Eq.16 with a procedure proposed in (Christofides, 2001) for obtaining estimates for the states of the approximate ODE model of Eq.14 from the measurements. While the estimation error leads to some loss in the size of the stability region obtained under state feedback, this loss can be made small by increasing the order of the ODE ap-
proximation and including more measurements. This approach therefore allows us to asymptotically (as \( \varepsilon \to 0 \)) recover the stability region associated with each control actuator configuration.

4. SIMULATION RESULTS

In this section, we demonstrate how the results of Theorem 1 can be used to deal with the problem of actuator failure, in the context of stabilizing the zero solution of the one-dimensional KSE with periodic boundary conditions and input constraints. In order to simplify the presentation of our results, we will consider the KSE in the space of square integrable odd functions that satisfy the boundary conditions of Eq.3 and have spatial zero mean. Linearizing the system of Eq.2 (with \( u \equiv 0 \)) around the spatially uniform steady-state, for \( v = 0.2 \), we observe that the system possesses two unstable eigenvalues. It was verified that for \( v = 0.2, \) the spatially uniform steady-state \( U(z,t) = 0 \) is unstable. Therefore, the control objective is to stabilize the system at this unstable steady state. To achieve this control objective, the controlled outputs are defined as:

\[
y_c(t) = \int -\pi^1 \sqrt{\sin(iz)U(z,t)} dz, i = 1, 2 \quad (20)
\]

and standard Galerkin’s method is initially used to derive a second-order model that describes the temporal evolution of the amplitudes, \( a_1 \) and \( a_2 \), of the first two eigenmodes, respectively, where \( x_i(t) = a_i(t)\phi_i(z) + a_2(t)\phi_2(z) \). The second order system is then employed for the synthesis of the controller using Eq.16, and the switching laws using Eq.18. The controller and switching laws are then implemented on a 20-order nonlinear ordinary differential equation model obtained from the application of Galerkin’s method to the system of Eq.2 (the use of higher-order Galerkin approximations led to identical numerical results).

Three pairs of point control actuators, situated at \((\xi_1 = 0.3\pi, \xi_2 = 0.7\pi)\) (configuration A) and \((\xi_1 = 0.1\pi, \xi_2 = 0.9\pi)\) (configuration B), and \((\xi_1 = 0.2\pi, \xi_2 = 0.8\pi)\) (Configuration C), respectively, are assumed to be available for stabilization. Configurations A and B have the same constraints of \( u_{\text{max},A} = u_{\text{max},B} = 2.0 \) while configuration C has constraints of \( u_{\text{max},C} = 0.5 \). Only one configuration is to be used for control at any given moment. The question to be addressed is how to decide which of the two “backup” actuator configurations can be used to maintain stability once the actuators in the third configuration fail.

We present the simulation results under state feedback first. Initially, Eq.18 with \( \rho = 0.001 \) is used to compute estimates of the stability regions for configurations A, B, and C. These regions are depicted in Figure 1 which shows the set of admissible initial conditions for the amplitudes, \( a_1(0) \) and \( a_2(0) \), of the first two eigenmodes, respectively, for configuration A (solid ellipse), configuration B (dashed ellipse), and configuration C (dotted ellipse) (note that \( x_i(0) = a_i(0)\phi_i(z) + a_2(0)\phi_2(z) \)).

From this figure, it is clear that for an initial condition \((a_1(0), a_2(0)) = (0.2, 1.5)\), only configuration A is feasible initially since the initial condition is outside the stability regions for B and C. Now, suppose that sometime after starting from this initial condition, a fault is detected in configuration A and it becomes necessary to switch to either configuration B or C. Without using the switching law of Theorem 1, it is not clear which of the two should be activated at this time. Figures 2a and 2b depict, respectively, the closed-loop state and manipulated input profiles when the failure of A occurs at \( t = 3.5 \) and configuration C is activated. We see in this case that the controller is unable to stabilize the system at the desired steady-state as the inputs stay saturated for all times. This is expected since the slow state at the time of switching lies outside the stability region for C (see point X on the solid trajectory in Figure 1—note that the solid trajectory settles at another steady state after switching at X). In contrast, by using the switching law of
Theorem 1 and monitoring the slow state evolution in time, we conclude that it is actuator B, not C, that should be activated because the trajectory at this time lies inside the stability region for B (see dashed trajectory in Figure 1 which converges to the origin). Figure 3 depicts the results for this case and shows that the controller successfully stabilizes the closed-loop system when configuration B is activated instead of C. The switching scheme therefore allows us to choose the appropriate alternative configuration.

For the case of output feedback, we use a couple of point sensors, located at \( z_1 = 0.35\pi \) and \( z_2 = 0.65\pi \), to obtain estimates of the first two eigenmodes which are then used for implementing the output feedback controller. As noted earlier, owing to the small discrepancy between the stability regions obtained under state and output feedback, we use the switching law of Theorem 1 only as an approximate guide in this case. The simulation results for this case are consistent with the state feedback results. These results are given in Figure 4 which shows that the closed-loop system is successfully stabilized by activating configuration B, instead of C, at the time that configuration A fails.

5. REFERENCES


