CONTROLLER DESIGN FOR THE IDENTIFICATION OF AN LFT MODEL

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Abstract: The aim of this paper is to identify an LFT model with frequency-domain data. Under a left invertibility condition inputs and outputs of the real structured model perturbation $\Delta$ can be deduced from the measured inputs and outputs, so that $\Delta$ can be identified with a Robust Least Squares method or an output error set-membership identification technique. The influence of a controller on the sensitivity of the identification scheme, face to neglected dynamics, measurement noise and other disturbances acting on the closed loop is studied. Using a classical convex closed loop design technique, a controller that reduces this sensitivity while satisfying stability and performance requirements is synthesized. An illustrative example is given.

Keywords: Parameter estimation, Frequency domains, Closed loop identification, Robust estimation, Sensitivity analysis.

1. INTRODUCTION

Glossary: FFT (Fast Fourier Transform), FPS (Feasible Parameter Set), LFT (Linear Fractional Transformation), LMI (Linear Matrix Inequality), LS (Least Squares), LTI (Linear Time Invariant), RLS (Robust Least Squares), SMI (Set-Membership Identification), UBB (Unknown But Bounded).

Identifying an LFT model is a difficult task which remains largely unexplored despite its interest: see especially Krause and Khargonekar (1990); Wolodkin et al. (1997). Indeed, in order to propose a unified framework which combines robust control, robustness analysis, identification and model validation (see Doyle et al. (1994)), a preliminary requirement is to use a common standard form. Moreover, most LTI models which depend on uncertain parameters can be put under the LFT form of figure 1(a): the transfer matrix $H(s)$ is fixed, $u$ and $y$ are the physical inputs and outputs, while the parametric uncertainties which are gathered inside the real structured model perturbation $\Delta$ (i.e. a fixed structure real matrix whose value is unknown) appear as a fictitious internal feedback $v = \Delta w$. If the state space representation is stemming from physical equations, uncertainties in $\Delta$ correspond to physical system parameters, which can be identified (see also Demourant and Ferreres (2001)). Besides this approach is particularly adapted in the identification for control context. A time domain method was especially proposed in Krause and Khargonekar (1990) to identify an LFT model: under a left invertibility condition which is not trivial to satisfy, the inputs and outputs $v$ and $w$ of $\Delta$ can be deduced from $u$ and $y$, so that $\Delta$ can be identified. In this paper a frequency domain variant of this method is proposed, which is easier to use since a transfer matrix on a finite frequency gridding is to be left inverted in the original technique of Krause and Khargonekar (1990), whereas just the frequency response of this transfer matrix (on a finite frequency gridding) is to be left inverted in our
variant. Moreover our framework allows to study the sensitivity of the identification scheme face to neglected dynamics, measurement noise and other disturbances acting on the closed loop. Using a classical convex closed loop design technique (see Boyd and Barratt (1991)), a controller that reduces this sensitivity while satisfying stability and performance requirements is synthesized. When identifying $\Delta$, the error on the estimated values of $v$ and $w$ can be bounded with the two or infinity norm, so that the output error model $v = \Delta w$ can be identified with a SMI technique (the issue is to build the FPS, i.e. the set of parameters which are compatible with the model structure, the assumptions made on UBB disturbances and all available measurements: see Walter and Piet-Lahanier (1990); Ferreres and M’Saad (1997)), or Robust Least Squares: see ElGhaoui and Lebret (1997). This technique provides a single optimal estimate, which minimizes a worst-case criterion, i.e. this robust version of Least Squares accounts for UBB perturbations on $A$ and $b$, when minimizing with respect to $x$ the two norm of $Ax - b$. The paper is organized as follows. Section 2 presents the identification technique, as well as the influence of a controller on the sensitivity of the closed loop scheme. An example is presented in section 3. Concluding remarks end the paper.

![Diagram](image)

(a) An LFT model $y = F_l(H, \Delta)u$.

(b) A closed loop identification problem.

Fig. 1. An LFT form.

2. AN IDENTIFICATION SCHEME

2.1 Problem statement

Consider the closed loop of figure 1(b), where $\Delta_1$ and $\Delta_2$ are two neglected dynamics satisfying the $H_\infty$ inequality $||\Delta_1(\cdot)||_\infty \leq 1$. The weighting functions $W_i$ are fixed, as well as the controller $K$. Note that the blocks of neglected dynamics could be located elsewhere, the issue is just to obtain bounds of the perturbation signals $\delta u$ and $\delta y$. $\delta u$ and $\delta y$ may also represent measurement noise, which is supposed to be UBB in the frequency domain.

Indeed, frequency domain measurements of signals $r$ (reference input), $u$ and $y$ (plant inputs and outputs) are supposed to be available, and the issue is to estimate with these signals uncertain parameters inside the real structured model perturbation $\Delta$. This is an open or closed loop identification problem, in the sense that the open loop transfer matrix between $u$ and $y$, or the closed loop one between $r$ and $y$ can be estimated, more precisely the value of $\Delta$ inside the open loop LFT model $y = F_l(H, \Delta)u$ or the closed loop one $y = F_l(H, \Delta)r$.

Indeed, when choosing $\delta u = \delta y = \delta z = 0$ in figure 1b, the transfer matrix between $r$ and $y$ can be rewritten as an LFT model $F_l(H, \Delta)$. As a final point $\delta z$, which is introduced after feedback, especially represents the effect of truncation when using FFT algorithms to transform time-domain measurements into frequency-domain ones.

We will proceed in two steps. Under a left invertibility condition we will show how to estimate $v$ and $w$, the inputs and outputs of $\Delta$, from the ideal plant inputs and outputs $\bar{u}$ and $\bar{y}$. Since $v$ and $w$ are estimated in practice with $u = \bar{u} - \delta u$ and $y = \bar{y} + \delta y + \delta z$, where $\delta u$, $\delta y$ and $\delta z$ are UBB, we will briefly show then how to estimate $\Delta$ with a RLS or SMI approach (see otherwise Demourant and Ferreres (2001) for the use of a SMI technique, which provides confidence intervals for estimated parameters). The sensitivity of the closed loop identification scheme is finally investigated, as well as the design of a controller which achieves a trade-off between the sensitivity of the scheme and closed loop stability and performance requirements.

2.2 Estimation of $v$ and $w$

Lemma 1 is the basis of our approach, as well as of the one in Krause and Khargonekar (1990).

**Lemma 1.** With reference to figure 1(a), let:

$$
\begin{bmatrix}
  y \\
  w
\end{bmatrix}
= \begin{bmatrix}
  H_{11} & H_{12} \\
  H_{21} & H_{22}
\end{bmatrix}
\begin{bmatrix}
  u \\
  v
\end{bmatrix} = H
\begin{bmatrix}
  u \\
  v
\end{bmatrix}
$$

(1)

If $H_{12}$ is left-invertible, i.e. there exists $H_{12}^\dagger$ with $H_{12}^\dagger H_{12} = I$, then:

1 A related but different problem is the influence of generating plant input and output signals $u$ and $y$, which are used to identify the open loop plant model, with open or closed loop experiments.
\[
\begin{bmatrix}
  w \\
  v
\end{bmatrix}
= \begin{bmatrix}
  H_{22} H_{12}^I & I \\
  H_{12}^I & 0
\end{bmatrix}
\begin{bmatrix}
  y - H_{11} u \\
  H_{21} u
\end{bmatrix}
\]

**Proof:** simply note that \( y = H_{11} u + H_{12} v \) implies \( v = H_{12}^I (y - H_{11} u) \).

Consider first the approach of Krause and Khargonekar (1990), which needs the left inversion of the transfer matrix \( H_{12}(s) \). Moreover \( H_{12}^I(s) \) must be stable. If \( H_{12}(s) \) is strictly proper or has unstable zeros, a solution proposed in Krause and Khargonekar (1990) is to introduce a scalar and proper transfer function \( F(s) \) in equation (1), so that \( F(s) H_{12}(s) \) is now to be inverted. Nevertheless this technique is not always applicable. In the following counter-example \( H_{12}(s) = \text{diag}((s + 2)/s, 1/(s + 1)(s + 3)) \), no scalar filter can make this transfer matrix proper. The basic idea of our variant is to left invert the matrix \( H_{12}(jw) \) at each point of a frequency gridding, a much easier task. Nevertheless, since \( H_{12}(s) \) is the transfer matrix between \( v \) and \( y \) in figure 1(a), a necessary condition to left invert \( H_{12}(jw) \) is that the number \( n_o \) of measured outputs is greater or equal to the number \( n_u \) of outputs of \( \Delta \). As a consequence, as done in Wolodkin et al. (1997); Demourant and Ferreres (2001), it is more interesting when possible to use a full real block \( \Delta \) instead of a classical diagonal model perturbation \( \text{diag}(\delta I_{n_y}) \) with possibly repeated uncertain parameters \( \delta_i \) since a full real block contains much more parameters.

**Lemma 2.** The transfer matrix between \( r \) and \( y \) in figure 1(b) (with \( \delta u = \delta y = \delta z = 0 \)) can be rewritten as an LFT model \( F_l(H_1, \Delta) \) with:

\[
\begin{align*}
\tilde{H}_{11} &= (I + H_{11} K)^{-1} H_{11} \\
\tilde{H}_{12} &= (I + H_{11} K)^{-1} H_{12} \\
\tilde{H}_{21} &= H_{21} [I - K (I + H_{11} K)^{-1} H_{11}] \\
&= H_{21} (I + K H_{11})^{-1} \\
\tilde{H}_{22} &= H_{22} - H_{21} K (I + H_{11} K)^{-1} H_{12}
\end{align*}
\]

As a key remark Lemma 1 may also be applied to:

\[
\begin{bmatrix}
  y \\
  w
\end{bmatrix}
= \begin{bmatrix}
  \tilde{H}_{11} & \tilde{H}_{12} \\
  \tilde{H}_{21} & \tilde{H}_{22}
\end{bmatrix}
\begin{bmatrix}
  r \\
  v
\end{bmatrix}
\]

i.e. \( v \) is deduced from \( r \) and \( y \) using \( v = \tilde{H}_{12}^I (y - \tilde{H}_{11} r) \), \( w = \tilde{H}_{21} r + \tilde{H}_{22} v \) and \( \tilde{H}_{12} = H_{12}(I + H_{11} K) \). The following Lemma proposes an expression of the estimation error for \( v \) and \( w \), at a given frequency \( \omega \). The \( \omega \) dependence of matrices \( H_{ij} \) and \( \tilde{H}_{ij} \) is dropped out in the following to alleviate the notations. See the appendix for the proof.

**Lemma 3.** Consider figure 1.b with non-zero values for \( \delta u, \delta y, \delta z \). When applying Lemma 1 to the closed loop LFT model, estimates of \( v \) and \( w \) are given by:

\[
\begin{align*}
v_e &= \tilde{H}_{12}^I (y - \tilde{H}_{11} r) = H_{12}^I (I + H_{11} K) y - H_{12}^I H_{11} r \\
w_e &= \tilde{H}_{21} r + \tilde{H}_{22} v_e
\end{align*}
\]

The estimate error is:

\[
\begin{align*}
\delta v &= v_e - v = H_{12}^I H_{11} \delta u + H_{12}^I \delta y + \\
&\quad + H_{12}^I (I + H_{11} K) \delta z \\
\delta w &= w_e - w \\
&= F_l(X, -K) \\
&= \begin{bmatrix}
  \delta u \\
  H_{12}^I (I + H_{11} K) \delta z
\end{bmatrix}
\end{align*}
\]

where \( X = \begin{bmatrix}
  X_{11} & X_{21} \\
  X_{21} & H_{11}
\end{bmatrix} \), \( X_{11} = [(H_{22} H_{12} - H_{21}) (H_{22} H_{12}^I - H_{22})] \) and \( X_{21} = [(H_{12} H_{12}^I - I) H_{11} (H_{12} H_{12}^I - I) H_{12}] \).

**Remarks:**

(i) To estimate \( v \) and \( w \) with the open loop LFT model (i.e. with plant inputs and outputs \( u \) and \( y \), instead of \( r \) and \( y \)), just choose \( K = 0 \) in the above Lemma.

(ii) Estimates of \( v \) and \( w \) are unbiased (\( \delta u = \delta y = \delta z = 0 \) implies \( \delta v = \delta w = 0 \)).

(iii) If \( H_{12}(s) \) is non-square, then most generally \( H_{12}(j\omega) H_{12}^I(j\omega) \neq I \).

Using equations (4) and a bound on \( \delta u, \delta y \) and \( \delta z \) it is possible to put our estimation problem under the standard form of a RLS or SMI problem, with the output error model \( v + \delta v = \Delta (w + \delta w) \). The \( H_{\infty} \) inequality \( \| \Delta(s) \|_{\infty} \leq 1 \) is used. The property of the \( H_{\infty} \) norm as the induced 2 norm allows to evaluate the 2 norm for \( \delta v \) and \( \delta w \) (see Demourant and Ferreres (2001)). The RLS solution can be obtained as the solution of an LMI problem, more precisely a second-order cone program (SOCP), see ElGhaoui and Lebret (1997).

### 2.3 Controller design

#### 2.3.1. Sensitivity of the identification scheme

Assume first that controller \( K \) is a priori given. The frequency range, inside which identification will be performed, is to be determined. With reference to Lemma 3, at a given frequency \( \omega \), \( \delta v \) and \( \delta w \) can be rewritten as:

\[
\begin{bmatrix}
  \delta v \\
  \delta w
\end{bmatrix}
= \begin{bmatrix}
  A_1(\omega) & A_2(\omega) & A_3(\omega) \\
  A_4(\omega) & A_5(\omega) & A_6(\omega)
\end{bmatrix}
\begin{bmatrix}
  \delta u \\
  \delta y \\
  \delta z
\end{bmatrix}
\]

where

\[
\begin{align*}
A_1(\omega) &= \frac{1}{H_{11}(\omega)} \\
A_2(\omega) &= \frac{H_{12}(\omega)}{H_{11}(\omega)} \\
A_3(\omega) &= \frac{H_{12}^I(\omega)}{H_{11}(\omega)} \\
A_4(\omega) &= \frac{H_{21}^I(\omega)}{H_{11}(\omega)} \\
A_5(\omega) &= \frac{H_{22}^I(\omega)}{H_{11}(\omega)} \\
A_6(\omega) &= \frac{H_{22}^I(\omega)}{H_{11}(\omega)}
\end{align*}
\]
It is interesting to visualize various norms of matrices $A_t(\omega)$ as a function of frequency. Assume first that disturbance vectors $\delta u$, $\delta y$ and $\delta z$ are bounded with the infinity (resp. two) norm. To predict the worst-case behavior of the identification scheme, i.e. the maximal error induced by these disturbances on the estimates of $v$ and $w$, the induced infinity (resp. two) norm of matrices $A_t(\omega)$ can be visualized as a function of frequency $\omega$. Frequency points at which this norm is minimal, or at least reasonable, are selected. Nevertheless it is also worth studying the average sensitivity of the identification scheme, with matrix norms which are not induced vector norms, e.g. the Frobenius one.

2.3.2. An open loop convex optimization problem

The frequency gridding $(\omega_i)_{i \in [1,N]}$ inside which identification will be performed, is a priori fixed. The issue is to minimize with the feedback controller $K(s)$ the effect of disturbances $\delta u$, $\delta y$ and $\delta z$ on the estimate errors $\delta v$ and $\delta w$. It seems impossible to globally minimize in equation (4) all 6 possible transfer matrices between disturbances and estimate errors, at least if we limit ourselves to solving convex optimization problems. Nevertheless, we show in the following that the issue of minimizing the transfer matrix, either between $\delta z$ and $\delta v$, or between $(\delta u, \delta y)$ and $\delta w$, can be recast as an open or closed loop convex optimization problem.

We first consider the estimation error $\delta v$ in equation (4). Only the effect of $\delta z$ can be minimized with respect to $K(s)$. Under a closed loop stability constraint, a solution is to minimize $\gamma$ with respect to $K(s)$, under convex constraints $\sigma(H_{12}(j\omega_1)(I + H_{11}(j\omega_1)K(j\omega_1))) \leq \gamma \sigma(H_{12}(j\omega_1))$. More precisely, when rewriting $K(s) = \sum \theta_i K_i(s)$, where filters $K_i(s)$ are fixed, the above equation defines a convex constraint with respect to the design parameters $\theta_i$. Note also that an other norm than $\sigma$ could be used, and that $\gamma = 1$ and $K = 0$ is an initial solution.

2.3.3. Youla parametrization and closed loop convex design

As a preliminary before section 2.3.4, consider the standard design problem of figure 2(a), where $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$ is an augmented plant. The closed loop transfer matrix $F_t(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}$ is a highly nonlinear function of controller $K$. Suppose an initial stabilizing controller $K_0(s)$, whose order is equal to the order of $P_{22}$, is available. Additional inputs and outputs $v$ and $e$ are introduced in $K_0$ (see figure 2(b)), with the key constraint that the transfer matrix between $v$ and $e$ is zero (a solution to achieve this property is to put $K_0$ under the form of an observed state feedback controller). When connecting then a free stable transfer matrix $Q$ to these additional inputs and outputs, $F_t(P, K)$ can be rewritten as $T_1 + T_2QT_3$, where fixed transfer matrices $T_i$ depend on $P$ and $K_0$, while $Q$ is the design parameter. Note then that a norm constraint on the closed loop transfer matrix $T_1 + T_2QT_3$ is convex with respect to $Q = \sum \theta_i Q_i$, where filters $Q_i$ are fixed. As a consequence, when constraining or minimizing the norm of various parts of the closed loop transfer matrix $T_1 + T_2QT_3$, a convex optimization problem with convex constraints is obtained. Optimal values of the design parameters $\theta_i$ are computed, $Q(s)$ is then deduced as well as $K(s)$ (see figure 2(b)).

The used approach is based on convex optimization. The minimization of $||F_t(P, K)||_\infty$ is well established in the context of $H_\infty$ control theory. But convex optimization offers two main advantages. It is possible to minimize $H_\infty$ norm of different SISO or MIMO transfer functions of $F_t(P, K)$ separately instead of minimizing a large MIMO transfer matrix $F_t(P, K)$. Moreover a limited frequency domain for the minimization of $H_\infty$ norm can be considered instead of the whole frequency domain what is less constraining and generally more interesting. These two advantages are fully exploited in our problem. A successful application of convex synthesis is given in Dardenne and Ferreres (1998).

![Fig. 2. The synthesis problem.](image)

2.3.4. A convex closed loop optimization problem

![Fig. 3. Fictitious and physical closed loop transfer matrices.](image)

The issue of minimizing the transfer matrix $T$ between $(\delta u, \delta y)$ and $\delta w$ in equation (4) is especially relevant. First note that the main disturbances in practical identification problems are
undermodelling and measurement noise, i.e. disturbances $\delta u$ and $\delta y$, rather than disturbances $\delta z$ appearing after feedback (typically the effect of truncating time-domain signals when using FFT algorithms). Moreover in a stochastic framework the estimate of an output-error model is usually biased: because of the relation $w + \delta w = \Delta (w + \delta w)$, reducing the magnitude of $\delta w$ will reduce the bias on the estimate of $\Delta$.

With reference to equation (4), $T$ is rewritten as

$$F_i \left( Y_{11} H_{21}, Y_{21} H_{11} \right), -K),$$

where:

$$Y_{11} = \left[ (H_{22} H_{12} H_{11} - H_{21}) H_{22} H_{12} \right]$$

$$Y_{21} = \left[ (H_{12} H_{12} - I) H_{11} (H_{12} H_{12} - I) \right]$$

This LFT transfer matrix is depicted in figure 3 (forget at the moment the additional input $r$ and output $y$). Its core is the interconnection of the nominal open loop plant model ($y = F_i (H(s), \Delta) u = H_{11} (s) u$ if $\Delta = 0$) with controller $K(s)$.

Minimizing $\mathcal{T}$ with respect to $K$ is a deeply non-convex problem, because of the nonlinear expression of $\mathcal{T}$ as a function of $K$. Nevertheless, using Youla parametrization $\mathcal{T}$ is rewritten as $\mathcal{T}_i + \mathcal{T}_2 Q \mathcal{T}_3$, where transfer matrices $\mathcal{T}_i$ are fixed, and minimizing a norm of $\mathcal{T}$ with respect to $Q$ becomes a convex optimization problem.

In order to apply the Youla parametrization technique to $\mathcal{T}$, space-state representations would be needed for $Y_{11}(s)$ and $Y_{21}(s)$, and thus for $H_{12}(s)$. Moreover, $H_{21}(s)$ is an open loop transfer matrix, which may be unstable. To avoid these problems, Youla parametrization is just applied to the transfer matrix $T_K(s)$ between $e$ and $s$ (see figure 3), i.e. $T_K(s)$ is rewritten under the form $T_1(s) + T_2(s) Q(s) T_3(s)$. Then at frequency $\omega_i$ the response of $\mathcal{T}$ can be rewritten as:

$$T(j\omega_i) = Y_{11}(j\omega_i) - H_{21}(j\omega_i) [T_1(j\omega_i) + T_2(j\omega_i) Q(j\omega_i) T_3(j\omega_i)] Y_{21}(j\omega_i)$$

which remains affine with respect to the design parameter $Q(s)$. Thus constraining or minimizing the norm of this closed loop response is a convex optimization problem, and this renders especially possible to compute the minimal sensitivity of the identification scheme, which is achievable by feedback. No local minimum can be obtained.

2.3.5. A trade-off between control and identification objectives

As a first point note that transfer matrix $\mathcal{T}$ between $(\delta u, \delta y)$ and $\delta w$ is a fictitious closed loop transfer matrix. Additional specifications on the physical closed loop transfer matrices can also be accounted for. As an example, redefine in figure

$3 T_K(s)$ as the closed loop transfer matrix between $(e, r)$ and $(s, y)$, where $r$ is the reference input and $y$ the measured output. Here again, $T_K(s)$ can be rewritten under the form $T_1(s) + T_2(s) Q(s) T_3(s)$.

As a consequence, when constraining simultaneously the fictitious transfer matrix $\mathcal{T}$ as well as physical closed loop transfer matrices, it is possible to rigorously study the trade-off between the sensitivity of the identification scheme and desirable closed loop stability and performance properties. More precisely the design objectives are as follows (see figure 1(b)):

1. Stability and satisfactory performance properties for the nominal closed loop.
2. Robust stability and performance face to neglected dynamics $\Delta_1$ and $\Delta_2$.
3. Robust stability and performance face to parametric uncertainties inside the full real block $\Delta$.

4. Reduction of the sensitivity of the closed loop identification scheme, compared to the sensitivity of the open loop one.

Specifications 1, 2 and 4 can be recast into the minimization of a convex objective, under convex constraints, see Boyd and Barratt (1991). Specification 3 is non-convex. A first solution is to solve a non-convex optimization problem including specifications 1 to 4. Otherwise, as often done in practice, specifications 1, 2 and 4 are accounted for in the design process, and specification 3 is checked a posteriori, since efficient computational tools exist to check robust stability and performance properties of an uncertain closed loop, see Ferreres (1999).

3. APPLICATION

For the sake of brevity a short example is now presented. See Demourant and Ferreres (2001) for an application to the aerodynamic model of an aircraft. We consider the following transfer function

$$y = f_i (H(s), [\delta_1 \delta_2], u).$$

Let $\alpha_0 = 1 + \delta_1$ and $\alpha_1 = 0.4 + \delta_2$, i.e. the denominator of the nominal transfer function is $s^2 + 0.4 s + 1$ (the associated poles have a damping ratio of 0.2 and a natural frequency of 1 rad/s). The uncertain transfer function is put under the LFT form $y = F_i (H(s), [\delta_1 \delta_2]; u)$, i.e. the fictitious input $v$ is a scalar signal, while the dimension of $w$ is 2 (the fictitious feedback in figure 1.a is $v = [\delta_1 \delta_2] w$).

Figure 4 shows (in dashed line) the magnitude of the physical transfer function of $T_{y,r}$ between $r$ and $y$, for a definition of these signals (see figure 3, with $K_0$), as well as the influence of $\delta y$ on

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2 The transfer matrix between $r$ and $y$ is a very classical sensitivity function $(I + H_{11} K)^{-1} H_{11}$ for the nominal closed loop plant.
\( \delta v \) (which is independent of feedback) and \( \delta w \). To decrease the peak of \( T_{uv} \), an observed state feedback controller is synthesized, which improves the damping ratio of the poles up to 0.7. Nevertheless, this increases the sensitivity on \( \delta w \) (see the dashes-dotted lines on figure 4).

The technique of sections 2.3.4 and 2.3.5 is applied, i.e. the closed loop properties are improved by adding in the observed state feedback controller a transfer matrix \( Q(s) = \sum_i \theta_i Q_i(s) \). The poles of \( Q(s) \) are chosen as \(-0.5 \pm 0.5 j, -1 \pm j, -3 \pm 3 j, -7 \pm 7 j\), the numerator of \( Q(s) \) is free. This basis contains 18 filters \( Q_i(s) \). A frequency gridding is used, i.e. 100 points between 0.1 and 10 rad/s. The magnitude of \( T_{uv} \) is constrained to remain below the magnitude of \( T_{uv} \) corresponding to the observed state feedback controller. Moreover, the magnitude of the closed loop transfer matrix \( T_{dw,dy} \) between \( \delta y \) and \( \delta w \) is constrained to remain below the magnitude of its open loop counterpart on the whole frequency gridding (this constraint is slightly relaxed with a factor of 1 \%). Finally the magnitude of \( T_{dw,dy} \) is to be minimized between 0.1 and 1 rad/s (trials show that minimizing \( T_{dw,dy} \) at higher frequencies is impossible). The results are quite satisfactory (continuous line on figure 4).

![Graph of the physical transfer function](image)

Fig. 4. Magnitude of the physical transfer function between \( r \) and \( y \) in the upper subfigure, and of the fictitious transfer matrix between \( \delta y \) and \( \delta v \) (resp. \( \delta w \)) in the middle (resp. lower) one. The matrix norm is the two norm in the upper subfigure, the induced infinity norm in the other ones. The dashed line corresponds to the open loop, the dash-dotted one to the observed state feedback controller, the continuous one to the improved controller.

When considering the middle and lower subfigures of figure 4, we choose to identify \( \delta_1 \) and \( \delta_2 \) only between 0.1 and 1 rad/s. A multiplicative noise is added on \( y \), with a maximal magnitude of 1\%

The true values are \( \delta_1 = 0.100 \) and \( \delta_2 = 0.100 \), the estimated ones (with a RLS method) are 0.103 and 0.092.

### 4. CONCLUSION

A two-step frequency domain method was proposed to identify a MIMO LFT model containing uncertain physical parameters. This identification method directly allows to identify physical parameters of a model. The inputs and outputs \( v \) and \( w \) of the real structured model perturbation \( \Delta \) are deduced from the physical inputs and outputs of the open or closed loop LFT model. Then \( \Delta \) can be estimated with a RLS or SMI technique. Indeed it is possible to quantify the effect of neglected dynamics, measurement noise and other disturbances on the estimated values of \( v \) and \( w \), and thus to study the sensitivity of the identification scheme.

Beyond the necessity to operate in closed loop when the open loop plant is unstable, feedback can also be used to reduce this sensitivity. More precisely this reduction is done in two complementary ways. First by avoiding to identify the LFT system at frequencies where this sensitivity is too high. Second by synthesizing with convex closed loop optimization a feedback controller, which achieves a trade-off between closed loop stability and performance requirements and the reduction of the sensitivity of the identification scheme.

### References


