CONTROLLER DESIGN FOR OFF-TRACKING ELIMINATION
IN MULTI-ARTICULATED VEHICLES

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Abstract: The motion of a multi-body autonomous robot as well as of a train-like multi-articulated transportation vehicle is characterized by the deviation of the path of each intermediate vehicle from that of the leading one (off-tracking). In this paper, we make use of an innovative junction technique, which allows the kingpin to slide along the axis of the leading vehicle, something that proved to be very effective in reducing off-tracking. We propose two controllers for the elimination of the off-tracking phenomenon, in both robotic and transportation multi-articulated vehicles; the one is heuristically derived while the other one is based on steady-state off-tracking when an n-trailer vehicle moves on a circular trajectory. Simulation results for various cases, without and with the sliding kingpin system, showed that significant off-tracking reduction or even elimination can be achieved.

Keywords: Transportation systems, multi-articulated vehicles, vehicle trains, off-tracking.

1. INTRODUCTION

In the autonomous robotics field, the goal is to build physical systems that accomplish useful tasks without human intervention while operating in unknown environments. On the other hand, in Intelligent Transportation Systems the goal is similarly to construct transportation vehicles, intelligent enough to be driven with as less as possible human intervention. In ground freight transportation, heavy-duty trucks may be combined to form a “truck train” or “road train”, consisting of a number of semi-trailers and a single high-power tractor [Manesis 1998]. A similar train is formed in multi-body autonomous robots. Off-tracking, that is the deviation of the path of each trailer from those of the leading vehicle or robot, is among the basic technical problems that must be solved [Bushnell et. al 1994], [Altafini and Gutman 1998].

The combination of many trailers with a single tractor forming a “road train” or “truck-train” is called hard platooning. Road trains may consist of a number of trailers (at least 3 and possibly up to 20) and a lead tractor. For such a road train of trucks to be safely driven in a multi-lane highway, a number of issues must be dealt with. Besides economic and political considerations, technical issues include the necessity for a set of traffic and driving rules that must be defined and obeyed, the solution of the path-following problem, the mechanical realization, and the space limitations outside a highway. The advantages of using truck trains in highway freight transportation are discussed in [Manesis 2001].
The motion of the n-trailer system is subject to nonholonomic constraints (rolling without slipping) so it has been studied as a class of nonholonomic systems by many researchers and has both theoretical and practical interest. The work [Kolmanovsky and McClamroch, 1995] is an excellent survey of recent advances in control of nonholonomic systems. The main problem that has attracted most of the attention is path following. We know of a few works only that consider the off-tracking problem. A closed-form expression for the off-tracking of the rear pivot point of a simple tractor-semi-trailer vehicle can be found in [Alexander and Maddocks, 1998] while off-tracking bounds for a car pulling trailers have been derived in [Bushnell et. al., 1994]. For example in [Altafini, 1998] the path-following problem with reduced off-tracking is addressed for the n-trailer system. This is achieved by keeping track of the error distance of each of the middle points of the axles of the vehicle from the path using different moving frames. In [Nakamura et. al. 2000] different passive steering mechanisms as well as control laws are presented for nonholonomic trailer systems. The main focus of such mechanisms is on reducing passive tracking error from tractor’s trajectory and little attention was paid on active motion control.

In Section II we describe the multi-articulated vehicle model and the off-tracking problem. Section III contains a brief description of the sliding kingpin system together with the state equations of the multi-articulated vehicle when sliding is applied. In section IV we describe two new controllers one of heuristic type and the other based on the compensation for the steady-state off-tracking when the leading vehicle moves in a circular trajectory, while in section V simulation results are presented with and without sliding. Section VI contains conclusions and discussion about the results and some future research problems.

2. THE MULTI-ARTICULATED VEHICLE

In this section we describe briefly the model of the multi-articulated vehicle that is common for both robotic and transportation vehicles. It is a long and complex vehicle system consisting of a high power tractor pulling a number of passive robot bodies or semi-trailers as shown in Fig.1. The state equations of the above system, called also n-trailer system, with a driving axle and hence a steering angle for the tractor are

\[
\begin{align*}
\dot{x}_0 &= U_1 \cos \theta_1 \\
\dot{y}_0 &= U_1 \sin \theta_1 \\
\dot{\phi} &= U_2 \\
\dot{\theta}_0 &= \frac{U_1}{L} \tan \phi \\
\dot{\theta}_i &= \frac{U_1}{L} \sin(\theta_{i-1} - \theta_i) \\
\end{align*}
\]

where \(x_0, y_0\) are the Cartesian coordinates of the leading vehicle (tractor) and \(U_1, U_2\) are the two control inputs, the linear velocity and the steering angle rate respectively [Laumond 1993]. The above equations are derived from algebraic manipulation of the \(2n\) holonomic constraints and the \(n+1\) nonholonomic constraints under the assumption of the same length for all trailers. The only difference between the multi-body robotic systems and truck-train is the magnitude of the different physical quantities (length, velocity, steering angle limits, weight, etc.)

Off-tracking is defined as the deviation of the semi-trailers’ axles or the kingpin hitch from the path of the steering axle of the leading vehicle. In the case of truck-trains, it is more imperative than in any other case that the last semi-trailer follow exactly the path of the lead tractor during a turn for lane change or a turn due to the curvature of the highway. Otherwise it will be possible for the last semi-trailer to violate the outer boundary of the highway or to crash with an adjacent car during a lane change although both keep invariant their relative velocity. It is known that the driver of any long truck-train, because of the off-tracking of the rear trailers, turns the tractor far towards the desired path in order to avoid this phenomenon. When we deal with mobile robots the major problems are to find an obstacle-free path and path following control. However, in the case of multi-articulated robotic vehicles we must take into consideration the off-tracking phenomenon when finding an obstacle-free path. The reason is that the last trailer may collide with obstacles if the vehicle attempts to follow the designed path for the leading vehicle with off-tracking neglected. One efficient way to solve this problem is to find an obstacle-free path for the leading vehicle, add a controller for path
following and use another kingpin controller for off-tracking elimination.

3. THE SLIDING KINGPIN SYSTEM

The off-tracking can be eliminated by sliding each trailer with respect to the previous one, a technique firstly described in [Manesis 1998]. According to this technique the kingpin hitch in each semi-trailer slides in a direction perpendicular to the longitudinal axle (i.e. along the rear axle) of the trailer by a distance $S_i$.

In this section we present briefly the sliding kingpin system and the state-equations of the multi-articulated vehicle when sliding is used, together with the assumptions that are made during the derivation of the equations. Consider two intermediate semi-trailers of a truck train as shown in Fig.2. The position of each semi-trailer $P_i$, is taken to be the middle point of the $ith$ semi-trailer’s rear axle.

$$
\text{Fig.2. The kingpin slides along the axle when the semi-trailer turns.}
$$

Position $P_i$ is defined by the pair $(x_i, y_i)$ in the Cartesian coordinates system while $\theta_i$ is the orientation of the $ith$ semi-trailer with respect to the horizontal axis. To simplify derivation of the truck train model we will not consider initially a steering angle for the tractor, since the extension of the model to cover this case is simple.

It has been pointed out [Bushnell et al 1994] that when the lead car of a single trailer system is traveling along a circle of radius $R_i$, then the trailer is traveling along a circle of radius $R_j$ with the same center, where $R_i < R_j$. In order to compensate for this path deviation of the trailer, we suppose that the kingpin hitching point slides from the point $P_i$ to the point $P_{si}$ by a distance $S$. The following assumptions are necessary for deriving the mathematical model:

a) All trailers have the same length $L$.
b) Each trailer is modeled as having only one axle.
c) Each trailer is assumed to be hooked up to the midpoint of the rear axle of the preceding trailer.
d) By sliding the location of the kingpin, the weight of the trailer shifts toward an outer direction, which doesn’t affect the kinematic behavior of the train.
e) The unbalanced pulling point (when the kingpin sliding is nonzero) does not cause skidding of the whole axle.
f) The sliding of the kingpin can be performed with the trailer fully loaded via a hydraulic mechanism.

In the general case of a $n$-trailer truck train, we have the classical $(n+1)$ nonholonomic constraints imposed by the rolling and non-slipping condition

$$
\dot{x}_i \sin \theta_i - \dot{y}_i \cos \theta_i = 0 \quad (2)
$$

and $2n$ holonomic equations introduced by the corresponding links, which, because of the sliding $S = \overrightarrow{P_iP_{si}}$ (see Fig. 2), are of the form

$$
\begin{align*}
x_{isi} &= x_i - L_i \cos \theta_{isi} + S_i \sin \theta_i \\
y_{isi} &= y_i - L_i \sin \theta_{isi} - S_i \cos \theta_i
\end{align*}
$$

Taking the derivatives of the holonomic Eq. (3), combining them with Eq. (2) and eliminating $\dot{x}_i, \dot{y}_i$ leads to a system of $(n+1)$ equations. The solution of this system combined with the equations of motion of the tractor with steering angle and under the assumption $\dot{S}_i = 0, \dot{L}_i = L$ yields

$$
\begin{align*}
\dot{x}_i &= \frac{U}{E} [L \cot \phi_i] \\
\dot{y}_i &= \frac{U}{E} [L \tan \phi_i] \\
\dot{\theta}_i &= \frac{U}{E} [L + S_i \tan \theta_i] \\
\dot{\phi}_i &= \frac{U}{E} [L \cos(\phi_i - \phi)] \\
\dot{\phi} &= \frac{U}{E} [L \cos(\phi_i - \phi)] \\
\dot{\phi} &= \frac{U}{E} [L \cos(\phi_i - \phi)] \\
\end{align*}
$$
where \(x_0, y_0\) are the Cartesian coordinates of the leading vehicle, \(\vartheta_0\) its orientation, \(\vartheta_j, i = 1, 2, ..., n\) the orientation angle of the \(i^{th}\) trailer, \(U_1, U_2\) the two control inputs linear velocity and steering angle rate respectively, \(L\) the length of each trailer and \(S\) the sliding distance, which is determined from the controller.

4. CONTROLLER DESIGN

Equations (1) and (4) describe the kinematic behavior of an \(n\)-trailer system without and with sliding, respectively. In a multi-articulated vehicle two different controllers are used the one for path following and the other for off-tracking elimination regulating the sliding distance in the sliding kingpin system. For path following issues the linear velocity and the steering angle rate of the leading vehicle are the control inputs. In the classical case, the “driver” regulates the above control inputs in such a way as to achieve kinematic stability and the desirable trajectory tracking. For an autonomous multi-body robot moving inside a limited laboratory or industrial environment, the embedded controller regulates the control inputs based on a control algorithm for path following. The overall structure of the control system for an \(n\)-trailer vehicle is depicted in Fig. 3. The first controller for off-tracking elimination that we use is heuristically found based on basic control engineering principles and is given by:

\[
S_i = K_i \delta \vartheta_0 \quad (5)
\]

where \(\delta \vartheta_0\) is the orientation of the leading vehicle.

Following the procedure below we derive the equations for the second “closed-loop” controller for each kingpin of the \(i^{th}\) trailer. It is known that the curve radius for a vehicle is given by \(r = \frac{U}{\vartheta} = \frac{U}{\delta \vartheta}\). So in general and for the \(i^{th}\) trailer will be given by \(r = \frac{U_i}{\delta \vartheta} \quad (6)\). From the set of equations (1) we have that

\[
\dot{\vartheta}_i = \frac{U_i}{L} \left( \prod_{j=1}^{i} \cos(\vartheta_{j-1} - \vartheta_i) \right) \sin(\vartheta_{i-1} - \vartheta_i) \quad (7)
\]

By combining (6), (7) and taking into consideration the relation

\[
U_s = U_i \prod_{j=0}^{i-1} \cos(\vartheta_j - \vartheta_{j+1}) \quad (8)
\]

and after some algebraic manipulation it yields that

\[
r_i = L \cot(\vartheta_{i-1} - \vartheta_i) \quad (9)
\]

We conclude from the last relation that the curve radii for different trailers are different so it is logical to introduce different sliding for each trailer. In [Bushnell et al. 1994] was proven that if the leading vehicle travels along a circular trajectory with radius \(r\) (whereas \(Lr\) > 0) then the trailer converges to a circular trajectory with radius \(R = \sqrt{L^2} - r\). In order for the leading vehicle and the semi-trailer follows the same circular trajectory we introduce the following lemma.

**Lemma 1**

If the kingpin sliding is given by \(S = \sqrt{r^2 + L^2} - r\) then the trailer in the steady state follows the circular trajectory with radius \(r\) of the leading vehicle,

**Proof**

According to [Bushnell et. al. 1994] the trailer in steady-state will travel a circular trajectory of radius...
\[ R_{ss} = \sqrt{r_z^2 - L_z^2} \] (Fig. 4), whereas \( r_z = r + S \). So we have that
\[ R_{ss} = \sqrt{r_z^2 - L_z^2} = \sqrt{(r + S)^2 - L_z^2} = \ldots \]
After some algebraic manipulations, we conclude that \( R_{ss} = r \).

The above lemma can be extended for the \( n \) trailers case following the same procedure, so the sliding for the \( i^{th} \) trailer will be given by
\[ S_i = \sqrt{r_i^2 + L_i^2} - r_i \]
(10)
By combining (9) and (10) we find that the different sliding distances that we must apply to the different trailers are given by the relation
\[ S_i = L \frac{1 - \cos(\theta_i - \theta_i^{*})}{\sin(\theta_i - \theta_i^{*})} \]
(11)

5. SIMULATION RESULTS

To test the controllers described in the last section, the Matlab/Simulink simulation environment was used and independently verified through Mathematica. Simulation results without the sliding kingpin mechanism are shown in Fig. 5. The individual trajectories of a truck train with 3 trailers traveling on a \( 90^\circ \) circular arc, emphasize the off-tracking deviation. Fig. 6 shows the corresponding trajectories derived after the application of the sliding kingpin mechanism, where the sliding distance is determined from controller (11). Fig. 7 shows the corresponding trajectories, when controller given by (11). Observing the figures we notice that the simulation results are much better when controller (11) is used than the controller (5). In all simulations we assume a multi-articulated vehicle with the same length for all trailers and tractor, equal to one unit length.
6. CONCLUSIONS

Off-tracking is one of the most significant problems occurring in articulated vehicles. The sliding kingpin is a technique for correcting such deviations. In this paper, a controller for adjusting the sliding distance of a kingpin sliding mechanism has been proposed based on the theoretical steady-state off-tracking when the leading vehicle moves in a circular trajectory. Its response was compared to another heuristically found controller that performs also well. Both designs have been validated through simulation, whose results showed satisfactory performance of both designs. However, the analytically designed controller fared better in the steady state part of the circular trajectory. The main topic for future research is to derive equations for the n-trailer system with the sliding kingpin mechanism without making the assumption that the derivative of sliding distance is zero and find out how the designed controller affects the behavior of the vehicle in this situation.

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Acknowledgements

This research work is partially supported by Karatheodori Program of the Research Commission of the University of Patras.