HIERARCHICALLY CONSISTENT CONTROLLED DISCRETE EVENT SYSTEMS

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Abstract: In the hierarchical control of discrete event systems (DES), the hierarchical consistency expresses the requirement that a control task is solvable within the model at a given level if it is in fact executable by the infrastructure one level down. In this paper we present a method for construction of a two level hierarchy of DES with hierarchical consistency. By application of a generalized model for controlled DES for the high level system, hierarchical consistency is obtained directly with no refinement of the hierarchy. This approach makes itself distinct from other approaches for hierarchical control for that they all add complexity by refining the the hierarchy to ensure hierarchical consistency.

Keywords: Discrete-Event Dynamic Systems, Hierarchical Control, Consistency

1. INTRODUCTION

In the Ramadge & Wonham (RW) framework for supervisory control of discrete event systems (DES), although the supervisor synthesis algorithms have polynomial complexity with the number of states of the system, the number of states grows exponentially with the number of system components (Ramadge and Wonham, 1989). The general idea of vertical decomposition of the system to reduce the overall complexity is considered in the hierarchical control of DES, first introduced in Zhong and Wonham (1990) and also subject of Wong and Wonham (1996), Pu (2000) and Hubbard and Caines (2002).

In the hierarchical control of DES, the hierarchical consistency expresses the requirement that a control task is solvable within the model at a given level if it is in fact executable by the infrastructure one level down. As it will be shown later, to achieve hierarchical consistency in a two level hierarchy, the above approaches impose some conditions to the hierarchy. If the conditions are not valid, the approaches propose refinements for the hierarchy by insertion of new high level events and modification of the low level system until the conditions are valid. Therefore this refinement adds complexity to build the hierarchy with hierarchical consistency.

In this work, hierarchical consistency is achieved for a two level hierarchy with no refinement for the hierar-
chy. This is done by modelling the high level system using a generalized framework for controlled DES introduced in Cury et al. (2001). It is also provided a constructive procedure to obtain the complete hierarchy, given the low level system and the high level set of events.

The paper has the following outline: section 2 presents the problem formulation; section 3 reviews the generalized framework for supervisory control of DES of Cury et al. (2001); section 4 introduces the proposed model for the high level system; section 5 contains the main results of the paper; section 6 presents the method to build the hierarchy illustrated by an example; section 7 makes some comments on related work, summarizes the contributions of the paper and give directions of future research.

2. PROBLEM FORMULATION

This section presents the basic problem of hierarchical control for DES and the concept of hierarchical consistency.

We introduce some preliminary language formalism to describe the models from Wonham (1999). Let $\Sigma$ be a finite set of symbols, $\Sigma^+$ be the set of all finite length strings formed by concatenation of symbols in $\Sigma$, and $\Sigma^*$ be the set $\Sigma^+$ plus the empty string $\epsilon$. Given two strings $s, t \in \Sigma^*$, $s$ is a prefix of $t$, $s \leq t$, if there exists $u \in \Sigma^*$ such that $s \cdot u = t$; also, $s$ is a strict prefix of $t$, $s < t$, if $s$ is a prefix of $t$ and $s \neq t$. Any subset $L$ of $\Sigma^*$ is a language on $\Sigma$. Given a language $L$ on $\Sigma$, the prefix closure of $L$ is a language on $\Sigma$, denoted by $\overline{L}$, that contains every prefix of strings in $L$. A language is said to be prefix closed if it is equal to its prefix closure.

Consider the two level hierarchical control scheme in figure 1, where the low level DES is $D_{lo}$ and the high level DES is $D_{hi}$ (Zhong and Wonham, 1990). The behavior of $D_{hi}$ is an abstraction of the behavior of $D_{lo}$, generated by an information channel $inf_{lohi}$. We consider that the control action of supervisor $f_{hi}$ is virtual, that is, it is in fact implemented by a supervisor $f_{lo}$ which controls $D_{lo}$ following the directives of supervisor $f_{hi}$, transmitted through by the command channel $com_{hi}$.

The model for $D_{lo}$ is the standard RW model for controlled DES (Ramadge and Wonham, 1989). Let $\Sigma_{lo}$ be the set of events of $D_{lo}$. $D_{lo}$ is defined by the pair $(L_{lo}, L_{m,lo})$, where $L_{lo} \subseteq \Sigma_{lo}$ is a prefix closed language representing every string that can be generated by $D_{lo}$: $L_{m,lo} \subseteq L_{lo}$ is a language of marked strings, that is, defining completed tasks for $D_{lo}$. The event control mechanism for $D_{lo}$ is defined by partitioning the alphabet $\Sigma_{lo}$ into a set of controllable events, $\Sigma_{lo,c}$, that can be inhibited, and a set of uncontrollable events, $\Sigma_{lo,u}$.

The following presents a summary of the supervisory control results of Ramadge and Wonham (1989). A supervisor for $D_{lo}$ is a map $f_{lo}: L_{lo} \rightarrow 2^{\Sigma_{lo}}$ for that given $s \in L_{lo}$, $f_{lo}(s)$ is the set of enabled events after $s$. The closed loop $f_{lo}/D_{lo}$ is characterized by the language $L(f_{lo}/D_{lo})$, the set of all strings in $L_{lo}$ that survive under supervision, and $L_m(f_{lo}/D_{lo}) = L(f_{lo}/D_{lo}) \cap L_{m,lo}$, the set of closed loop marked strings. A non-blocking supervisor $f_{lo}$ is one that $L_m(f_{lo}/D_{lo}) = L(f_{lo}/D_{lo})$, that is, all closed loop strings are prefixes of closed loop marked strings. For a language $E_{lo} \subseteq \Sigma_{lo}$, there is a non-blocking supervisor $f_{lo}$ for $D_{lo}$ such that $L_m(f_{lo}/D_{lo}) = E_{lo}$ if and only if $E_{lo}$ is $L_{m,lo}$-closed and controllable with respect to $D_{lo}$. Given $D_{lo}$ and $E_{lo} \subseteq \Sigma_{lo}$ there is a unique maximal controllable and $L_{m,lo}$-closed language contained in $E_{lo}$, denoted by sup $CF(E_{lo})$. Given $E_{lo} \subseteq \Sigma_{lo}$, if sup $CF(E_{lo})$ is non-empty, the supervisor that corresponds to the less restrictive behavior of $D_{lo}$ that follows $E_{lo}$ can be implemented by using a finite state generator for sup $CF(E_{lo})$.

Let $\Sigma_{hi}$ be the event set for $D_{hi}$. The information channel $inf_{lohi}$ is modelled by a reporter map $\theta : L_{lo} \rightarrow \Sigma_{hi}^*$, formally defined by the recursion $\theta(\epsilon) = \epsilon$ and $\theta(s \sigma) = \theta(s) \theta(\sigma)$, where $\epsilon$ represents the empty string on both alphabets, $s \in L_{lo}$, $\sigma \in \Sigma_{lo}$, and $\tau \in \Sigma_{hi}$ (Zhong and Wonham, 1990). In words, a reporter map notifies the occurrence of events in $D_{hi}$ by the observation of the sequences generated by $D_{lo}$. At this point, let $D_{hi}$ be a controlled DES generating the language $\theta(L_{lo}) = \{ t \in \Sigma_{hi}^* | (\exists s \in L_{lo}) \theta(s) = t \}$.

There is hierarchical consistency between $D_{lo}$ and $D_{hi}$ if and only if, for any $E_{hi} \subseteq \Sigma_{hi}^*$, if there exists a non-blocking supervisor $f_{hi}$ for $D_{hi}$ such that $L_m(f_{hi}/D_{hi}) = E_{hi}$, then there exists a non-blocking supervisor $f_{lo}$ for $D_{lo}$ such that $\theta(L_m(f_{lo}/D_{lo})) = E_{lo}$.

$^3 E \subseteq \Sigma_{lo}^*$ is said to be controllable with respect to $D_{lo}$ if $K \cdot \Sigma_{lo,u} \cap L_{lo} \subseteq K$. $K \subseteq L$ is said to be $L$-closed if $K \cap L = K$. 

Fig. 1. Scheme for hierarchical control of DES.
There is strong hierarchical consistency between $D_l$ and $D_{hi}$ if and only if for any $E_{hi} \subseteq \Sigma_{hi}$ there exists a non-blocking supervisor $f_{io}$ for $D_{hi}$ such that $L_m(f_{hi}/D_{hi}) = E_{hi}$ if and only if there exists a non-blocking supervisor $f_{io}$ for $D_l$ such that $\theta(L_m(f_{io}/D_l)) = E_{hi}$. The hierarchical control problem for DES is: given $D_{io}$ and $\Sigma_{hi}$, find $D_{hi}$ with (strong) hierarchical consistency.

3. A GENERALIZED MODEL FOR CONTROLLED DES

This section reviews the generalized model for controlled DES of Cury et al. (2001).

Given a set of events $\Sigma$, a controlled DES $D$ on $\Sigma$ is a tuple $(L, \Gamma) \subseteq (\Sigma^* \times 2^{\Sigma^*} \times (M, N))$, where $L \subseteq \Sigma^*$ is a prefix closed language and $\Gamma$ is a map of $s \in L$ into control sets $\Gamma(s) \subseteq 2^{\Sigma^*} \times \{M, N\}$. For the system $D$, the language $L$ represents the set of all strings in $\Sigma$ that can be generated by the system, and for a string $s \in L$, $\Gamma(s)$ is a set of controls $\gamma \in 2^{\Sigma^*} \times \{M, N\}$, where $\gamma$ is the set of enabled actions after $s$, and if $\# = M$, $s$ is considered marked, and if $\# = N$, $s$ is considered not-marked. The generalized model controlled DES above defined differs from the standard RW model in that the control set depends on the string generated by the system, and the marked strings are determined by the control rather than being a subset of all strings.

The following defines some operations for controls and control sets. For the set $\{M, N\}$, define the partial order $\geq$ and the operations $\lor$ and $\land$ as for the binary set $\{0, 1\}$, with $M$ playing the role of 1. For controls $\gamma_1 \geq \gamma_2$ in $2^{\Sigma^*} \times \{M, N\}$, $\gamma_1 \lor \gamma_2 \geq \gamma_2$, if and only if $\gamma_2 \geq \gamma_1$ and $\# \geq \#_1$; and the union is $\gamma_1 \lor \gamma_2 = (\gamma_1 \lor 2^{\gamma_2})(\#_1 \lor \#_2)$. For control sets $\Gamma_1$ and $\Gamma_2$ in $2^{\Sigma^*} \times \{M, N\}$, $\Gamma_1 \geq \Gamma_2$ if and only if for all $\gamma_2 \in \Gamma_2$ there is $\gamma_1 \in \Gamma_1$ such that $\gamma_2 \lor \gamma_2 \geq \gamma_1$; and the union $\Gamma_1 \lor \Gamma_2$ is a set of controls $\gamma \in 2^{\Sigma^*} \times \{M, N\}$ for which there is $\gamma_1 \lor \Gamma_2 \in \Gamma_1$ and $\gamma_2 \lor \Gamma_1 \in \Gamma_2$ such that $\gamma = \gamma_1 \lor \gamma_2$. The results for supervisory control in Cury et al. (2001) are the following. Given the CDES $D = (L, \Gamma)$, a supervisor for $D$ is a map $f : L \rightarrow 2^{\Sigma^*} \times \{M, N\}$ which maps $s \in L$ to the control $f(s) \in \Gamma(s)$. For $s \in L$, if $f(s) = \gamma_1 \geq \gamma_2$, the active event set in $L$ after $s$ is restricted to $\gamma \cap \Sigma_L(s)$ and $s$ is considered as marked if $\# = M$, otherwise, not-marked. The closed loop behavior $f(D) \subseteq L$, the strings in $L$ allowed by $f$ under supervision, and a marked language $L_m(f/D) \subseteq L(f/D)$, the strings in $L(f/D)$ where the supervisor chooses a control with the $M$ attribute. A non-blocking supervisor is one for which $L(f/D) = L_m(f/D)$. Given $D = (L, \Gamma)$ and $K \subseteq L$, $K$ is said to be $\Gamma$-compatible if and only if $K = \emptyset$ or for all $s \in K$, there is $\gamma_1 \lor \Gamma(s) = \gamma \cap \Sigma_L(s) = \gamma_K(s)$, and $\# = M$ if and only if $s \in K$, otherwise $\# = N$. Given $D = (L, \Gamma)$ and $L \subseteq L$, there is a non-blocking supervisor $f$ such that $L_m(f/D) = K$ if and only if $K$ is $\Gamma$-compatible. For the case that the control set $\Gamma(s)$ is closed for arbitrary unions of controls for all $s \in L$, there is a unique maximal $\Gamma$-compatible language contained in $K$, denoted by $sup(G(K))$. The maximal $\Gamma$-compatible language is used to implement the less restrictive supervisor for a desired behavior.

4. MODEL FOR THE HIGH LEVEL SYSTEM

This section introduces the model for $D_{hi}$ for the two level hierarchy in figure 1.

Let $w : L_{lo} \rightarrow \Sigma_{hi} \cup \{\tau_0\}$ be the tail map, defined recursively as $w(\epsilon) = \tau_0$, $w(s \sigma) = \tau_0$, if $\theta(s \sigma) = \theta(s)$, and $w(s \sigma) = \tau$, if $\theta(s \sigma) = \theta(s) \tau$, where $\tau_0$ is the silent event, a new symbol not in $\Sigma_{hi}$, representing that the reporter map has not notified the occurrence of a new event, $s \in L_{lo}$, $\sigma \in \Sigma_{lo}$ and $\tau \in \Sigma_{hi}$ (Zhong and Wonham, 1990). Define set $L_{voc} \subseteq L_{lo}$ as the set of vocal strings of $D_{lo}$, the strings $s \in L_{lo}$ for that $w(s) \neq \tau_0$ and the empty string $\epsilon$. A string in $L_{lo}$ that is not vocal is a silent string.

For the string $s \in L_{lo}$, define the following set $L(s) = \{u \in L_{lo} | (\forall u' \in \Sigma_{lo}^+) u' < u \Rightarrow w(s \cdot u') = \tau_0\}$. $L(s)$ is a prefix closed language on $\Sigma_{lo}$ that contains, besides the empty string, all the non-empty strings in $\Sigma_{lo}$ that, when concatenated with $s$, form either a silent string or a vocal string that occur in $D_{lo}$ after $s$ and until the the reporter map notifies the occurrence a new high level event. Define also the sets $L_{voc}(s) = \{v \in L(s) - \{\epsilon\} | w(s \cdot v) \neq \tau_0\}$ and $L_{m}(s) = L_{voc}(s) \cup \{u \in L(s) | s \cdot u \in L_{m,lo}\}$. $L_{voc}(s)$ contains the non-empty strings in $L(s)$ that correspond to vocal strings in $D_{lo}$, and $L_{m}(s)$ contains $L_{voc}(s)$ plus the strings in $L(s)$ that correspond to marked strings in $D_{lo}$. Finally, define the subsystem $D(s)$ as the controlled DES in the RW framework $(L(s), L_{m}(s))$, with the same control structure than $D_{lo}$, that is, defined by the partition of $\Sigma_{lo}$. The subsystem $D(s)$ corresponds to the behavior of $D_{lo}$ after the occurrence of $s$ and until the notification of a new high level event by the reporter map, and a task for the subsystem is to reach either a vocal or a marked string of $D_{lo}$.

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4 The strong hierarchical consistency appears in Wong and Wonham (1996) with the name control consistency.

5 For $L \subseteq \Sigma^*$ and $s \in L$, the active event set in $L$ after $s$ is $\Sigma_L(s) = \{\sigma \in \Sigma | s \sigma \in L\}$.
Also, for the string $s \in L_{lo}$ define the set $\Sigma_{voc}(s) = \{ \sigma \in \Sigma_{hi} \mid (\exists v \in L_{voc}(s)) \tau = w(s \cdot v) \}$, that contains the next events to be reported to the high level after the occurrence of $s$. Therefore, let $\Gamma_{voc}(s)$ be the set of high level controls $\gamma_{\#} \in 2^{\Sigma_{hi}} \times \{M, N\}$ such that (i) there is a non-blocking supervisor $f$ for the subsystem $D(s)$ such that $w(s \cdot [L_m(f/D(s)) \cap L_{voc}(s)]) = \gamma \cap \Sigma_{voc}(s)$, and (ii) if $\# = M$ then $L_m(f/D(s)) = L_{voc}(s) \neq \emptyset$, else $L_m(f/D(s)) = L_{voc}(s) = \emptyset$. The set $\Gamma_{voc}(s)$ is the control set that can be implemented in the high level by means of supervisory control after the occurrence of $s$. Notice that a control with attribute $M$ corresponds to a supervisor that allows marked strings in $D_{lo}$, and a control with attribute $N$ corresponds to a supervisor that don’t enable marked strings in $D_{lo}$. Call $\Gamma_{voc}(s)$ the vocal control set for $s$.

The computation of the vocal control set for $s \in L_{lo}$ is done by solving a supervisory control problem for the subsystem $D(s)$. Define the following specification languages: for the control $\gamma_M$, $E_s(\gamma_M) = L_m(s) - \{ u \in L_m(s) \mid w(s \cdot u) \not\in \gamma \}$, and for $\gamma_N$, $E_s(\gamma_N) = L_m(s) - \{ u \in L_m(s) \mid w(s \cdot u) \not\in \gamma \}$. The specification $E_s(\gamma_M)$ inhibits every vocal string after $s$ whose output is not an event in $\gamma$, and $E_s(\gamma_N)$, besides inhibiting every vocal string after $s$ whose output is not an event in $\gamma$, also inhibits the silent marked strings after $s$.

Proposition 1. For $s \in L_{lo}$ and $\gamma_{\#} \in 2^{\Sigma_{hi}} \times \{M, N\}$, if (i) $K = \sup CF(E_s(\gamma_{\#})) = \emptyset$, (ii) $w(s \cdot [K \cap L_{voc}(s)]) = \gamma \cap \Sigma_{voc}(s)$, and (iii) if $\# = M$, then $K - L_{voc}(s) = \emptyset$, else $K - L_{voc}(s) \neq \emptyset$, then $\gamma_{\#} \in \Gamma_{voc}(s)$.

By application of proposition 1, the vocal control set for $s \in L_{lo}$ is built by testing if each control $\gamma_{\#}$ in the set $2^{\Sigma_{hi}} \times \{M, N\}$ is an element of the vocal control set.

Finally, define $D_{hi}$ as the controlled DES in the generalized framework of Cury et al. (2001) ($L_{hi}, \Gamma_{hi}$) on $\Sigma_{hi}$. The language $L_{hi} \subseteq \Sigma_{hi}^{\omega}$ is given by $L_{hi} = \theta(L_{lo})$, the image of $L_{lo}$ by the reporter map. For $t \in L_{hi}$ define the inverse image map as $\theta^{-1}(t) = \{ s \in L_{lo} \mid \theta(s) = t \}$. For $t \in L_{hi}$ the control set $\Gamma_{hi}(t) = \bigcup \Gamma_{voc}(v)$, for $v \in \theta^{-1}(t) \cap L_{voc}$. Therefore, each control set for $t \in L_{hi}$ is the union of the vocal control sets for the vocal strings corresponding to $t$. It can be proved that $\Gamma_{hi}(t)$ is closed for the union of controls.

5. MAIN RESULTS

This section contains the proof that there is hierarchical consistency for the proposed two level hierarchy. Given a $\Gamma_{hi}$-compatible specification $E_{hi} \subseteq \Sigma_{hi}^*$ for $D_{hi}$, refer figure 1, there is a non-blocking supervisor $f_{hi}$ for $D_{hi}$ such that $L_m(f_{hi}/D_{hi}) = E_{hi}$. Define $f_{hi}$ for $t \in E_{hi}$ as $f_{hi}(t) = \gamma_{\#}$ for $t \in E_{hi}$ and $f_{hi}(t) = \gamma_N$ for $t \in E_{hi} - E_{hi}$, where $\gamma \cap \Sigma_{hi}(t) = \Sigma_{lo}(t)$. The control input $\gamma_{\#}$ is not actually applied to $D_{hi}$ by $f_{hi}$, it is in fact sent through $\text{com}_hi$ as a control directive for a supervisor $f_{lo}$ for $D_{lo}$. To follow the directive $\gamma_{\#}$ of $f_{hi}$ at $t$, the supervisor $f_{lo}$ for $D_{lo}$ decomposes the current string $s \in L_{lo}$ into two strings $s = v \cdot u$, where $v = \sup \{ v' \in L_{voc} \mid v' \leq s \}$, the greatest prefix of $s$ which is a vocal string, and $u \in L(v)$. Notice that $t = \theta(v) = \theta(s)$. The control policy of $f_{lo}$ at $s$ is the same that a supervisor for the subsystem $D(v)$ applies at $u$ to follow the directive of $f_{hi}$ at $t$, given by the optimal control $\gamma_{\#} \subseteq \Gamma_{voc}(v)$, $\gamma_{\#} \subseteq \Sigma_{lo}$, and the language $E_v(\gamma_{\#} \subseteq \Gamma_{voc}(v)) = \sup CF(E_v(\gamma_{\#} \subseteq \Gamma_{voc}(v)))$. Therefore, define the control action of $f_{lo}$ for $s \in L_{lo}$ as $f_{lo}(s) = \{ \sigma \in \Sigma_{lo} \mid \sigma \in E_v(\gamma_{\#} \subseteq \Gamma_{voc}(v)) \}$. It can be proved that, for the above supervisory scheme, if $E_{hi}$ is $\Gamma_{hi}$-compatible, then $f_{lo}$ is non-blocking and $\theta(L_m(f_{lo}/D_{lo})) = E_{hi}$. This is exactly the definition of hierarchical consistency (section 2), therefore:

Theorem 1. For the proposed two level hierarchy, there is hierarchical consistency between $D_{lo}$ and $D_{hi}$.

The next step is to determine a hierarchy with strong hierarchical consistency. When two or more vocal strings have the same image through the reporter map, the union of their vocal control sets may lead to the loss of some particular controls of some of the control sets. Therefore, some behaviors implementable by supervisor in $D_{lo}$ may be lost in the process of abstraction that builds $D_{hi}$. If any pair of vocal strings of $D_{lo}$ corresponds to different strings in $D_{hi}$, there is no such loss. Therefore, define the reporter map $\theta$ to be deterministic when for any $v_1, v_2 \in L_{voc}$, if $v_1 \neq v_2$ then $\theta(v_1) \neq \theta(v_2)$.

Theorem 2. If the reporter map $\theta$ is deterministic, then there is strong hierarchical consistency between $D_{lo}$ and $D_{hi}$.

6. EXAMPLE

This section presents the method for (strong) hierarchical consistency, illustrated by an example.

Let the state representation for $D_{lo}$ and the $inf_{lohi}$ be the Moore automaton $G_{lo}$ (Wonham, 1999), for that the recognized languages are the languages of $D_{lo}$ and the state output function defines $inf_{lohi}$. Consider, the DES in figure 2: the transition diagram follows the
conventional notation of (Wonham, 1999), where transitions with a tick correspond to controllable events and the output $\tau_0$ is not represented. Notice that the vocal strings of $D_{lo}$ correspond to the states in $G_{lo}$ with output and the initial state, called vocal states.

![Fig. 2. Moore automaton $G_{lo}$: $D_{lo}$ and $inf lo hi$](image)

Given a string $s \in L_{lo}$, the state representation for the subsystem $D(s)$ is the Moore automaton $G(x)$, where $x = [s]$, the state of $G_{lo}$ equivalent to $s$. $G(x)$ is built by taking the reachable component of $G_{lo}$, starting from $x$, and stopping when a state with an output or a visited state is found. The vocal control sets for the state $x$ are computed by application of proposition 1. Figure 3 displays the subsystems and the vocal control sets for the vocal states.

![Fig. 3. Subsystems for the vocal states of $G_{lo}$](image)

The state representation for $D_{hi}$ is the pair $(G_{hi}, \Gamma_{hi})$. $G_{hi}$ is obtained by first substituting the transition labels of $G_{lo}$ for the state output labels, and then taking the deterministic (considering $\tau_0$ null event) equivalent automaton (Wonham, 1999). $\Gamma_{hi}$ is a table relating each state of $G_{hi}$ to its control set. By section 4, each entry in $\Gamma_{hi}$ for a state $x$ is the union of the vocal control sets for the vocal states of $G_{lo}$ corresponding to $x$. The automaton $G_{hi}$ and $\Gamma_{hi}$ for the running example are shown in figure 4.

![Fig. 4. $D_{hi}$: automaton $G_{hi}$ and table $\Gamma_{hi}$](image)

Analyze the high level specification $E_{hi}$, recognized by the automaton in figure 5. $E_{hi}$ is $\Gamma_{hi}$-compatible, and the controls that implement a corresponding high level supervisor are also represented in figure 5. The

![Fig. 5. $E_{hi}$ and controls](image)

low level implementation for $E_{hi}$ is given by the language $E_{lo}$ recognized by the automaton in figure 6. When implementing a supervisor for $E_{lo}$ it is not necessary to implement the automaton in figure 6 directly. $E_{lo}$ is in fact implemented by loading the corresponding subsystem and implementing for each subsystem the control policy that follows the high level control directive. This process is shown by the indications of subsystem and high level controls in figure 6.

![Fig. 6. Language implemented in the low level](image)

Consider the second specification as $E_{hi}' = \{\epsilon, AB\}$. Although $E_{hi}'$ is not $\Gamma_{hi}$-compatible, there is $E_{lo}' = \{\epsilon, abc, d\}$ controllable with respect to $D_{lo}$, $L_{m,lo}$-closed, and such that $\theta(E_{lo}') = E_{hi}'$. From this example, there is not strong hierarchical consistency between $D_{lo}$ and $D_{hi}$.

To achieve strong hierarchical consistency, examine the outputs of $G_{lo}$ and modify the event labels to eliminate the non-determinism in the reporter map. By inspection of figure 2, the vocal states 4 and 5 correspond to the same state in $G_{hi}$, figure 4. Therefore, to make the information channel deterministic, a new instance of event $B, B'$, is created for state 4. The resulting high level system is shown in figure 7, with its corresponding control structure. Finally, it can be checked that the specification $E_{hi}' = \{\epsilon, AB\}$ is $\Gamma_{hi}$-compatible for the new $D_{hi}$.

7. CONCLUSION

This section summarizes the contributions of this paper and makes some comments regarding related work found in the literature.

In the works of Zhong and Wonham (1990), Wong and Wonham (1996), Pu (2000) and Hubbard and Caines
(2001), hierarchical consistency is achieved with no additional condition or refinement for
 hierarchical control problem are possible: multilevel hierar-
 chies, hierarchical coordination and the consideration of systems with more elaborated control structure, as discrete state abstractions of hybrid systems as in González et al. (2001).

Similar approach for the hierarchical control of DES is found in Torrico and Cury (2001), but for the state aggregation viewpoint.

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8. REFERENCES


Fig. 7. Refined $G'_{hi}$ and $\Gamma'_{hi}$

This paper provides a solution for the hierarchical control problem considering the general case of marking behavior for the systems. By application of the generalized model for controlled DES from Cury et al. (2001), hierarchical consistency is achieved with no additional condition or refinement for $D_{lo}$ and $inf_{lohi}$. Moreover, this work also provides a method to build the hierarchy with hierarchical consistency, considers the refinement necessary for the strong hierarchical consistency, and finally gives directions of construction of a low level supervisor, given a high level designed supervisor.

With an extension of the results presented in this paper, by considering a two level hierarchy with the same generalized model for controlled DES of Cury et al. (2001), the following extensions for the hierarchical control problem are possible: multilevel hierar-

\[\Gamma_{voc}(0) = \{\{A\}\} \]

\[\Gamma_{voc}(3) = \{\{A\}, \{B\}, \{B'\}\} \]

\[\Gamma_{voc}(4) = \{\{A\}, \{A,B'\}\} \]

\[\Gamma_{voc}(5) = \{\{\} , \{A\}\} \]