Abstract: We study a linear discrete-time partially observed system perturbed by white noises. The observations are transmitted to the controller via communication channels with irregular transmission times. Various measurement signals may incur independent delays, arrive at the estimator out of order, and be lost or corrupted. The estimator is given a dynamic control over the sensors data by selecting signals to be used from those arrived at the current time. The minimum variance state estimate and the optimal sensor control strategy are obtained. Ideas of model predictive control are employed to derive a non-optimal but implementable in real time method for sensor control.

Keywords: communication networks, optimal state estimation, optimal sensor control

1. INTRODUCTION

The paper considers the sensor control problem that is to estimate the state of an uncertain process based on measurements obtained over asynchronous communication channels from noisy controlled sensors.

The standard assumption in the classical estimation theory is that data transmission and information processing required by the algorithm can be performed instantaneously. However in a number of newly arisen engineering applications, the observations are sent over communication channels that produce considerable and randomly varying transmission times (Tepedelenlioglu and Giannakis, 2000; Ziegler and Cioffi, 1992; Hristu and Morgansen, 1999; Tsai and Ray, 1999; Wong and Brockett, 1997). Examples concern remote control of several mobile units, planetary rovers, arrays of microactuators, underwater acoustic, exploration seismology, and power control in mobile communication. Other examples are offered by complex networked sensor systems containing a large number of low power sensors, as well as complex dynamical processes like advanced aircraft, spacecraft, and manufacturing process, where time division multiplexed computer networks are employed for exchange of information between spatially distributed plant components. All these new engineering applications motivate development of a new chapter of control and estimation theory that deals with networked systems in which control and communication issues are combined together, and all the delays and limitations of the communication channels between sensors, actuators and controllers are taken into account.

In many of the above applications, only a limited number of sensors can be linked with the estimator via communication channels during any measurement interval. In such a case, the estimator can dynamically select which sensors use the channels. This gives rise to the sensor control problem. Such a problem also arises when measurements from a large number of sensors are available to the estimator, but the com-
putational power is such that only data from a small selection of the sensors can be processed at any given time hence forcing the estimator to dynamically select which sensor data are important for the task at hand.

Up to now, sensor scheduling has been addressed in the literature under the assumption that the information exchange is instantaneous and for continuous time systems. Stochastic ones were considered in (Baras and Bensoussan, 1989; Miller and Runggaldier, 1997; Rago et al., 1996). It was shown that the optimal sensor schedule is independent of the sensor data and can be computed before the experiment has commenced. In (Petersen and Savkin, 1999; Savkin et al., 2000), a problem of robust state estimation via sensor switching was studied for uncertain non-stochastic systems.

Unlike the above researches, a sensor control problem is studied for a networked system in this paper. We consider a discrete time linear partially observed system driven by a white noise. The observations are sent to the estimator over communication channels, which produce random transmission delays, may lose data due to, e.g., noise in the communication medium and protocol malfunctions, and do not keep the order of messages. The estimator selects no more than a limited number of messages from those arrived at the current time. Each message is marked with a "time stamp" indicating the time of the transfer beginning, which is characteristic of most of the channels. The statistics of the transmission delays is known a priori. The objective is to find a minimum variance state estimator. The minimum is over not only state estimates, but also non-anticipating data selection strategies.

The outline of the paper is as follows. Section 2 offers the problem statement. Sections 3 and 4 contain examples of the sensor control and the assumptions, respectively. The minimum variance state estimator and the optimal sensor control strategy are presented in Sections 5 and 6, respectively. In Section 7, we apply the model predictive control approach. The proofs of the results presented are available upon request, or can be found in (Matveev and Savkin, 2000).

2. PROBLEM STATEMENT

For \( t = 0, \ldots, T - 1 \), consider the following system:

\[
x(t + 1) = A(t)x(t) + \xi(t), \quad x(0) = a,
\]

\[
y(t) = C(t)x(t) + \chi(t), \quad t = 0, \ldots, T.
\]

Here \( x \in \mathbb{R}^n \) is the state, \( \xi \in \mathbb{R}^n \) is a process disturbance, \( y \in \mathbb{R}^k \) is the vector of sensors outputs, and \( \chi \) is a noise.

**Measurements transmission** To transfer \( y \), employed are \( q \) parallel communication channels (called observation ones) each transmitting a fixed portion \( y_v \) of \( y \):

\[
y = \text{col} \{ y_1, \ldots, y_q \}, \quad y_v \in \mathbb{R}^{f_v}.
\]

We denote \( \text{col} \{ M_1, \ldots, M_f \} := (M_1^t, \ldots, M_f^t)^t \) whenever \( M_i \) is a \( q_i \times s \) matrix for all \( i \). These channels provide independent transmission delays. So the estimator receives a tuple of the form

\[
(y_v[\theta])_{(\nu, \theta) \in S(t)}
\]

at the time \( t \). Here the set \( S(t) \) may be empty and

\[
(\nu, \theta) \in S(t) \Rightarrow \theta \leq t,
\]

\[
S(t_1) \cap S(t_2) = \emptyset \quad \text{whenever} \quad t_1 \neq t_2.
\]

**Selection of measurement signals** In general, the estimator is unable to employ the entire bulk of the newly arrived data. So it makes a selection from it via forming a vector \( z \), by which it is fed instead of (4),

\[
z(t) = \sum_{(\nu, \theta) \in S(t)} E_{v, \theta}[t, v(t)]y_v[\theta],
\]

\[
v(t) \in V(t).
\]

(Any sum over the empty set is put to be zero.) Here \( v \) is a control chosen by the estimator from a given variety \( V(t) \), and \( E_{v, \theta}(\cdot) \) are matrix-functions.

**Remark 2.1.** A typical selection procedure consists in choosing \( f' \leq f \) signals \( y_v \) in the situation where the estimator is able to work with no more than a fixed number \( f \) of them. Then \( v = v(t) = \{ s[1], \ldots, s[f'] \} \) determines a subset of \( S(t) \) to be employed, \( V(t) \) consists of all tuples \( v \subset S(t) \) with \( f' \leq f \), and \( z := (y_v[\nu])_{\nu=1}^f \) where \( y_v[\nu] \) is defined to be zero for \( \nu > f' \). Given \( v(t) \), the transformation \( (y_v[\theta])_{(\nu, \theta) \in S(t)} \rightarrow z \) is linear and so can be written in the form (6).

Other examples of data selection procedures will be given in the next section.

**Information about the communication medium** We suppose that the estimator receives an information characterizing the prior functioning of the channels.

**Notation 2.1.** The information about the bygone states of the communication network currently received by the estimator is organized in a tuple denoted by \( \delta(t) \).

This information may be based on the time stamps that enable one to calculate the time taken by the channel.
to transmit the message at the moment of its arrival. This information may also include a part concerning channels that compete with observation ones for the network resources. This part may be of interest for prognosis of future states of the observation channels. Though we do not specify the content of δ, some assumptions about it will be given in Section 4.

**Problem statement** The natural class of admissible control strategies is as follows

\[ v(t) = V[t, S(t - 1), \Delta(t)], \]

where \( S(t) := \text{col} \{s(0), \ldots, z(t)\} \) for \( t \geq 0 \),

\[ S(-1) := 0 \in \mathbb{R}, \quad \text{and} \]

\[ \Delta(t) := \text{col} \{\delta(0), \ldots, \delta(t)\}. \]  

(7)

The problem is to find a minimum variance estimate of the current state \( x(t) \) from (1). In other words, we look for a deterministic function of the observations

\[ \hat{x}(t|t) = X[t, S(t), \Delta(t)] \]  

that, along with the function \( V(\cdot) \) from (7), minimizes the total estimation error

\[ J := \sum_{t=0}^{T} E[e(t|t)]^2, \quad e(t|t) := x(t) - \hat{x}(t|t). \]  

(9)

3. EXAMPLES OF SELECTION PROCEDURES

In this section, we supplement Remark 2.1 with further examples of data selection procedures.

1. Consider the situation from the above remark. Suppose also that at any sampling instant, the estimator can employ no more than one signal among those arriving via a given observation channel. Then the set \( V(t) \) of admissible controls must be reduced as compared with that remark. Now it consists of all tuples

\[ v = \{s[1], \ldots, s[f]\}, \quad s[i] = (\nu_i, \theta_i) \in S(t) \]

such that \( f' \leq f \), and \( \nu_i \neq \nu_j \) whenever \( i \neq j \).

2. Now consider another situation where there is an upper bound \( g \) on the number of channels that can be simultaneously connected with the estimator. All signals arriving over these channels are employed. Then the set \( V(t) \) consists of all tuples \( v \) such that

\[ s = (\nu, \theta) \in S(t) \land \exists i : \nu = \nu_i \Rightarrow \exists j : s = s_j, \]

and the set \( \{\nu_i\} \) contains no more than \( g \) elements.

3. Now let any observation channel be equipped with a terminal device that forms a weighted average of the signals arriving simultaneously via this channel. Suppose that the scaling coefficients \( \alpha_{\nu, \theta}(t) \) can be dynamically adjusted by the estimator. Then

\[ v = v(t) = (\alpha_{\nu, \theta}(t))_{(\nu, \theta) \in S(t)}, \]

where \( \alpha_{\nu, \theta}(t) \geq 0 \), and

\[ \sigma_{\nu}(t) := \sum_{\theta : (\nu, \theta) \in S(t)} \alpha_{\nu, \theta}(t) = 1 \]

for no more than \( g \) channels with \( \sigma_{\nu}(t) = 0 \) for all the other ones. Furthermore \( z(t) = (z_{\nu}(t)) \), where

\[ z_{\nu}(t) := \sum_{\theta : (\nu, \theta) \in S(t)} \alpha_{\nu, \theta}(t)y_{\nu, \theta}(\theta). \]

Now suppose that any terminal device can operate only in a finite number of modes, each associated with a definite set of coefficients, and the estimator can switch the mode. Then the set \( V(t) \) consists of the tuples \( v \) related to these modes.

4. A generalization of the last example concerns the case where each terminal device serves a fixed collection of channels by forming a weighted average of all signals arriving over them. In this case, the estimator picks no more than a fixed number of such collections to be fed by the corresponding averaged signals.

4. ASSUMPTIONS

**Assumption 4.1**. The vectors \( a(0), \xi(t) \), and \( \chi(t) \) from (1) and (2) are random, Gaussian, and independent with \( E \xi(0) = 0 \) and \( E \chi(t) = 0 \). The mean \( Ea \) and the correlation matrices

\[ R_{aa} := E[a - Ea][a - Ea]^*, \]

\[ R_{\xi\xi}(t) := E \xi(t) \xi(t)^*, \quad R_{\chi\chi}(t) := E \chi(t) \chi(t)^* \]

are known. So are the matrices \( A(t), B(t), C(t) \) from (1) and (2). The tuples \( \{\delta(t)\} \) from Notation 2.1 are random and independent of \( a, \{\xi(t)\}, \{\chi(t)\}, \) and \( J_\nu R_{\chi\chi}(\theta) J_\eta^* = 0 \) whenever \( \nu \neq \eta \). Here \( J_\nu = (0, \ldots, 0, I, 0, \ldots, 0) \) is the \( I \times k \)-matrix partitioned in accordance with (3), for which the unit matrix \( I \) is at the \( \nu \)th place.

Tacitly assumed presence of the observation channels time stamps in the tuple \( \delta \) gives rise to the following.

**Assumption 4.2**. The estimator can determine the set \( S(t) \) from (4) at the time \( t \), i.e., \( S(t) = S[t, \delta(t)] \).

(Capital script letters denote deterministic, i.e., nonrandom, functions.) In the examples from Remark 2.1 and Section 3, the set \( V(t) \) of admissible controls depends on \( S(t) = S[t, \delta(t)] \). The next assumption takes this into account.

**Assumption 4.3**. The set \( V(t) \) from (6) is determined by the communication network monitoring data:

\[ V(t) = \mathbb{S}(t, \delta(t)). \]

The set \( \mathbb{S}(t, \delta) \) is finite for all \( t \) and \( \delta \). In some of the above examples, the matrix \( E_{\nu, \theta}(\cdot) \) from (6) also depends on \( S(t) = S[t, \delta(t)] \). This gives rise to the following assumption.
Assumption 4.4. The matrices from (6) may depend on the communication network monitoring data: $\tilde{E}_{\nu,\theta} = E_{\nu,\theta}[t, v(t), \delta(t)]$. Here $\tilde{E}_{\nu,\theta}[t, v, \delta]$ is a deterministic matrix-function of $t, \delta, v \in S[t, \delta]$, and $(\nu, \theta) \in S[t, \delta]$. Its size is $p_{\nu, \delta} \times l_{\nu}$, where $l_{\nu}$ is taken from (3).

Assumption 4.5. The delays in the observation channels are bounded by a known constant: $t - \theta \leq \sigma$ whenever $(\nu, \theta) \in S(t, \delta)$.

The following assumption is not of principle and is adopted only to simplify the formulations.

Assumption 4.6. The set $\Upsilon$ of the values that can be taken by the tuple $\delta$ from Notation 2.1 is finite.

The last assumption to follow means that the communication medium is a system with a finite aftereffect.

Assumption 4.7. Let $P_{t+1}(\delta|\delta_1, \ldots, \delta_t)$ stand for the conditional probability of the event $\delta(t+1) = \delta$ given $\delta(t) = \delta_1, \ldots, \delta'(t') = \delta'$. For any $t$ and $t'$ : 0, this probability is known in advance. There exists a known constant $\sigma = 0, 1, \ldots$, such that for any $t \geq \sigma$ and $\delta_0, \ldots, \delta_{t+1} \in \Upsilon$, the following relation (where $[\sigma] := \min \{\sigma, t\}$) holds

$$P_{t+1}(\delta_{t+1}|\delta_t, \ldots, \delta_0) = P_{t+1}(\delta_{t+1}|\delta_t, \ldots, \delta_{t-\sigma+1}).$$

Note that $P(\alpha = a|\beta = b) := 0$ whenever $P(\beta = b) = 0$, where $a \in \mathbb{A}$ and $b \in \mathbb{B}$ are random quantities, and $\mathbb{A}, \mathbb{B}$ are finite sets. Furthermore we assume that relations (5) hold almost surely, and restrict ourselves to consideration of the sensor control strategies (7) and state estimates (8) with measurable functions $V(\cdot)$ and $\tilde{X}(\cdot)$, respectively.

5. CONSTRUCTION OF THE STATE ESTIMATOR

In this section, we assume that the control (7) is chosen and fixed and find the minimum variance state estimate (8). As will be shown in Section 6, this estimate is a part of the solution of the primal problem. More precisely, after determining the optimal sensor control strategy (7), the complete solution results from supplementing it with the above estimate.

For any $N \times N$-matrix $\Lambda$, we denote by $\Lambda^+$ its pseudoinverse $W^{-1} \Lambda^T$. Here $\Lambda^T$ is the orthogonal projection of $\mathbb{R}^N$ onto $\text{Im} \Lambda$, the space $\mathbb{R}^N$ is equipped with the standard inner product, the operator $W$ is obtained by restricting $\Lambda$ on $(\ker \Lambda)^{-1}$, where $\perp$ stands for the orthogonal complement, and $W^{-1} : \text{Im} \Lambda \rightarrow (\ker \Lambda)^{-1}$. Evidently $\Lambda^+ = \Lambda^{-1}$ whenever $\Lambda^{-1}$ exists.

Denote by $\tilde{x}(j)$ the minimum variance estimate of $x(j)$ based on $z(0), \ldots, z(t), \delta(0), \ldots, \delta(t)$. Being coupled with certain $n \times n$ matrices $H_{ij}(t), \Pi_{ij}(t), i, j = 0, \ldots, \sigma$, where $\sigma$ is the constant from Assumption 4.5, the tuple of the estimates

$$\tilde{X}(t) = \left[ \tilde{x}(t), \tilde{x}(t-1), \ldots, \tilde{x}(t-\sigma) \right]$$

may be generated recursively by the following analog of the Kalman filter.

Recurrent state estimator

The next tuple $\tilde{X}(t+1)$ is generated by equations:

$$\tilde{x}(j|t+1) = \tilde{x}(j|t) + K_{t+1-j}(t+1)\tilde{z}(t+1) - \tilde{z}(t+1)$$

for $j = t+1, t, \ldots, t+1 - \sigma$, where

$$\tilde{z}(t+1) := A(t)\tilde{x}(t+1)$$

and

$$\tilde{z}(t+1) := \sum\limits_{i=0}^{\sigma} G_i(t+1) \tilde{x}(t+1 - i|t),$$

$$G_i(t) := \sum\limits_{\nu,\theta \in \Upsilon, i \in S(t)} E_{\nu,\theta}[t, v(t)] J_\nu C(t-i).$$

Here $C(\theta), E_{\nu,\theta}[t, v]$ and $J_\nu$ are the matrices from (2), (6), and Assumption 4.1, respectively.

The matrices $K_i(t), i = 0, \ldots, \sigma$ in (10) are given by

$$K_i(t) = \sum\limits_{j=0}^{\sigma} H_{ij}(t) G_j(t)^+ \Lambda^+ (t).$$

Here $\Lambda^+ (t)$ is the pseudoinverse of the matrix

$$\Lambda(t) := \sum\limits_{i,j=0}^{\sigma} G_{ij}(t) H_{ij}(t) G_j(t)^* + \sum\limits_{(\nu, \theta) \in \Upsilon} E_{\nu,\theta}[t, v(t)] J_\nu C(t-i) J_\nu^* E_{\nu,\theta}[t, v(t)]^*.$$

The matrices $\{H_{ij}(t)\}_{i,j=0}^{\sigma}, \{\Pi_{ij}(t)\}_{i,j=0}^{\sigma}$ are generated recursively by equations

$$H_{ij}(t+1) = A(t) \Pi_{ij}(t) A(t)^* + R_{ij}(t)$$

$$\Pi_{ij}(t+1) = H_{ij}(t+1) - K_i(t+1) \sum\limits_{\nu=0}^{\sigma} G_{\nu}(t+1) H_{\nu,j}(t+1).$$

The filter is initialized by the formulas:

$$A(-1) := I, \quad \tilde{x}(-1) := E\tilde{a}, \quad \tilde{x}(-1 - j) := 0 \quad \forall j = 1, \ldots, \sigma,$$

$$R_{ij}(-1) := R_{\nu \nu}, \quad \Pi_{ij}(-1) := 0.$$
the above estimator generates the sequence of minimum variance estimations, i.e.,
\[
\hat{x}(j|t) = E[x(j)|\mathcal{Z}(t), \Delta(t)]
\]
whenever \( t - \sigma \leq j \leq t \) and \( j \geq 0 \). Here \( \mathcal{Z}(t) \) and \( \Delta(t) \) are defined in (7).

6. OPTIMAL SENSOR CONTROL

To describe the optimal strategy of sensors control, note first that due to Assumptions 4.2 and 4.4, the matrix (11) is determined by \( \delta = \delta(t) \) and \( v = v(t) \):
\[
G_i(t) = G_i[t, \delta, v] := \sum_{\nu,\delta \in \mathbb{S}(t,\delta)} \mathcal{E}_{\nu,t-i}(t, v, \delta)J_\nu C(t-i),
\]
and the recursion (12), (13) can be shaped into
\[
\mathcal{H}(t+1) = \mathcal{H}(t+1), \mathcal{H}(t), \mathcal{H}(t+1), \mathcal{H}(t+1) \bigg\}, \quad (15)
\]
where \( \mathcal{H}(t) := [\mathcal{H}_{ij}(t)]_{i,j=0}^\sigma \) (The symbol \( [M_j]_{i,j=0}^\sigma \) denotes the \((\sigma+1) \times n \times (\sigma+1) \times n \) matrix whose partition into \( n \times n \) blocks is composed of the matrices \( M_{ij} \).) The map \( \delta(\cdot) \) acts as follows
\[
\alpha = \left[ t, \mathcal{H}, \delta, v \right] \rightarrow \mathcal{H} = \left[ \mathcal{H}_{ij} \right] \downarrow \Lambda \mathcal{H}_{ij} \rightarrow \mathcal{H}(\alpha) := \left[ \mathcal{H}_{ij} \right]
\]
Here \( H_{ij} \) is the right hand side of (12) with \( \mathcal{H}(t) := \mathcal{H} \), and
\[
\Lambda := \sum_{i,j=0}^\sigma G_i H_{ij} G_j^* + \sum_{(\nu,\theta) \in \mathbb{S}(t,\delta)} \mathcal{E}_{\nu,\theta}(t, v, \delta)J_\nu R_{XX}(\theta)J_\nu^* \mathcal{E}_{\nu,\theta}(t, v, \delta)^*,
\]
\[
\mathcal{H}_{ij} := H_{ij} - \sum_{\nu,\eta=0}^\sigma H_{\nu\eta} G^*_\nu \Lambda G_\eta H_{\eta j}, \quad (16)
\]
Now consider the following key procedure:
\[
\mathcal{M}_T \rightarrow \cdots \rightarrow \mathcal{M}_2 := [W_2(\cdot) \rightarrow W_2(\cdot) \rightarrow W_2^m(\cdot)] \rightarrow \mathcal{M}_1 \rightarrow \cdots \rightarrow \mathcal{M}_0 \quad (17)
\]
Here \( W_2(\cdot), W_2(\cdot), W_2^m(\cdot) \) are real functions of \( \mathcal{H}, \mathcal{Z} = [\delta_0, \ldots, \delta_{\mathcal{Z}(t)}], \quad v \) with \( W_2(\cdot) \) and \( W_2^m(\cdot) \) independent of \( v \). (We recall that \( \mathcal{Z}(t) \) is defined in Assumption 4.7.) The recursion (17) is initialized by putting \( W_T(\cdot) \equiv 0 \). Its step
\[
W_T(\cdot) \rightarrow W_T(\cdot) \rightarrow W_T^m(\cdot) \rightarrow W_{T-1}(\cdot)
\]
is as follows:
\[
W_T[\mathcal{H}, \mathcal{Z}, v] := W_T\left\{ \delta_{\mathcal{Z}(t), \delta_0, v}, \mathcal{Z} \right\} + \text{tr} \delta_{\mathcal{Z}(t), \delta_0, v},
\]
where \( \delta_{\mathcal{Z}(t), \delta_0, v} \) is the block of the matrix \( \delta(\alpha) \) and the symbol \( \text{tr} \) stands for the trace of a matrix;
\[
W_T^m[\mathcal{H}, \mathcal{Z}, v] := \min_{v \in \mathbb{S}(t,\delta_0)} W_T\left\{ \mathcal{H}, \mathcal{Z}, v \right\}, \quad (18)
\]
where \( \mathbb{S}(\cdot) \) is taken from Assumption 4.3;
\[
W_{T-1}[\mathcal{H}, \delta_0, \ldots, \delta_{\mathcal{Z}(t-1)}] := \sum_{\delta \in \mathcal{V}} \left\{ P_1[\delta | \Delta^t] \times W_{T-1}[\mathcal{H}, \delta, \delta_0, \ldots, \delta_{\mathcal{Z}(t-1)}] \right\}.
\]
The notation \( P_1[\cdot] \) was defined in Assumption 4.7. Finally we put \( V_T[\mathcal{H}, \mathcal{Z}] := v_*, \) where \( v_* \) is an element furnishing the minimum in (18) (which is attained due to the last claim from Assumption 4.3.) Since the functions \( W_T(\cdot), W_2(\cdot), \) and \( W_2^m(\cdot) \) are measurable, the function \( V_T(\cdot) \) can be chosen measurable as well.

The main result of the section is as follows.

**Theorem 6.1.** Consider the system (1)–(3), (6) and suppose that Assumptions 4.1–4.7 hold. Then the minimum value of the estimation error (9) over all sensor control strategies (7) and estimates (8) is attained at the sensor control strategy
\[
v(t) = V_T[\mathcal{H}[t-1], \delta[t], \ldots, \delta[t-\mathcal{Z}(t)]], \quad (19)
\]
along with the state estimate described in Section 5. We recall that \( \mathcal{Z}(t) := \min_{\theta, t} \mathcal{S}(t, \eta) \), where \( \mathcal{S}(\cdot) \) is taken from Assumption 4.7, and \( \mathcal{T}(t) \) is the sequence of matrices generated by the above state estimator.

By (15) and (19), the current optimal sensor control is determined on the base of the past history of the communication medium observations:
\[
v(t) = \mathcal{S}[t, \delta(t), \ldots, \delta(0)].
\]
Thus the possibility to depend on \( \mathcal{Z}(t-1) \) given by (7) is not utilized.

7. MODEL PREDICTIVE SENSOR CONTROL

The dynamical programming equations like those derived in Section 6 have been the subject of intensive research in the field of optimal control theory. In realistic situations, solution of such equations is often a hard task. In this section, we apply ideas of model predictive control (see e.g. (Camacho and Bordons, 1995)) to give a non-optimal but real-time implementable method for sensor control.

**Remark 7.1.** Due to Theorem 5.1, it is natural to restrict ourselves to consideration of the state estimates described in Section 5. Both this estimate and the related estimation error (9) are determined uniquely provided the sensor control strategy (7) is chosen.
Notation 7.1. For any sensor control strategy \( \mathcal{S} = \{ \mathcal{V}(\cdot) \}_{t=0}^{T} \) and \( t = 0, \ldots, T \), the summary estimation error over the time interval \([0, \tau]\) is denoted by
\[
\mathcal{J}^\tau := \mathcal{J}^\tau[\mathcal{S}] := \sum_{t=0}^{\tau} \mathbb{E}[x(t) - \hat{x}(t|t)]^2.
\]

Definition 7.1. A control strategy \( \mathcal{S}^0 \) is said to be one-step-ahead optimal if for any \( \tau = 0, \ldots, T \) and any other control strategy \( \mathcal{S} \) that equals \( \mathcal{S}^0 \) on \([0, \tau - 1]\) provided \( \tau \geq 1 \) the following inequality holds
\[
\mathcal{J}^\tau[\mathcal{S}^0] \leq \mathcal{J}^\tau[\mathcal{S}].
\]

Remark 7.2. The idea of Definition 7.1 is very straightforward: we wish to pick the current control so that the current state estimation error be minimal provided the previous controls are chosen and fixed.

To construct a one-step-ahead optimal control, note that due to (14), \( \Lambda = \Lambda(t, \mathcal{H}, \delta, v) \) in (16). We recall that in (16), the matrix \( H \) is defined to be the right hand side of (12), where \( \mathcal{H}_{ij}(t) := \mathcal{H}_{ij}(\tau) \). Now for \( t = 0, \ldots, T \), introduce the function \( \hat{\mathcal{V}}_t[\mathcal{H}, \delta] := v^0 \), where \( v^0 \) is an element furnishing the maximum
\[
\max_{v \in \mathcal{S}[t, \delta]} \mathcal{J}[t, \mathcal{H}, \delta, v], \quad \text{where } \mathcal{J}[v] := \sum_{\nu, \mu=1}^{\sigma} \text{tr}[A(t)\mathcal{H}_{0,\nu-1}G_{\nu}^* + \Lambda G_{\nu}A(t)\mathcal{H}_{0,0}A(t)^*] + \sum_{\nu=1}^{\sigma} \text{tr}[A(t)\mathcal{H}_{0,0}A(t)^*G_{\nu}^* + \Lambda G_{\nu}A(t)\mathcal{H}_{0,0}A(t)^*] + \text{tr}[A(t)\mathcal{H}_{0,0}A(t)^*G_{\nu}^* + \Lambda G_{\nu}A(t)\mathcal{H}_{0,0}A(t)^*],
\]
and \( G_t = G(t, \delta, v) \) is given by (14). The maximum here is attained since the set \( \mathcal{S}[t, \delta] \) is finite by Assumption 4.3. So far the function \( \hat{\mathcal{V}}_t(\cdot) \) is evidently measurable, the function \( \hat{\mathcal{V}}_t(\mathcal{H}, \delta) \) can be chosen measurable as well.

The main result of the section is as follows.

Theorem 7.1. Consider the system (1)—(3), (6) and suppose that Assumptions 4.1—4.7 are true. Then the following control strategy is one-step-ahead optimal:
\[
v(t) = \hat{\mathcal{V}}_t[\mathcal{H}(t-1), \delta(t)].
\]
Here \( \{ \mathcal{H}(t) \}_{t=0}^{T} \) is the sequence of matrices generated by the state estimator described in Section 5.

Remark 7.3. Unlike the optimal sensor control strategy, the one-step-ahead optimal one (20) does not employ the statistics of the delays and losses in the communication channels.

8. CONCLUSIONS

A solution to a sensor control problem was offered. This problem is to estimate the state of an uncertain process based on a limited selection of the sensor signals obtained over asynchronous delayed communication channels. The minimum variance state estimate and the optimal sensor data selection strategy were proposed. Ideas of model predictive control were employed to derive a non-optimal but implementable in real time method for sensor control.

9. REFERENCES


