SYSTEMATIC NONLINEAR CONTROLLER DESIGN
FOR A POWER FACTOR CORRECTOR

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Abstract: A nonlinear controller for a power factor corrector is systematically constructed via Lyapunov-based controller design approach for the bilinear state averaged model. First, a nonlinear gain of the controller is derived to shape the source current and the output voltage be desired form respectively via nonlinear $H_\infty$ control system design approach. Second, a source current reference generator is constructed, which consists of a feedforward loop given by steady state analysis and a feedback loop with output voltage error amplifier. This paper, finally, shows efficiencies of the approach through computer simulations.

Keywords: power supplies, switching rectifiers, bilinear systems, H-infinity control, Lyapunov function, convex programming

1. INTRODUCTION

There has been a steady growth of interest in control of power electronic circuits (e.g., (Kassakian et al., 1991; Banerjee and Verghese, 2001)). Many works (e.g., (Kassakian et al., 1991)) discuss linear feedback control problems for power electronic circuits on the basis of linearized state averaged model of the circuits. The work (Banerjee and Verghese, 2001) is beginning to discuss nonlinear feedback control problems for the circuits via treating those as nonlinear systems.

In this paper, a nonlinear controller for a power factor corrector is systematically constructed on the basis of a bilinear state averaged model. The controller design approach consists of the following two steps.

First, a nonlinear gain of the controller is derived via nonlinear $H_\infty$ control system design approach by using a convex programming technique. The nonlinear gain is designed to (1) shape a source current be sinusoidal and in phase with a source voltage and (2) keep an output voltage constant.

Second, a source current reference generator is derived, which consists of a feedforward loop and a feedback loop. The feedforward loop is given by a steady state analysis for the corrector model. The feedback loop includes an output voltage error amplifier.

This paper is organized as follows. Section 2 gives a power factor corrector model. Section 3 derives a nonlinear gain and a source current generator. Section 4, finally, demonstrates efficiencies of the approach through computer simulations. The simulation uses the same circuit parameters as in the work (Escobar et al., 2001).

2. POWER FACTOR CORRECTOR MODEL

This paper constructs a power factor corrector control system as shown in Fig. 1. First, the power factor corrector model is derived. An inductor current $i$ and a capacitor voltage $v$ are treated as states of the system. A functions $\mu$ is regarded as a controller output to drive switches as shown in...
which operates against source voltage and load resistance variations.

3.1 Bilinear State Averaged Model and Its Steady State

First, the switched model \((\Sigma_S)\) gives a state averaged model \((\Sigma_{SA})\) of the form

\[
\frac{d}{dt} \begin{bmatrix} i \\ v \end{bmatrix} \approx \begin{bmatrix} -\frac{r}{L} & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i \\ v \end{bmatrix} + \begin{bmatrix} 1 \\ L \end{bmatrix} v_s \\
+ \left\{ \begin{bmatrix} 0 \\ 1/C \end{bmatrix} + v \begin{bmatrix} -1 \\ L \end{bmatrix} \right\} \mu
\]

where \(i, v\) and \(\mu\) denote moving averages of \(i, v\) and \(\mu\), respectively (Kassakian et al., 1991).

Here, a steady state of the model is analyzed. Given a source voltage \(v_s = \sqrt{2}V_c \sin \omega t\), assume that the capacitor voltage is \(v = v_r\). Then, dc components of the system in the steady state decide a source current \(i = \sqrt{2}I_c \sin \omega t\), which is sinusoidal and in phase with the source voltage, given as

\[
I_c = \frac{V_c}{2r} - \sqrt{\frac{V_c^2}{4r^2} - \frac{v_r^2}{r^2}} =: I_c(v_r).
\]

The decision process is discussed in the works (Escobar et al., 1999; Escobar et al., 2001). In the following section, the effective value \(I_c(v_r)\) is used for a source current reference generator. Note that the effective value \(I_c(v_r)\) depends on an effective value of source voltage \(V_c\) and a load resistance \(R\) which vary in practical circuits.

Next, for a Lyapunov-based controller design, the state averaged model \((\Sigma_{SA})\) moves to a model \((\Sigma_A)\) around a specified set point \([\bar{i} \; \bar{v} \; \bar{\mu}] = [0 \; E \; 0]\), which is given as the form

\[
\frac{d}{dt} \begin{bmatrix} \bar{i} \\ \bar{v} \end{bmatrix} \approx \begin{bmatrix} -\frac{r}{L} & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} \bar{i} \\ \bar{v} \end{bmatrix} + \begin{bmatrix} 1 \\ L \end{bmatrix} 0 \\
+ \left\{ \begin{bmatrix} E \\ 0 \end{bmatrix} + \bar{i} \begin{bmatrix} 0 \\ 1/C \end{bmatrix} + \bar{v} \begin{bmatrix} 1 \\ L \end{bmatrix} \right\} \bar{\mu}
\]

where \([\bar{i} \; \bar{v} \; \bar{\mu}] = [\bar{i} \; \bar{v} \; \bar{\mu}] - [0 \; E \; 0]\). The model \((\Sigma_A)\) is rewritten as the form

\[
\dot{x}_p = A_p x_p + B_p w_p + B_{p3}(x_p) \bar{\mu}
\]

where

\[
x_p := [i \; \bar{v}]^T, \quad w_p := [v_s \; E]^T,
\]
For the bilinear state averaged model \((\Sigma_A)\), a nonlinear controller is constructed to meet the following design specification:

**S1** The source current \(\tilde{i}\) is sinusoidal and in phase with the source voltage \(v_s\) \((i_r \to \omega_e)\).

**S2** The output voltage \(\tilde{v}\) should track a voltage reference \(v_r\) \((v_r \to \omega_e)\).

The specification gives a design model \((\Sigma_G)\), as shown in Fig. 2, of the form

\[
\dot{x} = Ax + B_1w + B_2(x)u, \quad z = C_1x + D_{12}u, \quad (6) \text{ and } (7)
\]

where

\[
x = [x^T_w \ x^T_p]^T, \quad w = [i_r \ v_r \ u_p^T]^T, \quad \text{and } u = \tilde{u}, \quad z = [z^T_x \ z^T_z \ z_u^T]^T, \quad \text{and } A = \begin{bmatrix} A_w & -B_w \\ 0 & A_p \end{bmatrix}, \quad B_1 = \begin{bmatrix} B_w \\ 0 & B_{p1} \end{bmatrix},
\]

\[
B_2(x) = \begin{bmatrix} 0 \\ 0 & B_{p20} + x_4 \begin{bmatrix} 0 \\ B_{p21} \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ B_{p22} \end{bmatrix} \end{bmatrix}, \quad C_1 = \begin{bmatrix} W_w \ 0 \\ 0 & W_x \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 \\ W_u \end{bmatrix}.
\]

\(x_w, A_w, B_w, C_w\) are state and coefficient matrices of state space description of the form

\[
\dot{x}_w = A_w x_w + B_w \begin{bmatrix} \tilde{i}_r \\ v_r \end{bmatrix} - x_p, \quad x_w = 0, \quad (8)
\]

\[
y_w = C_w x_w, \quad (9)
\]

where

\[
A_w = \begin{bmatrix} 0 & \omega_n & 0 \\ -\omega_n & 0 & 0 \\ 0 & 0 & -\varepsilon \end{bmatrix}, \quad B_w = \begin{bmatrix} 0 & 0 \\ 0 & k_i \\ k_v & 0 \end{bmatrix}, \quad (10)
\]

\[
C_w = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (11)
\]

for weighting functions

\[
W_i(s) = \frac{k_i \omega_n}{s^2 + \omega_n^2}, \quad W_u(s) = \frac{k_v}{s + \varepsilon}, \quad (12)
\]

with respect to the specification. The weighting functions \(W_i(s)\) and \(W_u(s)\) are used for the current and voltage to track the references \(i_r\) and \(v_r\), respectively. Parameters \(k_i\) and \(k_v\) adjust gains of the weighting functions for the tracking to be achieved. \(\omega_n\) is a principal value of angular frequency in the voltage source. Matrices \(W_w, W_x\) and \(W_u\) are weighting parameters to be used only for the controller design.

### 3.3 Nonlinear Gain

On the basis of the design model \((\Sigma_G)\), this section gives concretely a nonlinear gain, that is a block from \(x\) to \(u\) in Fig. 2, via nonlinear \(H^\infty\) control system design approach. The work (Sasaki and Uchida, 1998) says that for a given \(\gamma\) in a specified domain of \(x\), a matrix \(Y\) satisfying inequalities of the form

\[
\begin{align*}
& X \geq 0, \quad Y \geq 0, \\
& A^T Y A - Y 
\end{align*}
\]
(1) \( Y > 0, \)
\[
\begin{bmatrix}
AY + YAT + \gamma^{-2}B_1B_1^T & YC_1^T \\
-B_2(x)(D_1^T D_{12})^{-1}B_2(x)^T & -I
\end{bmatrix} < 0
\] (13)

gives the nonlinear gain of the form
\[
u = -(D_1^T D_{12})^{-1}B_2(x)^T Y^{-1} x.
\] (15)

For any state \( x \), which is current and voltage in a specified domain, the matrix \( Y \) satisfying the inequalities is concretely given by using a convex programming technique (Sasaki and Uchida, 1998). The gain guarantees a closed-loop system stability and a tracking performance for output voltage and source current references. Note that the nonlinear gain (15) does not depend on a structure to generate the source current reference \( i_r \).

3.4 Source Current Reference Generator

This section gives a mechanism to generate a source current reference \( i_r \). For a given output voltage reference \( v_r \), in a steady state of the system, a source current amplitude is given by the equation (3). The source current amplitude, however, depends on the source voltage \( v_s \) and the load resistance \( R \) which vary in practical circuits. The source and load variations need change an amplitude of current reference \( i_s \), so that the power balance between ac and dc ports are not violated. For this purpose, a gain block \( K(s) \) is used, which is
\[
K(s) := k_p + \frac{k_l}{s} + k_d s
\] (16)
where \( k_p, k_l, \) and \( k_d \) are constant parameters decided by a system designer. Then, the source current reference \( i_s \) is given by the form
\[
i_s = \sqrt{2} \left[ I_e(v_r) + K(s)\{v_r - \bar{v}\} \right] \frac{1}{\sqrt{2}V_e} v_s.
\] (17)

Note that the \( v_s/\sqrt{2V_e} \) means that for a voltage source the generator (17) requires only an angular frequency of the source.

As shown in Fig. 2, a feedback loop with the block \( K(s) \) adjusts the amplitude of current reference such that the output voltage keeps constant even if the source or load changes, thus operating as output voltage error amplifier. The feedback loop is the same as a conventional loop used in many works (e.g., Redl, 1994)).

The \( I_e(v_r) \) (i.e., the equation (3)) in the equation (17) can be directly changed if the source and load changes are exactly measured. The works (Escobar et al., 1999; Escobar et al., 2001) directly adjust the \( I_e(v_r) \) only to load variations via an adaptive control technique.

\[ \text{Fig. 3. Bode plots of output voltage error amplifier } K(s) \]

4. COMPUTER SIMULATIONS

This section shows efficiencies of the approach through computer simulations. A software package; MATLAB, Simulink, LMI Control Toolbox is used for the simulations. Circuit parameters are used as
\[
L = 1 \text{ [mH]}, \quad C = 2200 \text{ [\mu F]}, \quad R = 240 \text{ [\Omega]},
\]
\[
r = 2.2 \text{ [\Omega]}
\]

which are the same as in the work (Escobar et al., 2001). The load resistance \( R = 240 \) denotes the nominal value.

For the circuit, a nonlinear controller is derived. First, a nonlinear gain (15) is designed. Design parameters for the gain are chosen as
\[
W_e = \text{diag} \left[ 10^2, 10^{-4} \right], \quad W_u = 10,
\]
\[
W_x = \text{diag} \left[ 10^{-10}, 10^{-10} \right], \quad \omega_n = 100\pi,
\]
\[
\varepsilon = 10^{-4}, \quad k_i = 50, \quad k_v = 0.003, \quad \gamma = 0.98
\]
where \( \omega_n \) focuses on a voltage source with a frequency 50 hertz and \( W_x \) is set such that the tracking performance for source current has more weight than that for output voltage. Now, consider that a set point is \( [0 \text{ ampere, } 180 \text{ volts}] \) and the current and voltage are varying as \pm 6 amperes, \pm 50 volts (i.e., \(-6 \leq i \leq 6, \quad 130 \leq v \leq 230\)). Then the convex programming technique in the section 3.3 gives the solution \( Y \), as shown in (18), to obtain the nonlinear gain (15). Next, a voltage error amplifier \( K(s) \) is designed. The gain is chosen in order to be low around the input line frequency (i.e., 50 hertz) as shown in Fig. 3, whose parameters are given as
\[
k_p = 0.3, \quad k_l = 2.5, \quad k_d = 10^{-5}.
\]

For an output voltage tracking, the smaller the parameter \( k_p \) is, the longer the transient response time is and the smaller the overshoot is.

Now, the nonlinear controller gives computer simulation results. Figs. 4 to 7 show responses for the switched model \( (\Sigma_S) \) in the following cases,
where the initial state is $[i \ v] = [0 \ 150]$ and the switching frequency is 13 kilohertz;

(C1) An output voltage reference $v_r$ changes from 160 volts to 200 volts at 0.3 seconds for a source voltage $v_s = 150 \sin 100\pi t$ volts and a load resistance $R = 240$ ohms;

(C2) A load resistance $R$ changes from 240 ohms to 80 ohms at 0.3 seconds for a source voltage $v_s = 150 \sin 100\pi t$ volts;

(C3) An amplitude of source voltage $v_s$ changes from 150 volts to 120 volts at 0.3 seconds for a load resistance $R = 240$ ohms.

Figs. 4, 6 and 7 show responses by the nonlinear gain. Fig. 5 shows responses by a linear gain.

Fig. 4 shows responses by the nonlinear gain because a source current in Fig. 4 includes smaller high frequency components than in Fig. 5. Fig. 4 and Fig. 5 show that the nonlinear gain gives a slightly better performance than the linear gain because a source current in Fig. 4 includes smaller high frequency components than in Fig. 5 and for an output voltage tracking an overshoot in Fig. 4 is smaller than in Fig. 5.

Than in the work (Escobar et al., 2001), in all cases, the overshoots are greater and so the transient response times are much shorter. Moreover, the work (Escobar et al., 2001) does not have any adaptation for source voltage variations.

The above computer simulation results show that the nonlinear controller works very well. Efficiencies of the systematic nonlinear controller design approach was shown.

5. CONCLUSION

A power factor corrector control system was clearly constructed via systematic nonlinear controller design approach. A nonlinear gain was derived via a nonlinear $H^\infty$ control system design approach by using a convex programming technique. The gain guarantees a closed-loop system stability and a tracking performance for output voltage and source current references. A source current reference generator was composed of a feedforward loop derived by steady state analysis for the averaged model and a feedback loop with output voltage error amplifier. Computer simulations demonstrate efficiencies of the approach.

The systematic approach had been also applied to design the other power converter systems in the works (Sasaki et al., 1999; Sasaki and Inoue, 2000).

6. REFERENCES


Fig. 4. (C1) Output voltage reference $v_r$ changes from 160 volts to 200 volts

Fig. 5. (C1) Under linear gain, output voltage reference $v_r$ changes from 160 volts to 200 volts

Fig. 6. (C2) Load resistance $R$ changes from 240 ohms to 80 ohms

Fig. 7. (C3) Source voltage changes from 150 volts to 120 volts