Abstract: A flexible robot arm with strain gauges distributed on it is used as a sensing unit in determination of endpoint position and force of flexible manipulator. The position and orientation of the flexible arm is expressed as a function of the local curvature. An interpolation technique gave a continuous curvature function from a finite set of measurements made with strain gauges. The endpoint force and moment depend not only on the local strains but also on the positions and orientations of the endpoint and those points where the strain gauges are located. Using the measured strains, one obtained the endpoint position and orientation as well as force and moment of the flexible arm. The experimental results demonstrated the effectiveness and accuracy of the proposed approach.

Keywords: Flexible arm, Endpoint, Position, Force, Detection algorithms, Tests.
tion of nonlinear kinematic model for flexible beam element can be found in a technical report (Gu, 2001b).

With the exception of position detection, strain gauge has also been used to detect the endpoint force of flexible manipulator (Richter and Pfeiffer, 1991; Kim et al., 1996). However, the previous studies used only one strain gauge location at the root of each link and employed a simplified model to determine the endpoint force. Actually, the flexible link in a space flexible manipulator can encounter very complicated loading situations and exhibit complex deformation configuration. Using only one strain gauge location cannot obtain enough information and provide accurate measurement of the endpoint force and moment. In this paper, the strain gauges are placed at the four positions of each flexible link and detect both bending and torsion strains at the each position. These four sets of strain measurements are then used to determine the endpoint position and force of the flexible link. One can determine the endpoint position and force of a flexible manipulator by considering the kinematics and statics of both rigid and flexible links. Because the positions and orientations of flexible links are considered in the force transformation, accurate endpoint force and moment are obtained. The experimental results demonstrated the effectiveness and accuracy of the proposed approach.

According to the beam bending and torsion theory, the curvatures of a flexible link is proportional to the strain vector as:

$$\mathbf{\kappa} = \mathbf{C}_a \mathbf{i}\mathbf{\epsilon} \quad i = 1, 2, \cdots, N_e \quad (2)$$

where

$$\mathbf{\kappa} = \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}, \quad \mathbf{C}_a = \begin{bmatrix} 1/c_{a} & 0 & 0 \\ 0 & 1/c_{y} & 0 \\ 0 & 0 & 1/c_{z} \end{bmatrix} \quad (3)$$

The constants $c_{a}$, $c_{y}$ and $c_{z}$ in Eq. 3 are the active radius that depends on the distance from the neutral axis of the link to the associated strain gauges. For a cylindrical flexible beam with outside diameter of $D_0$, this radius is $D_0/2$.

The internal forces associated with the strain measurements include two bending moments and a torsion torque, and are expressed as:

$$\mathbf{iN} = \begin{bmatrix} \mathbf{iN}_x \\ \mathbf{iN}_y \\ \mathbf{iN}_z \end{bmatrix} \quad i = 1, 2, \cdots, N_e \quad (4)$$

A proportional relation between the moment vector $\mathbf{iN}$ and the local strain vector $\mathbf{i\epsilon}$ can be found as:

$$\mathbf{iN} = \mathbf{C}_e \mathbf{i\epsilon} \quad i = 1, 2, \cdots, N_e \quad (5)$$

where the strain-to-force gains are given by

$$\mathbf{C}_e = \begin{bmatrix} G I_x/c_{a} & 0 & 0 \\ 0 & E I_y/c_{y} & 0 \\ 0 & 0 & E I_z/c_{z} \end{bmatrix} \quad (6)$$

$E I_z, E I_y$ and $G I_x$ represent the bending and torsion stiffness of the link cross section.

2. FLEXIBLE LINK UNIT

A cylindrical flexible link used as a position and force measurement unit is shown in Fig. 1. There are $N_e$ positions where the strain gauges are located. In each position, the strain gauges detect two bending strains ($\epsilon_{y}, \epsilon_{z}$) and one torsion strain ($\epsilon_{a}$). The measured bending and torsion strains can be expressed as $N_e$ local strain vectors:

$$\mathbf{i\epsilon} = \begin{bmatrix} \mathbf{i\epsilon}_a \\ \mathbf{i\epsilon}_y \\ \mathbf{i\epsilon}_z \end{bmatrix} \quad i = 1, 2, \cdots, N_e \quad (1)$$

These strain vectors provide the basic information of the link deformations. It is our purpose to use this information to determine the endpoint position and orientation as well as endpoint force and moment of the flexible link. To achieve this purpose, the relations between the local strains and the curvatures as well as the internal forces of link sections are determined.

3. POSE DETERMINATION

To determine the position and orientation of a flexible link, a moving local frame $\{C_i\}$ is defined along the link as shown in Fig. 1. The moving frame has its $x$ axis tangent to the neutral axis and can move along the neutral axis. At any instant, the position and orientation of the moving frame are a function of the space variable $s$, which represents the arc length of the neutral axis measured from the base point of the link to an any given point at the neutral axis. When $s = 0$, the moving frame is located at the base point and called as the base frame $\{C_0\}$. When $s = L$, the moving frame is located at the end point and called as the endpoint frame $\{C_n\}$.

The position and orientation of a flexible link has a one-to-one relation with the curvature of the link. Recalling the kinematics developed by Piedboeuf (Piedboeuf, 1995), the rotation matrix and position
the algorithm that determine the endpoint position and order to show the coupling between the bending and orientation up to the second order: torsion, the second order term in Eq. 7 and Eq. 8 must derive the term of the endpoint position and orientation. In Piedboeuf and Miller (Piedboeuf and Miller, 1994), we obtain an interpolation algorithm using the polynomial function. The algorithm considered only the first order of the endpoint position and orientation as long as the curvatures are determined from the curvature and the position matrix \( N_{i} \). Determination of endpoint position and orientation is straightforward using Eq. 10 - Eq. 23, which involves only simple matrix computation. The position and orientation of any point (frame \( \{C_{i}\} \) ) on the flexible link can be determined by replacing \( L \) with \( \delta \), the arc length from frame \( \{C_{0}\} \) to \( \{C_{i}\} \).

4. STATICS

The external forces are considered to apply at the both ends of the flexible link as shown in Fig. 1.
and moments and setting them equal to zero results in:

\[ \mathbf{T}^n \mathbf{F} = \mathbf{n} \mathbf{F}_n \]  

(24)

with

\[ \mathbf{i} \mathbf{F} = \begin{bmatrix} \mathbf{i} \mathbf{V} \\ \mathbf{i} \mathbf{N} \end{bmatrix}, \quad \mathbf{n} \mathbf{F}_n = \begin{bmatrix} n_{f_1} \\ n_{n_1} \end{bmatrix} \]  

(25)

and

\[ \mathbf{i} \mathbf{T}_F = \begin{bmatrix} \mathbf{i} \mathbf{p}_n \times \mathbf{i} \mathbf{R} & 0 \end{bmatrix} \]  

(26)

where \( \mathbf{i} \mathbf{p}_n \) and \( \mathbf{i} \mathbf{R} \) represent the position and orientation of the endpoint frame \( \{C_n\} \) relative to the frame \( \{C_l\} \). For simplicity, the gravity of the flexible link is not considered in the above formulation. The internal force \( \mathbf{i} \mathbf{F} \) consists of a force vector \( \mathbf{i} \mathbf{V} \) and a moment vector \( \mathbf{i} \mathbf{N} \). Because the strain gauges detect only the bending and torsion strains, the internal moment vector is the available portion in Eq. 24 and one can write the following equation:

\[ \mathbf{i} \mathbf{N} = \begin{bmatrix} \mathbf{i} \mathbf{p}_n \times \mathbf{i} \mathbf{R} & \mathbf{i} \mathbf{R} \end{bmatrix} \mathbf{n} \mathbf{F}_n, \quad i = 1, 2, \ldots, N_e \]  

(27)

Substituting the strain-force relation given in Eq. 5, we obtain a linear relation between the endpoint force and the measured strains as:

\[ \Delta \mathbf{e} = \mathbf{C}_n \mathbf{n} \mathbf{F}_n \]  

(28)

where

\[ \Delta \mathbf{e} = [\Delta \mathbf{e}^1 \ldots \Delta \mathbf{e}^i \ldots \Delta \mathbf{e}^{N_e}]^T \]  

(29)

\[ \mathbf{C}_n = \begin{bmatrix} \mathbf{C}^{-1}_e \mathbf{p}_n \times \mathbf{i} \mathbf{R} & \mathbf{C}^{-1}_e \mathbf{i} \mathbf{R} \\ \vdots & \vdots \\ \mathbf{C}^{-1}_e \mathbf{n}_e \mathbf{p}_n \times \mathbf{n}_e \mathbf{i} \mathbf{R} & \mathbf{C}^{-1}_e \mathbf{n}_e \mathbf{i} \mathbf{R} \end{bmatrix} \]  

(30)

\( \Delta \mathbf{e} \) denotes the change of the strains caused by the endpoint force \( \mathbf{n} \mathbf{F}_n \). The actual measured strain \( \mathbf{e} \) also includes the strains caused by the gravity of the flexible link itself. \( \mathbf{i} \mathbf{p}_n \) and \( \mathbf{i} \mathbf{R} \) can be determined using the following formulation:

\[ \mathbf{i} \mathbf{p}_n = ^0 \mathbf{R}^T (^0 \mathbf{p}_n - ^0 \mathbf{p}_l) \]  

(31)

\[ \mathbf{i} \mathbf{R} = ^0 \mathbf{R}^T v_n \mathbf{R} \]  

(32)

Using Eq. 28, the endpoint force can be expressed as a function of the measurement strains:

\[ \mathbf{n} \mathbf{F}_n = ^n \mathbf{C}_F \Delta \mathbf{e} \]  

(33)

where the force measurement matrix \( ^n \mathbf{C}_F \) is given by:

\[ ^n \mathbf{C}_F = (\mathbf{C}_n^T \mathbf{C}_n)^{-1} \mathbf{C}_n^T \]  

(34)

Eq. 33 gives the endpoint force and moment that are expressed in the endpoint frame \( \{C_n\} \). The endpoint frame is a local frame and its position and orientation vary as the endpoint position and orientation change. In some cases, it is required to know the endpoint force that is expressed in the base frame \( \{C_0\} \) of the flexible link. This endpoint force may be denoted by \( ^0 \mathbf{F}_n \). Using rotation matrix \( ^0 \mathbf{R} \), the endpoint force \( ^n \mathbf{F}_n \) can be transform to the base frame \( \{C_0\} \) and the endpoint force \( ^0 \mathbf{F}_n \) is expressed as:

\[ ^0 \mathbf{F}_n = ^0 \mathbf{R} ^n \mathbf{F}_n \]  

(35)

It is clear that the determination of the endpoint force and moment of a flexible link requires the knowledge of the positions and orientations of the endpoint as well as those points where the strain gauges are located.

5. VALIDATION

The use of strain gauges to obtain the endpoint position and force is especially useful for the flexible manipulator implementing a hybrid position and force control. This is the case of a space flexible manipulator performing a complex operation such as assembly of a component for the space station. Under this situation, the flexible link usually undergoes both bending and torsion deformations. To validate our approach, the position and force tests were conducted on a flexible link as shown in Fig. 3. The flexible link is a cylindrical beam of 1.4 meters long and has the strain gauges located at the four positions. In each position, the strain gauges detect the two bending strains and a torsion strain. Using these four sets of strain measurements, the endpoint position and force were then determined using the algorithms developed in Section 3 and Section 4.

Experimentally, the flexible beam was held at its base by a six degrees of freedom rigid robot. An external

Fig. 3. Flexible beam using for the validation
force was applied to the endpoint of the flexible beam by
hanging a weight at the end of an extension bar that is fixed to
the endpoint of the beam as shown in Fig. 4. The loading
configuration mainly results in a vertical force, a bending
moment and a torsion torque acting on the endpoint of the flexible link.

The actual endpoint position and orientation may be different
because of the gravity of the flexible link. The orientation is
represented using the Euler angles that are obtained from the rotation
matrix (Gu and Piedboeuf, 2001). Because the load is applied in
the vertical direction, large endpoint deflection appears in the
vertical direction, large endpoint deflection appears in the
different because of the gravity of the flexible link. The
orientation is represented using the Euler angles that are
obtained from the rotation matrix (Gu and Piedboeuf, 2001). Because
the load is applied in the vertical direction, large endpoint
deflection appears in the horizontal direction (y axis) and a small
rotation about y axis. This can be explained from
Eq. 7 and Eq. 8, in which the coupling second order terms in
$p_z$ and $\theta_y$ depend on the torsion about z-axis and the bending in y
direction. It can be seen in Fig. 5 that the corresponding
$p_z$ is about 1 mm and $\theta_y$ is about 0.08 degrees. The foreshortening of
the beam, $p_z$ is caused by the beam bending and is about
1.4 mm, which is well detected by the strain gauges.

The endpoint orientation, $\theta_x$ is mainly caused by the
torsion of the beam, and have a value of 1.25 degrees. The
detection results in Fig. 5 has been shown to be compatible to the one obtained using
the Optotrak vision system (Gu and Piedboeuf, 2001).
Fig. 7. Large noise appears in $F_x$ due to the poor observability when using the bending and torsion strains to observe the axial force. To improve the measurement of $F_x$, the axial strain that associated with the internal axial force $V_x$ may be required. The axial strains can be obtained using the same set of the strain gauges as used in the bending strain measurements but different bridge configurations (Gu, 2001a).

To illustrate the accuracy of the detection, the follow-
ing equation is used to calculate the relative error for the endpoint force and moment:

$$ e_r = \frac{\text{actual force} - \text{detected force}}{\text{actual force}} \times 100\% (36) $$

Table 3 lists the relative errors for $F_y$, $M_x$ and $M_z$ under the two loading weights. It can be seen that the errors for the loading weight $F_y$ are within 2 percent.

### Table 1. Endpoint force under load 1.357 kg

<table>
<thead>
<tr>
<th>Force</th>
<th>$F_x$</th>
<th>$F_y$</th>
<th>$F_z$</th>
<th>$M_x$</th>
<th>$M_y$</th>
<th>$M_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta F_n$</td>
<td>1.17</td>
<td>-12.48</td>
<td>-1.20</td>
<td>-8.28</td>
<td>-0.30</td>
<td>-1.21</td>
</tr>
<tr>
<td>$\delta F_n$</td>
<td>-0.54</td>
<td>-13.55</td>
<td>-0.40</td>
<td>-8.28</td>
<td>0.28</td>
<td>-1.19</td>
</tr>
</tbody>
</table>

In order to compare the detected endpoint forces $\delta F_n$ and $\delta F_n$ with the actual endpoint force. We obtained the actual endpoint force and moment according to the mass of the weight and the dimensions of the endpoint fixture and extension bar as shown in Fig. 4. The endpoint orientation of the flexible link has also been considered. For convenience, the actual force is denoted as $\delta F_n$.

### Table 2. Endpoint force Under load 2.273 kg

<table>
<thead>
<tr>
<th>Force</th>
<th>$F_x$</th>
<th>$F_y$</th>
<th>$F_z$</th>
<th>$M_x$</th>
<th>$M_y$</th>
<th>$M_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta F_n$</td>
<td>2.64</td>
<td>-20.48</td>
<td>-2.21</td>
<td>-13.75</td>
<td>-0.85</td>
<td>-2.16</td>
</tr>
<tr>
<td>$\delta F_n$</td>
<td>-0.54</td>
<td>-22.04</td>
<td>-0.55</td>
<td>-13.72</td>
<td>0.32</td>
<td>-2.02</td>
</tr>
</tbody>
</table>

Table 5 and Table 2 summarize the endpoint forces $\delta F_n$, $\delta F_n$ and $\delta F_n$ for the two loading weights. The units for the force and moment in the Tables are Newton (N) and Newton per meter (N/m) respectively. The actual endpoint force $\delta F_n$ is comparable to the detected endpoint force $\delta F_n$ since they are expressed in the same frame. From Table 5 and Table 2 we can see they are close to each other. The small errors are basically coming from the coordinate misalignment and measurement noise. However, the detected endpoint force $\delta F_n$ exhibits some difference to the actual endpoint force $\delta F_n$ because they are expressed in the different frames. This indicates that if a force sensor at end-effector is used for force control of flexible manipulator, it is still required to know the endpoint position and orientation of the flexible link when mapping the endpoint force from the endpoint frame $\{C_n\}$ to the base frame $\{C_0\}$. Otherwise, a misunderstanding of the endpoint force/moment will lead to an error in force control, especially for the flexible manipulator undergoing a large elastic deformation.

To illustrate the accuracy of the detection, the following equation is used to calculate the relative error for the endpoint force and moment:

$$ e_r = \frac{\text{actual force} - \text{detected force}}{\text{actual force}} \times 100\% (36) $$

Table 3 lists the relative errors for $F_y$, $M_x$ and $M_z$ under the two loading weights. It can be seen that the errors for the loading weight $F_y$ are within 2 percent.

### Table 3. Relative errors (in %)

<table>
<thead>
<tr>
<th>Loading weight</th>
<th>$e_x - F_x$</th>
<th>$e_y - M_x$</th>
<th>$e_z - M_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.357 kg</td>
<td>1.88</td>
<td>1.85</td>
<td>1.68</td>
</tr>
<tr>
<td>2.237 kg</td>
<td>1.08</td>
<td>0.95</td>
<td>3.06</td>
</tr>
</tbody>
</table>

### 6. CONCLUSION

We have demonstrated the use of flexible link as an endpoint position and force detection unit. The experimental results validate the proposed approach. Because this approach is dynamic-model free and involves only simple matrix calculations, it is easy to apply to a real system and does not require substantial computational resources. The position and force obtained in real-time can be directly used in kinematic transformation, static balance as well as hybrid position and force control of a flexible manipulator.

### 7. REFERENCES


