ON HIDDEN COUPLING IN MULTIPLE MODEL BASED PID CONTROLLER NETWORKS

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Abstract: The dynamics of conventional gain scheduling and local controller networks are subject to hidden coupling which introduces the requirement that the weighting functions be slow-varying. In this paper, two local PID controller network structures are developed, from the velocity-based and conventional local model network frameworks, respectively, which eliminate this hidden coupling term so that the dynamics of the global controller are not restricted by the blending functions. Thus the local properties can be guaranteed by the free choice of weighting functions. Simulation studies on a continuous stirred tank reactor confirm the analytical results.

Keywords: nonlinear control, PID control, local controller networks, local model networks

1. INTRODUCTION

Controller design for nonlinear plant remains difficult due to the inherent complexity of such systems. However, since approaches to the design and analysis of linear time-invariant (LTI) systems are well developed, it is attractive to solve the nonlinear control problem using linear system techniques. In practice gain scheduling is the most popular approach to nonlinear control problems (Rugh and Shamma, 2000; Leith and Leithead, 2000). First, one designs local linear controllers based on linearizations of the plant at several different operating points. A global nonlinear controller for the plant is then obtained by interpolating the gains of these local controllers. Surprisingly although gain scheduling has been successfully applied in many real world applications, it is difficult to obtain the analytical results on stability and robustness for this approach. The main problem is that the influence of the scheduling functions has to be taken into account. (Shamma and Athans, 1990) proved that this can be neglected only if the scheduling variable varies very slowly. In fact, the scheduling functions introduce an extra term into the global control law, which is referred to as hidden coupling (Lawrence and Rugh, 1995). In the fast scheduling case, this term may be very large and can then significantly affect the dynamics of the global controller.

Since (Johansen and Foss, 1992) proposed the local model network (LMN) for nonlinear dynamic modelling, the approach has been widely studied for modelling and control. The local controller network (LCN) is directly based on the local model structure, whereby the local controllers are designed for each local model and then blended together with the same weighting functions as in the LMN. Unfortunately, as in the conventional gain scheduling, there is also hidden coupling in these global controllers, which confines their use to slow-varying weighting functions. However, most weighting functions in LMNs in fact can vary rapidly in some operating regions.

The velocity-based approach to LMN was proposed recently (Leith and Leithead, 1999). Based on velocity-based linearization theory, the dynamics of nonlinear plant can now be approximated by the velocity-based LMN. As before, the velocity-
based global controller in the form of a LCN follows from the velocity-based LMN.

This paper presents two types of local PID controller network for a SISO nonlinear plant in which the restriction of slow-varying weighting functions can be relaxed. Analysis shows that hidden coupling is absent in the global controllers produced by these two approaches. The paper is organized as follows. In section 2, the conventional and velocity-based LMNs are briefly discussed. Section 3 addresses the design of two new PID controller networks. These are then analyzed and the absence of undesirable hidden coupling established. Supporting results from simulation studies on a Continued Stirred Tank Reactor are given in section 5, followed by conclusion in section 6.

2. NONLINEAR MODELLING WITH MULTI-MODEL APPROACH

Consider the single input and single output (SISO) nonlinear plant

\[ \dot{x} = f(x, u) \]
\[ y = g(x) \]  

(1)

where \( f(\cdot, \cdot) \) and \( g(\cdot) \) are nonlinear functions, \( x \in \mathbb{R}^n \) is the state vector, \( u \) is the control input and \( y \) is the measured output. In equ.(1) \((x_i^e, u_i^e, y_i^e), i = 1, 2, ..., N \) are a family of equilibrium points of the nonlinear plant (1) such that

\[ 0 = f(x_i^e, u_i^e) \]
\[ y_i^e = g(x_i^e) \]  

(2)

Linearizing the nonlinear plant in (1) at the point \((x_i^e, u_i^e, y_i^e)\), gives a family of local linear models.

\[ \dot{x}_i^e = A_i x_i^e + B_i u_i^e \]
\[ y_i^e = C_i x_i^e \]  

(3)

where the deviation variables are defined as \( x_i^e = x - x_i^e \), \( u_i^e = u - u_i^e \), \( y_i^e = y - y_i^e \), and the coefficients matrices are computed from,

\[ A_i = \frac{\partial f}{\partial x}(x_i^e, u_i^e) \]
\[ B_i = \frac{\partial f}{\partial u}(x_i^e, u_i^e) \]
\[ C_i = \frac{\partial g}{\partial x}(x_i^e) \]  

Equ.(3) is now rewritten in the form of a global state local model by substituting the deviation variables into equ.(3) to give

\[ \dot{x} = A_i (x - x_i^e) + B_i (u - u_i^e) \]
\[ y = C_i (x - x_i^e) + y_i^e \]  

(5)

This can be expressed as

\[ \dot{x} = A_i x + B_i u + M_i \]
\[ y = C_i x + N_i \]  

(6)

where

\[ M_i = -A_i x_i^e - B_i u_i^e \]
\[ N_i = -C_i x_i^e + y_i^e \]  

(7)

A conventional modelling approach for the nonlinear plant (1) is to combine the local models (6) with appropriate weighting functions \( \mu_i(\sigma) \),

\[ \dot{x} = \sum_{j=1}^{N} \mu_j(\sigma) (A_j \dot{x} + B_j u + M_j) \]
\[ \dot{y} = \sum_{j=1}^{N} \mu_j(\sigma) (C_j \dot{x} + N_j) \]  

(8)

Here \( \dot{x} \) is the state vector in the global model. The scheduling variable \( \sigma \), which can be a vector or a scalar, should embody the nonlinearity of the plant (1).

The velocity-based LMN was proposed by (Leith and Leithead, 1999). Differentiating both sides of equ.(6), the \( i \)th velocity-based local model is given by

\[ \dot{x} = A_i \dot{x} + B_i \dot{u} \]
\[ \dot{y} = C_i \dot{x} \]  

(9)

Note that in equ.(9), the model is linear in the variables \( \dot{x} \) and \( \dot{u} \). A velocity-based LMN is now constructed by combining the local models in equ.(9) with appropriate weighting functions \( \mu_i(\sigma) \).

\[ \dot{x} = \sum_{j=1}^{N} \mu_j(\sigma) \left( A_j \dot{x} + B_j \dot{u} \right) \]
\[ \dot{y} = \sum_{j=1}^{N} \mu_j(\sigma) (C_j \dot{x}) \]  

(10)

(Shorten et al., 1999) and (McLoone and Irwin, 2001) analyzed the dynamics of the conventional and velocity-based LMNs, respectively. By comparing the eigenvalues of linearizations for the nonlinear plant and the LMNs, it was found that the velocity-based LMN would provide better approximation to the dynamics of nonlinear plant.

3. MULTI-MODEL BASED PID CONTROLLERS

In this section, we consider the tracking control for nonlinear system in equ. (1). The control objective is to track the reference input \( r \) while the output \( y \) meets some performance requirement.
3.1 Conventional Gain Scheduled PID Control

For each local model (3), a PID controller can be designed as,

\[ u_i(t) = k_{P_i} e(t) + k_{I_i} \int_0^t e(v)dv + k_{D_i} \dot{e}(t) \]  

(11)

where \( k_{P_i}, k_{I_i}, k_{D_i} \) are the PID parameters and \( e = r - y \) is the tracking error. The conventional gain scheduled PID controller is a combination of local controllers with some weighting functions \( \rho_i(\sigma) \),

\[ u(t) = \sum_{j=1}^N \rho_j(\sigma) u_j(t) \]  

(12)

Note that in general, the control weighting function \( \rho_i \) can differ from the LMN one.

To investigate the local property of the closed-loop system, the local controller is written in the form of a transfer function with an approximate derivative component,

\[ G_{ci}(s) = k_{P_i} + \frac{k_{I_i}}{s} + k_{D_i} \frac{s}{N_s s + 1} \]  

(13)

where \( N_s \) is a large positive number. The controller (13) can be converted to the state form,

\[ \dot{x}_{ci} = F_i x_{ci} + G_i e \]  

(14)

\[ u_i = H_i x_{ci} + E_i e \]

where \( x_{ci} \in \mathbb{R}^{2 \times 1} \) is the state of the local PID controller, and

\[ F_i = \begin{bmatrix} -N_s & 0 \\ 1 & 0 \end{bmatrix} \quad G_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \]

\[ H_i = \begin{bmatrix} k_{I_i} - k_{D_i} N_s^2 & k_{I_i} N_s \end{bmatrix} \]

\[ E_i = N_s k_{D_i} + k_{P_i} \]  

(15)

Thus the global controller (12) is given by

\[ \dot{x}_g = F x_g + G e \]  

(16)

\[ u = \sum_{j=1}^N \rho_j(\sigma) (H_j x_c + E_j e) \]

In the steady-state, \( e^c = 0 \). From equ. (16), it can be seen that \( x_g^c \neq 0 \) when \( u^c \neq 0 \). Linearizing the controller (16) at an equilibrium point \( (x_g^c, e^c, u^c) = (x_g^c, 0, u^c) \) and letting the corresponding steady-state of \( \sigma \) be \( \sigma^c \), gives,

\[ \dot{x}_g^c = F x_g^c + G e^c \]  

(17)

\[ u^c = \sum_{i=1}^N \rho_i(\sigma^c) (H_i x_g^c + E_i e^c) + \sum_{i=1}^N \frac{\partial \rho_i}{\partial \sigma} (\sigma^c) H_i x_g^c \sigma^c \]

where \( x_g^c = x_g - x_g^c, e^c = e - e^c = e, u^c = u - u^c \) and \( \sigma^c = \sigma - \sigma^c \) are deviation variables. It can be seen that in the linearized controller, there is an extra term which is related to the rate of change of the scheduling functions. This term, which is referred to as hidden coupling, can significantly and detrimentally affect the local dynamics of the closed-loop system, particularly if the weighting functions vary rapidly.

3.2 Velocity-based Local PID Controller Network

The velocity-based LMN in equ.(10), within which each local model is linear, is an approximation to the nonlinear plant in equ.(1). The controller design for this plant therefore can be based on the LMN (10).

A PID controller for the local model of equ.(9) is shown in Fig. 1, where the input of the local PID controller is the derivative of tracking error, i.e.,

\[ \dot{e} = \frac{d}{dt} (r - y) = \dot{r} - \dot{y} \]  

(18)

The PID controller therefore can be described as

\[ \dot{u}_i(t) = k_{P_i} \dot{e}(t) + k_{I_i} \int_0^t \dot{e}(v)dv + k_{D_i} \ddot{e}(t) \]  

(19)

where \( \dot{u}_i \) is the controller output, as well as the input to the local model of equ.(9).

![Fig. 1. Local closed-loop system](image)

Following the LCN approach, the global controller is a combination of local controllers with the same weighting functions as in the LMN. Thus

\[ \dot{u}(t) = \sum_{j=1}^N \mu_j(\sigma) \dot{u}_j(t) \]  

(20)

The controller output in equ.(20) is in velocity form. Applying the global controller to the nonlinear plant in equ.(1), produces the closed-loop system shown in Fig. 2. Here it can be seen that, since

![Fig. 2. Velocity-based global feedback interconnection](image)

the controller is in velocity form, the output of the nonlinear plant is differentiated to obtain the
derivative of the tracking error and the controller output is integrated back to the original form. Further, for the scheduler in Fig. 2. to capture the nonlinearity in the plant in equ. (1), the scheduling variables should be the same as in equ. (1), i.e., not in velocity form.

As in Fig. 2, the tracking error is differentiated to provide the controller input. In practice, this cannot be implemented exactly. One solution is to use \( \frac{\text{d}x}{\text{d}t} \) to approximate pure differentiation \( \frac{\text{d}x}{\text{d}t} \), as is common practice when implementing the D term in PID controllers.

3.3 Mixed-Blended PID Control

Based on the LMN (8), the PID controller network can be constructed as

\[
u(t) = \sum_{j=1}^{N} \mu_j(\sigma) \left( k_{Pj} \dot{e}(t) + k_{Dj} \ddot{e}(t) \right) + \int_0^t \sum_{j=1}^{N} \mu_j(\sigma) k_{Ij} e(v) \, dv
\]

(21)

In this control structure, the weighted tracking error is integrated. This is different to the conventional gain scheduled control, in which only the tracking error is integrated. Thus, controller (21) is referred to as the mixed-blended PID controller.

4. HIDDEN COUPLING ANALYSIS

4.1 Velocity-based PID Controller Network

Rewriting the local controller (19) in state equation form, gives

\[
\begin{aligned}
\dot{x}_{ci} &= F_i x_{ci} + G_i \dot{e} \\
\dot{u}_i &= H_i x_{ci} + E_i \dot{e}
\end{aligned}
\]

(22)

Here the matrices \( F_i, G_i, H_i, E_i \) are as defined in equ. (15). In the steady-state, \( \dot{u}_i = 0, \dot{e} = 0 \). Therefore, from the state equation (22) it can be seen that the steady-state \( x_{ci}^* \) is zero. The global controller is a combination of the local controllers in equ.(22) with weighting functions \( \mu_i(\sigma) \). Thus,

\[
\begin{aligned}
\dot{x}_c &= F x_c + G \dot{e} \\
\dot{u} &= \sum_{j=1}^{N} \mu_j(\sigma) \left( H_j x_c + E_j \right) \dot{e}
\end{aligned}
\]

(23)

From the above analysis, the equilibrium point \((x_c^*, \dot{e}^*, \dot{u}^*)\) in equ.(23) is \((0, 0, 0)\). Although there is only one equilibrium point, since the scheduling variable covers the whole operating range of the nonlinear plant in equ.(1), the PID parameters are updated to provide the appropriate controller at the current operating point. Linearizing the global controller at the equilibrium point \((0, 0, 0)\) gives

\[
\begin{aligned}
\dot{x}_c^* &= F x_c^* + G \dot{e}^* \\
\dot{u}^* &= \sum_{j=1}^{N} \mu_j(\sigma) \left( H_j x_c^* + E_j \right) \dot{e}^*
\end{aligned}
\]

(24)

\[
\begin{aligned}
\dot{\dot{e}}^* &= \sum_{j=1}^{N} \mu_j(\sigma) \left( H_j x_c^* + E_j \dot{e}^* \right) + \\
&+ \left[ \left( \sum_{j=1}^{N} \frac{\partial}{\partial \sigma} \mu_j(\sigma) H_j \right) x_c^* \right] \sigma^* + \\
&+ \left[ \left( \sum_{j=1}^{N} \frac{\partial}{\partial \sigma} \mu_j(\sigma) E_j \right) \dot{e}^* \right] \sigma^*
\end{aligned}
\]

where \( x_{ci}^*, \dot{e}^*, \dot{u}^* \) and \( \sigma^* \) are the deviation variables for \( x_{ci}, \dot{e}, \dot{u} \) and \( \sigma \), respectively. It can be seen that since the weighting functions are parameter-varying functions, extra terms related to the deviation of the scheduling variable \( \sigma \) have been introduced. However, since the linearization point \((x_{ci}^*, \dot{e}^*, \dot{u}^*)\) is \((0, 0, 0)\), these extra terms are eliminated, i.e.,

\[
\begin{aligned}
\left[ \left( \sum_{j=1}^{N} \frac{\partial}{\partial \sigma} \mu_j(\sigma) H_j \right) x_c^* \right] \sigma^* &= 0 \\
\left[ \left( \sum_{j=1}^{N} \frac{\partial}{\partial \sigma} \mu_j(\sigma) E_j \right) \dot{e}^* \right] \sigma^* &= 0
\end{aligned}
\]

which proves that there is no hidden coupling.

4.2 Mixed-Blended PID Controller.

Controller (21) can be written in state form,

\[
\begin{aligned}
\dot{x}_{c1} &= -N_u x_{c1} + e \\
\dot{x}_{c2} &= \sum_{j=1}^{N} \mu_j(\sigma) k_{Ij} e \\
u &= \sum_{j=1}^{N} \mu_j(\sigma) \left( -k_{Dj} N_u^2 x_{c1} + x_{c2} + (N_u k_{Dj} + k_{Pj}) e \right)
\end{aligned}
\]

(25)

where \( x_{ci}, i = 1, 2 \) are the states in the mixed-blended PID controller. In the steady-state, the tracking error \( e = 0 \). Thus the steady-state of \( x_{c1} \) is \( x_{c1}^* = 0 \). Let the steady-state of \( x_{c2} \) be \( x_{c2}^* \). With the same approach as in the analysis of the velocity-based PID controller network, linearizing the state equation (25) at the equilibrium point \((x_{c1}^*, x_{c2}^*, e^*)\) is \((0, x_{c2}^*, 0)\), the terms related with the deviation of scheduling variable \( \sigma^* \) can be computed to be zero, which again means there is no hidden coupling in the global controller.

In the absence of hidden coupling, the scheduling functions in velocity-based and mixed-blended PID controllers can be allowed to vary rapidly. The weighting functions in a LMN can therefore be directly applied to the global controller so that \( \rho_i = \mu_i \) in equ. (12). On the other hand, it also provides the freedom to design the weighting functions so that the local property can be guaranteed near any equilibrium points.
5. CSTR SIMULATION EXAMPLE

Consider now the control of a simulated Continuous Stirred Tank Reactor (CSTR) plant,

\[
\begin{align*}
\dot{T}(t) &= \frac{\dot{Q}}{V} (T_f - T(t)) + K_1 C(t) e^{-\frac{E}{RT}} + \nonumber \\
&+ K_2 q(t) \left[ 1 - e^{-\frac{K_1}{Vt}} \right] (T_c - T(t)) \quad (26) \\
\dot{C}(t) &= \frac{\dot{Q}}{V} (C_f - C(t)) - K_0 C(t) e^{-\frac{E}{RT}} 
\end{align*}
\]

This is a two-state SISO highly nonlinear system in which the input \(q_c(t)\) is the flow rate of a coolant, the output \(C(t)\) is the concentration of a product compound, and state \(T(t)\) is the temperature of the solution. The parameter values and further details on the nonlinear behavior of the CSTR can be found in (McLoone et al., 1998).

The operating range used in the simulation of (26) was: \(C \in [0.04, 0.13]\), over which the CSTR plant remains stable. The performance specification was defined as: settling time \((2\%) \leq 5\text{ min}\), percentage overshoot \(\leq 10\%\), steady-state error = 0. Two linearization points \((C^*_i, T^*_i, q^*_i), i = 1, 2\) were chosen as follows:

\[
\begin{align*}
EQ1 : (0.0439, 456.2452, 80) \\
EQ2 : (0.1298, 432.9487, 110)
\end{align*}
\]

Linearizing the CSTR plant at these two points, produced two local models given by

\[
\begin{align*}
G_{LM1}(s) &= \frac{0.04877}{s^2 + 15.4s + 31.84} \quad (28) \\
G_{LM2}(s) &= \frac{0.03801}{s^2 + 0.518s + 6.878}
\end{align*}
\]

The PID parameters, selected so that the performance requirements were met for each operating points, were,

\[
k_{P1} = 90.5039, k_{I1} = 867.7892, k_{D1} = 0 \quad (29) \\
k_{P2} = 64.48, k_{I2} = 145.3, k_{D2} = 191.95
\]

Two groups of weighting functions were used to construct the global controller, with the plant output \(C\) as the scheduling variable. Fig. 3 shows slow- and fast-varying weighting functions.

Fig. 4 shows the tracking responses for the conventional LCNs applied to (26) with slow-varying and fast-varying weighting functions. The tracking responses for the velocity-based LCN and mixed-blended PID LCN are shown in Fig. 5 and Fig. 6 respectively.

It can be seen that, for the slow-varying weighting functions, the performance requirement is met across the whole operating range for all types of LCN. However, when they are fast-varying, the response of the conventional LCN is sluggish midway between the two linearization points (Fig. 4(b)), where the weighting functions vary most rapidly. For the velocity-based LCN, the overshoot is large in some regions mainly because of a large setpoint change. If this step size in setpoint is kept very small, the controller produces good performance globally. This also demonstrates that the local property is guaranteed without hidden coupling. For the mixed-blended PID controller, both responses meet the performance requirement. Robustness of both proposed controllers was investigated by varying \(q_f\) from 60 to 170, the nominal value of which is 100/min. Due to the space reasons, the results are not included in this paper. It was shown in all cases that the closed-loop systems were stable for the parameter uncertainties under consideration.
Fig. 6. Tracking responses of mixed-blended PID controller network


7. REFERENCES

