ON THE SENSITIVITY OF THE COUPLED CONTINUOUS-TIME RICCATI EQUATION

Adam Czornik * Aleksander Nawrat **
Andrzej Swierniak *

* Department of Automatic Control, Silesian Technical University, Akademicka 16, 44-101 Gliwice, Poland
** Department of Mathematics, Silesian Technical University, Kaszubska 23, 44-101 Gliwice, Poland

Abstract: The sensitivity of coupled continuous-time Riccati equations is analyzed through the norm of the inverse Lyapunov-like operator. This analysis leads to bounds on the perturbation of the solution. As a byproduct some properties of coupled Lyapunov equation and results on the stability of linear systems with jumps are presented.

Keywords: Coupled Riccati equation, sensitivity, stability, jump linear systems;

1. INTRODUCTION AND NOTATIONS

Coupled Riccati equation plays an important role in optimization control of linear systems parameters of which obey abrupt changes. In real words model parameters are not known precisely, for example they come from certain estimation procedures. In such a situation we are interested in the difference between the solution with true parameters and the solution with approximate ones. In the paper we present upper bounds for the norm of this difference in terms of degree of accuracy of the coefficients. Similar problem for standard Riccati equation has been investigated in (Gahinet and Laub, 1990). The problem of sensitivity of coupled Lyapunov equation which is a special case of coupled Riccati equation in the discrete time case has been discussed in (Czornik and Swierniak, 2001).

The paper is organized as follows. In the next section, we introduce the coupled Lyapunov equation and review some basic concepts concerning stability of jump linear systems. The coupled Riccati equation is presented in Section 3. The main result is included in Section 4 where the sensitivity results for coupled Riccati equation are presented. In Section 5 our results are illustrated by a numerical example. Finally, Section 6 contains conclusions.

The following notation and definitions are used in the paper.

We denote by $M_{n;m}(\mathbb{R})$ the linear space of all $n \times m$ real matrices. In case $m = n$ we write simply $M_n(\mathbb{R})$. We will write $t$ for transpose. The Euclidian norm in $\mathbb{R}^n$ will be denoted by $|\cdot|$. We denote $M_n(R)^+ = \{L \in M_n(R) : L = L^t, L \geq 0\}$. In $M_n(R)$ we consider the spectral norm $\|A\| = \max_{|x|=1} |Ax|$. Let $\mathcal{H}_n (\mathcal{H}_n^+)$ be the linear space made up of all $s$-length sequences of matrices $H = (H_1, \ldots, H_s)$, $H_i \in M_n(R)$ ($H_i \in M_n(R)^+$). In these spaces we will consider the following three norms $\|H\|_1 = \sum_{i=1}^s \|H_i\|$, $\|H\|_2 = \sqrt{\sum_{i=1}^s \|H_i\|^2}$, $\|H\|_\infty = \max \{\|H_i\| : i = 1, \ldots, s\}$. For a linear operator $X : \mathcal{H}_n \rightarrow M_n(R)$ we will consider also three norms $\|\cdot\|_1$, $\|\cdot\|_2$, $\|\cdot\|_\infty$ induced re-

---

1 The paper was supported by the Polish Committee for Scientific Researches (Grants 4T11A 012 22 and 8T11A 012 19).
respectively by \( \| \cdot \|_1, \| \cdot \|_2, \| \cdot \|_\infty \) in \( \mathcal{H}_n \) and \( \| \cdot \| \) in \( \mathcal{M}_n(\mathbb{R}) \). We will also consider three norms \( \| \cdot \|_1, \| \cdot \|_2, \| \cdot \|_\infty \) of a linear operator \( Y: \mathcal{H}_n \rightarrow \mathcal{H}_n \) and \( \| \cdot \| \) in \( \mathcal{M}_n(\mathbb{R}) \).

Definition 1. System (4) is stable, if for all \((x_0,i_0)\) in \( \mathbb{R}^n \times S \), we have

\[
\int_0^\infty E \| x(t,x_0,i_0) \|^2 dt < \infty.
\]

In that case we call the pair \((A,\Pi)\), where \( A = (A_1,\ldots,A_s) \in \mathcal{H}_n \) stable.

We assume throughout this paper that \((A,\Pi)\) is stable. For \( A \in \mathcal{H}_n \) and \( \Pi \in \Pi \) define the following Lyapunov operator

\[
\mathcal{L}_{A,\Pi} = \left( \mathcal{L}_{A,\Pi}^{(1)},\ldots,\mathcal{L}_{A,\Pi}^{(s)} \right)
\]

\[
\mathcal{L}_{A,\Pi}^{(i)}(x(t)) = A_i^t x_i + H_i A_i + \sum_{j \in S} \pi_{ij} H_j.
\]

Theorem 2. The system (4) is stable if and only if for any positive definite \( Q \in \mathcal{H}_n^+ \), there exists a unique positive definite \( H \in \mathcal{H}_n^+ \) such that

\[
\mathcal{L}_{A,\Pi} H = -Q.
\]

The following theorem establishes relationships between the norm of the Lyapunov operator and the solution of the relevant Lyapunov equation and the proof can be obtained as in (Czornik and Swierniak, 2001).

Theorem 3. Let \((A,\Pi)\) be stable. If \( \mathcal{P} \) is the unique solution of

\[
\mathcal{L}_{A,\Pi} \mathcal{P} = I_n,
\]

then

\[
\left\| \mathcal{L}_{A,\Pi}^{-1} \right\|_\infty = \left\| \mathcal{P} \right\|_\infty .
\]

Lemma 4. For \( A,\Delta A \in \mathcal{H}_n \), \( \Pi \in \Pi \) and \( \Delta \Pi = [\Delta \pi_{ij}]_{i,j \in S} \in \mathcal{M}_n(\mathbb{R}) \) such that \( \Pi + \Delta \Pi \in \Pi \),
and positive definite $Q \in \mathcal{H}_n^+$ suppose that $(A, \Pi)$ is stable and that
\[ \frac{\lambda_n(Q)}{2\|H\|_1} \geq \max \left\{ \left\| \Delta A_i + \frac{1}{2} \Delta \pi_{ij} I_n \right\|, \right. \\
\left. \; \max_{j \in S} \frac{1}{2} \left| \Delta \pi_{ij} \right| \right\}, \tag{11} \]
where $H$ is the unique solution of (6), then $(A + \Delta A, \Pi + \Delta \Pi)$ is stable.

Observe that if $\Delta \pi_{ii} = 0$ for all $i \in S$ (and consequently $\Delta \pi_{ij} = 0$ for all $i, j \in S$), then inequality (11) guarantees that $\bar{Q}_i > 0$ if
\[ \| \Delta A_i \| \leq \frac{\lambda_n(Q)}{2\|H\|_1} \]
Lemma 5. For $A, \Delta A \in \mathcal{H}_n^+, \Pi \in \Pi$ and positive definite $Q \in \mathcal{H}_n^+$ suppose that $(A, \Pi)$ is stable and that
\[ \frac{\lambda_n(Q)}{2\|H\|_1} \geq \| \Delta A_i \|, \; i \in S \tag{12} \]
then $(A + \Delta A, \Pi)$ is stable, where $H$ is the unique solution of (6).

3. THE COUPLED RICCATI EQUATION

When we consider controlled linear system
\[ \dot{x}(t) = A_{\tau(t)}x(t) + B_{\tau(t)}u(t), t \geq 0, \tag{13} \]
and the quadratic cost functional to be minimized
\[ J(x_0, u) = E \int_0^\infty \left\{ Q_{\tau(t)}x(t), x(t) \right\} + \left\langle \tilde{R}_{\tau(t)}u(t), u(t) \right\rangle \; dt, \tag{14} \]
where $Q = (Q_1, \ldots, Q_s) \in \mathcal{H}_n^+$, $\tilde{R} = (\tilde{R}_1, \ldots, \tilde{R}_s) \in \mathcal{H}_n^{++}, \tilde{R}_i > 0, i \in S$ the following coupled Riccati equation (CRE) arises
\[ Q_i + P_i A_i + A^*_i P_i - P_i R_i P_i + \sum_{j \in S} \pi_{ij} P_j = 0, \tag{15} \]
where $R_i = B_i \tilde{R}_i^{-1} B^*_i$. To present conditions for the existence of a certain type of solution of (CRE) we introduce two definitions.

Definition 2. System (13) is stabilizable if, there exists a sequence $K = (K_1, \ldots, K_s), K_i \in M_{n,m}(R), i \in S$ such that $(A - BK, \Pi)$ is stable. In this case we call the triplet $(A, B, \Pi)$, stabilizable.

Definition 3. The jump linear system
\[ \begin{align*}
\dot{x}(t) &= A_{\tau(t)}x(t) \\ y(t) &= C_{\tau(t)}x(t),
\end{align*} \]
where $C_i \in M_{n,m}$, is detectable, if there exists a sequence $K = (K_1, \ldots, K_s), K_i \in M_{n,i}(R), i \in S$ such that $(A - KC, \Pi)$ is stable. In this case we call the triplet $(A, C, \Pi)$ detectable.

The proof of the next theorem may be found in (Mariton, 1990).

Theorem 6. If $(A, R, \Pi)$ is stabilizable and $(A, Q, \Pi)$, is detectable then (15) has a unique solution $P \in \mathcal{H}_n^+$ Moreover $(A - RP, \Pi)$ is stable.

A solution $P \in \mathcal{H}_n^+$ of (15) such that $(A - RP, \Pi)$ is stable is called a stabilizing solution.

4. SENSITIVITY OF COUPLED RICCATI EQUATION

In this paragraph we will investigate the following perturbed version of (15)
\[ Q_i + \Delta Q_i + V_i (A_i + \Delta A_i) \]
\[ + (A_i + \Delta A_i)\dagger V_i - V_i (R_i + \Delta R_i) V_i \]
\[ + \sum_{j \in S} (\pi_{ij} + \Delta \pi_{ij}) V_j = 0, \tag{16} \]
where the perturbations are as follows
\[ \Delta A = (\Delta A_1, \ldots, \Delta A_s) \in \mathcal{H}_n, \]
\[ \Delta R = (\Delta R_1, \ldots, \Delta R_s) \in \mathcal{H}_n^+, \]
\[ \Delta Q = (\Delta Q_1, \ldots, \Delta Q_s) \in \mathcal{H}_n^+ \]
and $\Delta \Pi = [\Delta \pi_{ij}]_{i,j \in S} \in M_{n,R}$ is such that $\Pi + \Delta \Pi \in \Pi$.

To formulate next Lemma denote by $\Pi$ and $H$ the unique solutions of
\[ \mathcal{L}_{A-RK, \Pi} \Pi = -I \]
and
\[ \mathcal{L}_{A-LC, \Pi} H = -I \]
for stable $(A - RK, \Pi)$ and $(A - LC, \Pi)$, respectively.
Lemma 7. Suppose that \((A, R, \Pi)\), is stabilizable, \((A - R K, \Pi)\) is stable, \((A, Q, \Pi)\), is detectable and \((A - L Q, \Pi)\), is stable for certain \(K = (K_1, \ldots, K_s) \in \mathcal{H}_n\), and \(L = (L_1, \ldots, L_s) \in \mathcal{H}_n\). If the perturbations are such that

\[
\frac{1}{2} \|H\|_1 \geq \max \left\{ \varphi_1, \max_{j \in S} \frac{1}{2} |\Delta \pi_{ij}| \right\}
\]

and

\[
\frac{1}{2} \|H\|_1 \geq \max \left\{ \varphi_2, \max_{j \in S} \frac{1}{2} |\Delta \pi_{ij}| \right\},
\]

then (16) has a unique stabilizing solution, where

\[
\varphi_1 = \left\| \Delta A_i + \Delta R_i K_i + \frac{1}{2} \Delta \pi_i I_n \right\|
\]

\[
\varphi_2 = \left\| \Delta A_i + \Delta Q_i L_i + \frac{1}{2} \Delta \pi_i I_n \right\|
\]

Proof. We have

\[
(A_i + \Delta A_i) + (R_i + \Delta R_i) K_i = A_i + R_i K_i + \Delta A_i + \Delta R_i K_i.
\]

Assumption (17) together with Lemma 4 leads to the conclusion that \((A + \Delta A (R + \Delta R) K, \Pi + \Delta \Pi)\) is stable and consequently \((A + \Delta A, R + \Delta R, \Pi + \Delta \Pi)\) is stabilizable. Similarly assumption (18) implies detectability of \((A + \Delta A, Q + \Delta Q, \Pi + \Delta \Pi)\). The conclusions of the Lemma follow now from Theorem 6.

In the case when \(\Delta \Pi = 0\) we can use Lemma 5 in the proof of Lemma 7 and obtain the following result.

Lemma 8. Suppose that \((A, R, \Pi)\), is stabilizable, \((A - R K, \Pi)\) is stable, \((A, Q, \Pi)\), is detectable and \((A - L Q, \Pi)\), is stable for certain \(K = (K_1, \ldots, K_s) \in \mathcal{H}_n\), and \(L = (L_1, \ldots, L_s) \in \mathcal{H}_n\). If the perturbations are such that \(\Delta \Pi = 0\),

\[
\frac{1}{2} \|H\|_1 \geq \left\| \Delta A_i + \Delta R_i K_i \right\|, \ i \in S
\]

and

\[
\frac{1}{2} \|H\|_1 \geq \left\| \Delta A_i + L_i \Delta Q_i \right\|, \ i \in S
\]

then

\[
Q_i + \Delta Q_i + V_i (A_i + \Delta A_i) + (A_i + \Delta A_i)' V_i
\]

\[
- V_i (R_i + \Delta R_i) V_i + \sum_{j \in S} \pi_{ij} V_j = 0
\]

has a unique stabilizing solution.

To present next theorem the following notation will be useful

\[
M = \|A\|_\infty + \|R\|_\infty \|P\|_\infty, \quad b_\infty = \|Q\|_\infty
\]

\[
+ 2 \|A\|_\infty \|P\|_\infty + \|R\|_\infty \|P\|_\infty^2
\]

\[
b_1 = \|Q\|_1 + 2 \|A\|_2 \|P\|_2 + \|R\|_\infty \|P\|_2^2
\]

\[
l_\infty = \left\| \mathcal{L}_{A - R P, \Pi}^{-1} \right\|_\infty, \quad l_1 = \left\| \mathcal{L}_{A - R P, \Pi}^{-1} \right\|_1
\]

\[
N_\infty = l_2^2 \|R\|_\infty^8 b_\infty, \quad N_1 = l_2^2 \|R\|_\infty^8 b_1
\]

\[
\alpha_1 = \min_{\delta \in S} \frac{1}{2 \|H\|_1 \left( \|A_i\| + \|R_i\| \|K_i\| \right)}
\]

\[
\alpha_2 = \min_{\delta \in S} \frac{1}{2 \|H\|_1 \left( \|A_i\| + \|Q_i\| \|L_i\| \right)}
\]

\[
\delta_0 = \min (\alpha_1, \alpha_2),
\]

where \(K_i\) and \(L_i\) are defined in the statement of Lemma 8.

Theorem 9. Suppose that \((A, R, \Pi)\), is stabilizable and \((A, Q, \Pi)\), is detectable and let \(P\) be the unique stabilizing solution of (15). Moreover suppose that the perturbations in (20) are such that \(R + \Delta R, Q + \Delta Q \in \mathcal{H}_n\) and

\[
\left\| \Delta A_i \right\| \leq \delta \left\| A_i \right\|, \left\| \Delta R_i \right\| \leq \delta \left\| R_i \right\|, \\
\left\| \Delta Q_i \right\| \leq \delta \left\| Q_i \right\|, \Delta \Pi = 0
\]

where

\[
\delta < \min \left( \frac{1}{1 + 4 (l_2 M + N_2)} \delta_0 \right)
\]

then (20) has a unique stabilizing solution and

\[
\|V - P\|_* \leq \frac{1 - 2 l_2 M \delta - \beta}{2 l_2 \|R\|_* (1 + \delta)}
\]

where

\[
\beta = \sqrt{(1 - 2 l_2 M \delta)^2 - 4 N_2 \delta (1 + \delta)}
\]

and \(\delta \) is 1 or \(\infty\).

Proof. The fact that for \(\delta < \delta_0\) the perturbed equation (16) has a unique stabilizing solution follows from Lemma 8. Denote \(\Delta P = \)
The last inequality is equivalent to
\[ \delta^2 - 2 \delta + \frac{1}{2} \geq 0. \]
where \( \delta = \frac{1}{2} \left( L \left( M^2 - N \right) - N \right) \). Subtracting (15) from (16) we have
\[ \Delta P = \mathcal{L}_{\Delta R, R}^{-1} \left( -X \right), \]
where \( X = (X_1, ..., X_s) \)
\[ X_i = \Delta Q_i + \Delta A_i P_i + P_i \Delta A_i - P_i \Delta R_i P_i + \Delta P_i \left( R_i + \Delta R_i \right) \Delta P_i + \left( \Delta A_i - \Delta R_i P_i \right)' \Delta P_i + \Delta P_i \left( \Delta A_i - \Delta R_i P_i \right) \]
Using assumption (27) we obtain
\[ \left\| X_i \right\| \leq \delta \left\| Q_i \right\| + 2 \delta \left\| A_i \right\| \left\| P_i \right\| + \delta \left\| R_i \right\| \left\| P_i \right\| + \left( 1 + \delta \right) \left\| R_i \right\| \left\| \Delta P_i \right\| \]
\[ + 2 \delta \left( \left\| A_i \right\| + \left\| R_i \right\| \left\| P_i \right\| \right) \left\| \Delta P_i \right\| \]
and
\[ \left\| X \right\| \leq \delta \left\| Q \right\| + 2 \delta \left\| A \right\| \left\| P \right\| + \delta \left\| R \right\| \left\| P \right\| + \left( 1 + \delta \right) \left\| R \right\| \left\| \Delta P \right\| + 2 \delta \left( \left\| A \right\| + \left\| R \right\| \left\| P \right\| \right) \left\| \Delta P \right\| \]
Definition of a norm of a linear operator implies
\[ \left\| \Delta P \right\|_* \leq \left\| \mathcal{L}_{\Delta R, R}\left( \left\| \Delta P \right\| \right) \right\|_* \left\| X \right\|_* \]
where * is \( \infty \) or 1. Combining (32) with (30) and (32) with (31) leads after rearranging to
\[ l_* \left( 1 + \delta \right) \left\| R \right\|_\infty \left\| \Delta P \right\|_* = \left( 1 - 2 l_* M \delta \right) \left\| \Delta P \right\|_* + l_* b_* \delta \geq 0. \]
It is easy to check that the last inequality can be rewritten as
\[ l_* \left( 1 + \delta \right) \left\| R \right\|_\infty \times \left( \left\| \Delta P \right\|_* - \frac{1 - 2 l_* M \delta}{2 l_* \left( 1 + \delta \right) \left\| R \right\|_\infty} \right)^2 \]
\[ + l_* b_* \delta - \frac{(1 - 2 l_* M \delta)^2}{4 l_* \left( 1 + \delta \right) \left\| R \right\|_\infty} \geq 0. \] (33)
Now we show that under assumption (28)
\[ l_* b_* \delta - \frac{(1 - 2 l_* M \delta)^2}{4 l_* \left( 1 + \delta \right) \left\| R \right\|_\infty} < 0. \] (34)
The last inequality is equivalent to
\[ 4 \left( l_*^2 M^2 - N \right) \delta^2 - 4 \left( l_* M + N \right) \delta + 1 > 0. \]
If \( l_*^2 M^2 - N > 0 \), then by (28)
\[ 4 \left( l_*^2 M^2 - N \right) \delta^2 - 4 \left( l_* M + N \right) \delta + 1 > 0. \]
For \( l_*^2 M^2 - N < 0 \), consider the quadratic function
\[ h \left( \delta \right) = 4 \left( l_*^2 M^2 - N \right) \delta^2 - 4 \left( l_* M + N \right) \delta + 1. \]
Its value is 1 at \( \delta = 0 \), and it goes to \( -\infty \) when \( |\delta| \to \infty \), since \( l_*^2 M^2 - N < 0 \). Therefore \( h \) is strictly positive on any interval \( \left[ 0, \delta \right] \) such that \( h \left( \delta \right) > 0. \) But now,
\[ h \left( \delta \right) = 4 \left( l_*^2 M^2 - N \right) \delta^2 - 4 \left( l_* M + N \right) \delta + 1. \]
Hence, \( h \) is strictly positive on any interval \( \left[ 0, \frac{1}{1 + 4 \left( l_* M + N \right)} \right] \), and (34) holds. By (34) we conclude from (32) that either
\[ \left\| \Delta P \right\|_* \leq \frac{1 - 2 l_* M \delta - \beta_1}{2 l_* \left\| R \right\|_\infty \left( 1 + \delta \right)} \]
or
\[ \left\| \Delta P \right\|_* \geq \frac{1 - 2 l_* M \delta + \beta_2}{2 l_* \left\| R \right\|_\infty \left( 1 + \delta \right)}, \]
where \( \beta_1 = \sqrt{(1 - 2 l_* M \delta)^2 - 4 M \delta^2} \) and \( \beta_2 = \sqrt{(1 - 2 l_* M \delta)^2 - 4 M \delta^2} \). In Czornik (2000) it has been shown that the solution of (15) is a continuous function of its coefficients. Therefore \( \left\| \Delta P \right\|_* \to 0 \) when \( \delta \to 0 \), this is however in contradiction to (36), because the right hand side of the last inequality tends to a positive constant when \( \delta \to 0 \). Consequently only (35) is true.

The next theorem contains the main result of this paper and solves the following problem: Suppose that the exact value II and approximations \( A + \Delta A, R + \Delta R, Q + \Delta Q \) of the given \( A, R, Q \) are known, along with the exact solution \( V \) of (20). We would like to estimate the difference \( P - V \), where \( P \) is the solution of CRE (15) with true parameters. Such a situation is often met in practice when the parameters are known with a nonnegligible level of inaccuracy.

**Theorem 10.** Assume that exact value II and approximate values \( A + \Delta A^{(0)}, R + \Delta R^{(0)}, Q + \Delta Q^{(0)} \) of the parameters \( A, R, Q \) of (15) are known. Assume also that \( (A + \Delta A^{(0)}, R + \Delta R^{(0)}, II), \) is stabilizable and \( (A + \Delta A^{(0)}, Q + \Delta Q^{(0)}, II) \) is detectable and consequently (20) has a unique stabilizing solution \( V \). Finally, let \( M, b_*, l_*, \) and \( \delta_0 \) be the counterparts of \( M, b_* \),
The right hand side of (37) is denoted \( \langle 37 \rangle \) and put

\[
\frac{1}{1 + 4 (\lambda_0 + N_0)} \delta_0
\]

then (15) has a unique stabilizing solution \( P \) and

\[
\| V - P \|_\infty \leq \frac{1 - 2 \hat{L} \hat{M} \delta - \gamma}{2 \hat{L} \| R + \Delta R \|_\infty (1 + \delta)}, \quad (37)
\]

where \( \gamma = \sqrt{(1 - 2 \hat{L} \hat{M})^2 - 4 \hat{N} \hat{\delta} (1 + \delta)}. \)

5. NUMERICAL EXAMPLE

In this section a numerical example is presented. It illustrates the upper bound of Theorem 10 and the effect of the growing sensitivity of (15) when the coefficients \( (A, B) \) are close to unstabilizable.

**Example 1.** Consider matrices

\[
A_1 = \begin{bmatrix} 1.5 & 1 \\ 0 & 0.5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.5 & 0 \\ 1 & 1.5 \end{bmatrix},
\]

\[
R_1 = \begin{bmatrix} \varepsilon & 0 \\ 0 & 0 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},
\]

\[
Q_1 = Q_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

and put \( \pi_{11} = \pi_{22} = -1, \pi_{21} = \pi_{12} = 1. \) System \((A, Q, H)\), is detectable and \((A, R, H)\) is stabilizable for \( \varepsilon \neq 0 \) and is not stabilizable for \( \varepsilon = 0. \) We consider (15) with \( \varepsilon = 0.2 \) and (16) with \( \varepsilon = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, \) and \( 10^{-5}. \)

In the next tables (Table I and Table II) we present the norms \( \| V - P \|_1 \) and \( \| V - P \|_\infty \) and the upper bounds for them given by (37).

The right hand side of (37) is denoted \( ub_1 \) and \( ub_\infty \) for \( * = 1 \) and \( \infty, \) respectively.

**Table I: Norms \( \| V - P \|_1 \) and bounds**

<table>
<thead>
<tr>
<th>( \varepsilon )</th>
<th>( | V - P |_1 )</th>
<th>( ub_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>2.732583 × 10</td>
<td>3.231482 × 10</td>
</tr>
<tr>
<td>0.01</td>
<td>4.687537 × 10^2</td>
<td>5.796516 × 10^2</td>
</tr>
<tr>
<td>0.001</td>
<td>4.634255 × 10^3</td>
<td>3.526851 × 10^4</td>
</tr>
<tr>
<td>0.0001</td>
<td>4.549897 × 10^4</td>
<td>2.137133 × 10^6</td>
</tr>
<tr>
<td>0.00001</td>
<td>4.516519 × 10^5</td>
<td>5.524018 × 10^8</td>
</tr>
</tbody>
</table>

**Table II: Norms \( \| V - P \|_\infty \) and bounds**

<table>
<thead>
<tr>
<th>( \varepsilon )</th>
<th>( | V - P |_\infty )</th>
<th>( ub_\infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.922517 × 10</td>
<td>7.831416 × 10</td>
</tr>
<tr>
<td>0.01</td>
<td>3.223297 × 10^2</td>
<td>5.333214 × 10^2</td>
</tr>
<tr>
<td>0.001</td>
<td>3.128348 × 10^3</td>
<td>8.13339 × 10^4</td>
</tr>
<tr>
<td>0.0001</td>
<td>3.046341 × 10^4</td>
<td>9.126314 × 10^5</td>
</tr>
<tr>
<td>0.00001</td>
<td>3.015227 × 10^5</td>
<td>5.114416 × 10^7</td>
</tr>
</tbody>
</table>

6. CONCLUSION

An analysis of the sensitivity of the solution to the continuous coupled Riccati equation has been presented. Different measures of the sensitivity are considered and it is shown that they can be assessed through the spectral norm of the associated Lyapunov operator. It has appeared that the crucial property for the sensitivity is the stabilizability of the system. In the deterministic case presented results reduce to those from (Gahinet and Laub, 1990). Also a numerical example is presented to show the influence of the stabilizability of the system on the sensitivity of the Riccati equation as well as quality of obtained bounds.

REFERENCES


