A NONLINEAR SWITCHING CONTROL METHOD FOR MAGNETIC BEARING SYSTEMS
– MINIMIZING THE POWER CONSUMPTION –

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Abstract: In this paper, a nonlinear switching control method is proposed for a magnetic bearing system which has a strong nonlinearity in the magnetic actuator. The main purpose is to use minimal electrical power to achieve good response. By focusing on the structure of this system, it is found that the rotor part of the system can be stabilized by a virtual magnetic force first, which is given by a linear feedback. Then, the minimal currents necessary for the realization of this magnetic force is determined. As a result, it is discovered that the minimal currents are given by a set of switching rules. After that, the electromagnetic part is stabilized by using either backstepping method or high-gain feedback. The control input obtained is simpler compared with other related approaches, and the response of the rotor can be assigned directly. No bias current is used.

Finally, numerical simulations are implemented and the numerical simulations show that this method is extremely effective.

Keywords: Magnetic Bearing systems, Virtual Input, Switching Control, Power Consumption, Magnetic Force

1. INTRODUCTION

In magnetic bearing systems the magnetic force has a strong nonlinearity, which is proportional to the square of electric current and inversely proportional to the square of distance between the magnetic bearing and the rotor. For this reason, in conventional magnetic control systems using linear control approaches large current biases have to be applied to a pair of electromagnets in order to guarantee that the magnetic force acting on the rotor can be approximated as a linear function of the currents. This current bias does not contribute to control of magnetic bearing, thus is a waste of power.

Based on this observation, Ariga and Nonami (1999), Ariga et al. (2000) and Ariga et al. (2001) proposed a nonlinear control method based on backstepping to get rid of the biases of current. These researches are important and valuable for magnetic bearing systems. However, there are several drawbacks need to be addressed. First of all, since the backstepping was applied naively in (Ariga and Nonami, 1999; Ariga et al., 2000; Ariga et al., 2001), it is not clear whether the resulting control minimizes the electrical power in any sense, although no bias of current is used. Secondly, since backstepping is used recursively for several times the resulting control law is pretty complicated and its relationship with time response is not clear. As a result, control parameters have to be tuned by trial and error. Further, theoretically there are also some problems with the proof of stability.

In this paper, attention is focused on the dynamics property of this system, i.e. the magnetic bearing system is composed of a rotor and an actuator of electromagnets. The rotor is controlled by the magnetic
2. MODEL OF MAGNETIC BEARING SYSTEM

As the simplest case, let us consider the one-degree-of-freedom magnetic bearing system shown in Fig. 1. In this figure, \( x \) denotes the displacement of the rotor axle from the center, \( X_0 \) the gap between bearing and rotor at the equilibrium state, \( i_1, i_2 \) are the currents flowing through electromagnets 1 and 2 respectively. The voltage inputs applied to each electromagnet circuit are \( u_1 \) and \( u_2 \).

To express the dynamics in state space form, the state variables are chosen as

\[
x_1 = x, 
\dot{x}_1 = i_1, 
\dot{x}_2 = i_2
\]

Then, the state space model is described by the following equations

\[
\dot{x}_1 = x_2
\]

\[
\dot{x}_2 = \frac{1}{M} f
\]

\[
\dot{\xi}_1 = \frac{1}{L_1} \left( -R \xi_1 - \frac{\partial L_1}{\partial x_1} \xi_1 x_2 + u_1 \right)
\]

\[
\dot{\xi}_2 = \frac{1}{L_2} \left( -R \xi_2 - \frac{\partial L_2}{\partial x_1} \xi_1 x_2 + u_2 \right)
\]

In which the electromagnetic force \( f \) and the inductances \( L_1, L_2 \) are given by

\[
f = k \left\{ \frac{\xi_2^2}{(X_0 - x_1)^2} - \frac{\xi_1^2}{(X_0 + x_1)^2} \right\}
\]

\[
L_1 = \frac{2k}{X_0 - x_1}, \quad L_2 = \frac{2k}{X_0 + x_1}.
\]

In the above equations, \( M \) is the mass of rotor and \( R \) the resistance of each circuit. Moreover, \( k = \frac{N S}{\pi \mu_0} \) in which \( \mu_0 \) is the ratio of magnetic conductivity in vacuum, \( N \) the number of windings of wire, \( S \) the slice area of the iron core. Eqs. (2), (3) describe the dynamics of electric circuits of the two electromagnets. These model can be easily found in any standard textbooks on magnetic dynamics, such as (JSME, 1995; IEEJ, 1993).

![Fig. 1. Model of one-degree-of-freedom magnetic bearing system](image)

3. CONTROL DESIGN

As is clear from the state equation, if the magnetic force \( f \) is regarded as a virtual control input, then the dynamics of the rotor subsystem becomes linear. So linear control can be applied to stabilize this subsystem. After that, voltage input needs to be designed so as to realize the magnetic force designed in the 1st step. That is, the design process can be decomposed into 2 steps. They are described in the following sections respectively.

3.1 Design of Magnetic Force

Let us assume that the magnetic force \( f \) can be manipulated directly and consider the control of rotor by this virtual input first. Let \( x = [x_1, x_2]^T \) denote the state vector, then the state equation of rotor can be written as

\[
\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{f}{M}
\]

Obviously, this system is controllable, thus can be stabilized by linear state feedback. Now, define a function as

\[
s(x) := -\alpha x_1 - \beta x_2
\]

and set the virtual input \( f \) as

\[
f^* = Ms(x)
\]

Since the characteristic polynomial of the closed loop subsystem equals

\[
x^2 + \beta s + \alpha,
\]
the transient response of rotor can be assigned by choosing suitable $\alpha, \beta$. In the selection of $\alpha, \beta$, we equate (11) to
\begin{equation}
\dot{\xi}^2 + 2\zeta\omega_n\dot{\xi} + \omega_n^2 \xi = 0
\end{equation}
with damping ratio $\zeta$ and natural frequency $\omega_n$, and determine $\alpha, \beta$ from $\zeta, \omega_n$.

Next, let us find the corresponding electric currents. It is clear from Eq. (6) that the currents $\dot{\xi}_1, \dot{\xi}_2$ cannot be determined uniquely. As the total consumption power is proportional to $\dot{\xi}_1^2 + \dot{\xi}_2^2$, let us find $\dot{\xi}_1, \dot{\xi}_2$ that minimize the power index
\[ J = \min\{\xi_1^2 + \xi_2^2\} \]
subject to the constraint $f^* = Ms(x)$. It is easy to see that the optimal solution must satisfy
\begin{align}
\dot{\xi}_1 &= 0, \quad s(x)/a = -\frac{\xi_2^2}{(X_0 + x_1)^2} \quad (s(x) < 0) \\
\dot{\xi}_1 &= \dot{\xi}_2 = 0 \quad (s(x) = 0) \\
\dot{\xi}_2 &= 0, \quad s(x)/a = \frac{\xi_2^2}{(X_0 - x_1)^2} \quad (s(x) > 0)
\end{align}
where
\[ a = \frac{k}{M} \]

That is, this control uses only the current $\dot{\xi}_2$ of magnet 2 when $s(x) < 0$ and the current $\dot{\xi}_1$ of magnet 1 when $s(x) > 0$. From these equations, the required currents $\xi_1^*, \xi_2^*$ are obtained as:

1. when $s(x) < 0$
   \[
   \begin{cases}
   \dot{\xi}_1^* = 0 \\
   \dot{\xi}_2^* = (X_0 + x_1)\sqrt{-s(x)/a}
   \end{cases}
   \]
2. when $s(x) = 0$
   \[
   \begin{cases}
   \dot{\xi}_1^* = 0 \\
   \dot{\xi}_2^* = 0
   \end{cases}
   \]
3. when $s(x) > 0$
   \[
   \begin{cases}
   \dot{\xi}_1^* = (X_0 - x_1)\sqrt{s(x)/a} \\
   \dot{\xi}_2^* = 0.
   \end{cases}
   \]

Obviously when $x_1 = x_2 = 0$, we have $\dot{\xi}_1^* = \xi_2^* = 0$. This control law is a switching control law and is physically extremely natural. The function $s(x)$ which is proportional to the magnetic force $f^*$ acts as the switching surface.

It is worth noting that output feedback can also be used to construct $f^*$ if necessary.

3.2 Design of Voltage Inputs

In the design of voltage inputs so as to stabilize the whole system, two kinds of methods can be used. One is backstepping, another is high-gain feedback. The details are given separately.

3.2.1 Backstepping In order to do backstepping (Krstić et al., 1995), a Lyapunov function needs to be found for the closed loop subsystem of rotor. As this system is linear, one can try the following quadratic positive definite function
\[ V_1(x) = \frac{1}{2}x^T \begin{bmatrix} \alpha & 0 \\ 0 & 1 \end{bmatrix} x. \]

The derivative $\dot{V}_1$ along the trajectory is found to be
\[ \dot{V}_1(x) = -\beta \dot{x}_2^2 \]
via simple calculation. Hence, $V_1$ is non-positive. Further, $V_1 \equiv 0$ yields $x_2 \equiv 0$ which in turn gives $x_1 \equiv 0$. In view of LaSalle’s invariance principle (Khalil, 1996), this $V_1$ function can be used as Lyapunov function.

Next, the voltage inputs are designed to stabilize the whole system. As in standard backstepping, the error between real currents $\dot{\xi}_1, \dot{\xi}_2$ and required currents $\dot{\xi}_1^*, \dot{\xi}_2^*$ are defined as
\[ e_1 = \dot{\xi}_1 - \dot{\xi}_1^* \]
\[ e_2 = \dot{\xi}_2 - \dot{\xi}_2^*. \]

Let the candidate of control Lyapunov function for the whole system be
\[ V = V_1 + \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2. \]

The problem now is to find $u_1, u_2$ that make $V$ non-positive. In fact, inputs $u_1, u_2$ are determined as such that
\[ V = -\beta \dot{x}_2^2 - c_1 e_1^2 - c_2 e_2^2. \]

Here $c_1, c_2$ are tuning parameters and are both positive. The global asymptotic stability is guaranteed by LaSalle’s invariance principle.

The process of derivation is straightforward, so the detail is omitted. The control law derived is:

1. when $s(x) < 0$
   \[ u_1 = -L_1 c_1 e_1 + R_1 \frac{\partial L_1}{\partial x_1} \dot{\xi}_1 x_2 - a L_1 \frac{x_2 \dot{\xi}_1}{(X_0 - x_1)^2} + L_2 x_2 \sqrt{-s(x)} \left\{ \frac{\alpha k x_2}{(X_0 - x_1)^2} \left( \dot{\xi}_2 + \dot{\xi}_1 x_2 \right) \right\} \]
   \[ + \sqrt{-s(x)/a} \left\{ \frac{\beta k}{(X_0 - x_1)^2} \left( \dot{\xi}_1^2 \right) \right\} \]

2. when $s(x) = 0$
   \[ u_1 = -L_1 c_1 x_1 + R_1 \frac{\partial L_1}{\partial x_1} \dot{\xi}_1 x_2 - a L_1 \frac{x_2 \dot{\xi}_1}{(X_0 - x_1)^2} \]

3. when $s(x) > 0$
   \[ u_1 = -L_1 c_1 e_1 + R_1 \frac{\partial L_1}{\partial x_1} \dot{\xi}_1 x_2 - a L_1 \frac{x_2 \dot{\xi}_1}{(X_0 - x_1)^2} \]
\[ u_2 = -L_2 c_2 e_2 + R \xi_2 + \frac{\partial L_2}{\partial x_1} \xi_2 x_2 + aL_2 \frac{x_2 \xi_2}{(X_0 + x_1)^2} \]

(3) when \( s(x) > 0 \)

\[ u_1 = -L_1 c_1 e_1 + R \xi_1 + \frac{\partial L_1}{\partial x_1} \xi_1 x_2 - aL_1 \frac{(\xi_1 + \xi_2) x_2}{(X_0 - x_1)^2} \]

\[ -L_1 x_2 \sqrt{s(x)} - \frac{\alpha k x_2}{\sqrt{s(x)/a}} \left\{ \frac{\xi_1^2}{(X_0 - x_1)^2} - \frac{\xi_2^2}{(X_0 + x_1)^2} \right\} \]

\[ u_2 = -L_2 c_2 e_2 + R \xi_2 + \frac{\partial L_2}{\partial x_1} \xi_2 x_2 + aL_2 \frac{x_2 \xi_2}{(X_0 + x_1)^2} \].

3.2.2. High-Gain Feedback

As an easier and simpler method, high gain feedback can be used in the circuit loop to ensure that the real currents \( \xi_1 \), \( \xi_2 \) track the required currents \( \xi_1^* \), \( \xi_2^* \) almost instantly because the dynamics of electric circuit can be made much faster than the mechanical part.

The control law is

\[ u_1 = -c_1(\xi_1 - \xi_1^*) \]

\[ u_2 = -c_2(\xi_2 - \xi_2^*). \] (18)

One of the nice properties of high gain control is that it is robust to the parameter uncertainty of the circuits. In fact, one does not even need to know the parameters of the circuits. Thus the gain can be tuned on line easily.

4. NUMERICAL SIMULATIONS

Simulations are performed using the designed control laws. The results are explained separately. In both cases,

\[ \omega_n = 75[\text{rad/s}], \quad \zeta = 1.3 \]

are used in the determination of the desired electromagnetic force \( f^* \). This corresponds to a settling time of about 0.04[s] for the rotor subsystem.

Parameters of the plant are provided in the following table.

<table>
<thead>
<tr>
<th>Table 1. Table of parameters</th>
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<tbody>
<tr>
<td>( M )</td>
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<tr>
<td>( X_0 )</td>
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<td>( \mu_0 )</td>
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<td>( N )</td>
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<td>( S )</td>
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<td>( R )</td>
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In simulations, the initial value of displacement \( x_1 \) is set as \( x_1(0) = -2 \times 10^{-4}[\text{m}] \).

4.1 Backstepping Case

In this case, it is confirmed that the 5th term of \( u_1 \) for \( s(x) > 0 \) and the 5th term of \( u_2 \) for \( s(x) < 0 \) diverge at the instant of switching of the input. For this reason, they are replaced by the following formulae. As in both cases the formulae are similar, here only the new 5th term for \( u_1 \) is shown

\[ \frac{\alpha k}{\sqrt{as(x)}} \left( \frac{s(x)}{s(x) + ae} \right)^2. \] (19)

In simulation \( \varepsilon = 15 \) is used. Instead of asymptotic stability, boundedness is guaranteed by this change.

The results are shown in Figs. 2-5. Here, controller parameters are selected as \( c_1 = c_2 = 90 \).

The displacement and velocity of rotor settle in about 0.08[s], the currents and voltages of electromagnets settle to zero in about 0.2[s]. Further, as can be seen from Figs. 4 and 5 the control switches and only one of the two magnets is active. The switching works pretty well because it is determined based on the change of direction of magnetic force necessary for the control of rotor.

**Fig. 2. Response of displacement \( x_1 \)**

**Fig. 3. Response of velocity \( x_2 \)**
4.2 High-Gain Feedback Case

The responses of high-gain feedback case are shown in Figs. 6-9. The gains of controller used in the simulation are

\[ c_1 = c_2 = 12. \]

As is seen from these results, the simple high-gain feedback also works quite well. The response performance is comparable to that of the backstepping case. Although the maximum amplitudes of currents and voltages are larger.

Since high-gain feedback is easy to implement, this method may be preferred in practice.

5. CONCLUSION

In this paper, a nonlinear switching control method for one-degree-of-freedom magnetic bearing systems has been proposed, in which the consumption power is minimized in certain sense. Numerical experiments show the proposed control is very effective and no bias current is needed.

This approach can be extended to systems with general static actuator nonlinearity easily. It also works when the subsystem from the output of actuator to the output of plant is nonlinear if this subsystem can be stabilized. These generalizations are under way.

In the application aspect, it is possible to extended the method to the control of multi-degree-of-freedom
magnetic bearing systems, magnetic levitation systems and so on.

Further research is needed to find how to get over the singularity of the input resulting from backstepping.

6. REFERENCES


