A TABULAR ALGORITHM FOR TESTING THE ROBUST HURWITZ STABILITY OF CONVEX COMBINATIONS OF COMPLEX POLYNOMIALS

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Abstract: In this paper we present a tabular algorithm for testing the Hurwitz property of a segment of complex polynomials. The algorithm consists in constructing a parametric Routh-like array with polynomial entries and generating Sturm sequences for checking the absence of zeros of two real polynomials in the interval (0, 1). The presented algorithm is easy to implement. Moreover, it accomplishes the test in a finite number of arithmetic operations because it does not invoke any numerical root-finding procedure. Copyright ©2002 IFAC

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1. INTRODUCTION

For a linear system subject to parametric uncertainties, the robust $D$ stability analysis is to determine if all the roots of its characteristic polynomial remain in a simply-connected domain $D$ in the complex plane for all parameter variations in given intervals. Since the appearance of seminal Kharitonov’s theorem of robust Hurwitz stability of interval polynomials (Kharitonov, 1978) the robust stability analysis of linear systems has become an active subject of research and a flurry of results have been published. Kharitonov (1978) showed that for a polynomial whose coefficients take values independently in their respective intervals, the roots of the whole polynomial family remain in the left-half plane if and only if the roots of four specially constructed vertex polynomials are all in the left-half plane. This extreme-point result for robust stability of interval polynomials has motivated several researchers to present more general extreme-point stability tests for ensuring the robust $D$-stability of a larger class of polynomial families, such as polytopic polynomials, diamond polynomials, and ellipsoidal polynomials. Among others, the Edge theorem proposed by (Bartlett et al., 1988) is most elegant. It states that if the coefficients of the polynomial...
Checking the absence of zeros of the polynomial \( p(s; q) = a_0(q) + a_1(q)s + \cdots + a_{n-1}(q)s^{n-1} + a_n(q)s^n \), \( q = (q_0, q_1, \ldots, q_{m-1}) \) depend affine linearly on the uncertain parameters \( q_0, q_1, \ldots, q_{m-1} \), then the polytopic polynomial family \( p(s; Q) = \{ p(s; q) : q \in Q \} \) is robust Hurwitz stable if and only if all its exposed edge polynomials are Hurwitz stable. Here, an exposed edge polynomial corresponds to a convex combination of two polynomials taking adjacent vertex parameter values, which can be in general represented by

\[
A(s; \lambda) = (1 - \lambda)p_0(s) + \lambda p_1(s), \quad \lambda \in [0, 1]
\]  

where \( p_0(s) \) and \( p_1(s) \) are vertex polynomials of the same degree.

As the test for the robust stability of convex combinations of polynomials plays a crucial role in the robust stability analysis of linear systems, several authors have made efforts towards developing efficient test methods. Bialas (1985) showed that all polynomials in the family \( A(s; [0,1]) \) have their zeros in the left-half plane if and only if the matrix \( H_0H_1^{-1} \) has no real eigenvalues in \((-\infty,0)\), where \( H_0 \) and \( H_1 \) are the Hurwitz matrices associated with the real polynomials \( p_0(s) \) and \( p_1(s) \). Based on the zero set concept (Zeheb and Walach, 1981) or the classical zero exclusion principle (Barmish, 1994), methods of checking the absence of zeros on a polynomial on the stability boundary curve has been developed (Blondel, 1996; Bollepalli and Pujara, 1994; Ozturk, 1992; Zeheb, 1989) for the robust \( D \)-stability analysis of convex combinations of real and complex polynomials. These methods typically involve the “nonfinite” root finding procedure. Bose (1989) presented a unified approach of using resultant theory to test the robust Schur and Hurwitz stability of convex combinations of real and complex polynomials. As shown by Bose (1989), the test involves determining the determinant \( d(\lambda) \) of a symbolic matrix of the form \( \mathbf{A} + \lambda \mathbf{B} \) and checking the absence of zeros of the polynomial \( d(\lambda) \), which is of degree \( n - 1 \) for real coefficient case and degree \( 2n \) for complex coefficient case.

The purpose of this paper is to present a tabular algorithm for testing, along with the Sturm theorem, the robust Hurwitz stability of convex combinations of complex polynomials. The algorithm is a generalization of the ones (Hwang and Yang, 2001; Jeltsch, 1979) for generating fraction-free Routh array associated with a real polynomial. Since the algorithm is developed on the basis of using the Euclidean algorithm to extract the greatest common divisor of two complex polynomials associated with the parametric polynomial \( A(s; \lambda) \), it can be viewed as an algorithmic implementation of the resultant approach given by Bose (1989). The advantage of the presented algorithm is that it accomplishes the test in a finite number of arithmetic operations without having to invoke any “nonfinite” numerical root-finding procedure.

2. MAIN RESULTS

Consider an \( n \)-th degree complex-coefficient polynomial given by

\[
A(s) = \sum_{k=0}^{n} (a_k + jb_k)s^k, \quad j = \sqrt{-1}
\]

It can be decomposed as

\[
A(s) = A_e(s) + A_o(s)
\]

where

\[
A_e(s) = \frac{1}{2} [A(s) + A^*(s)] = a_0 + j b_1s + a_2s^2 + j b_3s^3 + \cdots + \begin{cases} a_ns^n, & \text{if } n \text{ is even} \\ j b_ns^n, & \text{if } n \text{ is odd} \end{cases}
\]

\[
A_o(s) = \frac{1}{2} [A(s) - A^*(s)] = j b_0 + a_1s + j b_2s^2 + a_3s^3 + \cdots + \begin{cases} j b_ns^n, & \text{if } n \text{ is even} \\ a_ns^n, & \text{if } n \text{ is odd} \end{cases}
\]

and \( A^*(s) \) denotes the conjugate reciprocal polynomial of \( A(s) \), i.e.,

\[
A^*(s) := \overline{A(-\bar{s})}
\]

We note that \( \bar{s} \) denotes the complex conjugate of \( s \), and that \( A_e(s) \) is even and \( A_o(s) \) is odd, i.e.,

\[
A_e^*(s) = A_e(s) \quad \text{and} \quad A_o^*(s) = -A_o(s)
\]

The following lemma relates the Hurwitz polynomial \( A(s) \) and the zeros of \( A_e(s) \) and \( A_o(s) \).

Lemma 1 (Bose, 1989): The complex-coefficient polynomial \( A(s) \) given in (3) is Hurwitz (all its zeros are in LHP) if and only if \( a_na_{n-1} + b_nb_{n-2} > 0 \) and the zeros of \( A_e(s) \) and \( A_o(s) \) are simple and interlace on the imaginary axis.

Now, let us consider the family of complex polynomials

\[
A(s; \lambda) = (1 - \lambda)p_0(s) + \lambda p_1(s), \quad \lambda \in [0, 1]
\]

where \( p_0(s) \) and \( p_1(s) \) are both \( n \)-th degree complex polynomials given by

\[
p_0(s) = \sum_{k=0}^{n} (p_{0,0} s^k + j p_{0,1} s^k) \]

\[
p_1(s) = \sum_{k=0}^{n} (p_{1,0} s^k + j p_{1,1} s^k)
\]

and \( p_{i,0} \) and \( p_{i,1} \) for \( i = 0, 1 \) and \( k = 0, 1, \cdots, n \), are real. Writing the parametric polynomial \( A(s; \lambda) \) in the form

\[
A(s; \lambda) = \sum_{k=0}^{n} (a_k(\lambda) + jb_k(\lambda)) s^k
\]
we have

\[ a_0(\lambda) = (1 - \lambda)p_{0,r,k} + \lambda p_{1,r,k} \]  \hspace{1cm} (11a)
\[ b_0(\lambda) = (1 - \lambda)p_{0,r,k} + \lambda p_{1,r,k} \]  \hspace{1cm} (11b)

Also, like the decomposition given in (3)-(6), \( A(s; \lambda) \) can be decomposed as

\[ A(s; \lambda) = A_v(s; \lambda) + A_\nu(s; \lambda) \]  \hspace{1cm} (12)

For testing the Hurwitz property of the convex combinations of two complex polynomials, Bose (1989) established the following theorem.

**Theorem 1 (Bose, 1989):** \( A(s; \lambda) \) is Hurwitz stable if and only if

(i) Both \( p_0(s) \) and \( p_1(s) \) are Hurwitz.

(ii) \( a_0(\lambda)a_{n-1}(\lambda) + b_0(\lambda)b_{n-1}(\lambda) > 0 \) for all \( \lambda \in (0, 1) \).

(iii) The resultant for \( A_v(s; \lambda) \) and \( A_\nu(s; \lambda) \) has no zeros in the open interval \( \lambda \in (0, 1) \).

It is noted that condition (iii) in Theorem 1 is equivalent to the fact that \( A_v(s; \lambda) \) and \( A_\nu(s; \lambda) \) have no common factors for all \( \lambda \in (0, 1) \). Based on this equivalence we can construct a fraction-free Routh-like array as shown in Table 1 for testing the robust Hurwitz stability of the polynomial family \( A(s; \{0, 1\}) \). The construction of the table is related to the extraction of the greatest common divisor (gcd) of the even and odd polynomials \( A_v(s; \lambda) \) and \( A_\nu(s; \lambda) \) by the Euclidean algorithm. Letting

\[ A_0(s; \lambda) = A_v(s; \lambda) \]  \hspace{1cm} (13a)
\[ B_0(s; \lambda) = \frac{1}{2}A_\nu(s; \lambda) \]  \hspace{1cm} (13b)

we can generate two descending-degree polynomial sequences \( \{ A_i(s; \lambda) \}_{i=0}^{n-1} \) and \( \{ B_i(s; \lambda) \}_{i=0}^{n-1} \) by the following long division operations:

\[ \frac{A_i(s; \lambda)}{B_i(s; \lambda)} = \frac{A_{i+1}(s; \lambda)}{B_{i+1}(s; \lambda)} + \frac{j \omega_{i+1}(\lambda)}{B_{i+1}(s; \lambda)} = \frac{A_{i+1}(s; \lambda)}{B_{i+1}(s; \lambda)} \]  \hspace{1cm} (14a)
\[ \frac{B_i(s; \lambda)}{A_{i+1}(s; \lambda)} = \frac{B_{i+1}(s; \lambda)}{A_{i+1}(s; \lambda)} + \frac{j \omega_{i+1}(\lambda)}{A_{i+1}(s; \lambda)} = \frac{B_{i+1}(s; \lambda)}{A_{i+1}(s; \lambda)} \]  \hspace{1cm} (14b)

where \( i = 0, 1, \ldots, n-1 \), and

\[ d_i(\lambda) = \begin{cases} 1, & \text{if } l < 2 \\ b_{l-2,0}(\lambda), & \text{if } l \geq 2 \end{cases} \]  \hspace{1cm} (14c)
\[ e_i(\lambda) = \begin{cases} 1, & \text{if } l < 2 \\ a_{l-2,0}(\lambda), & \text{if } l \geq 2 \end{cases} \]  \hspace{1cm} (14d)

It follows from (14) that both \( A_i(s; \lambda) \) and \( B_i(s; \lambda) \) are \((n-j)\)th-degree complex polynomials and satisfy the following relations:

\[ A_{i+1}(s; \lambda) = \frac{1}{j \omega_{i+1}(\lambda)} (B_{i+1}(s; \lambda)A_i(s; \lambda)) \]  \hspace{1cm} (15a)
\[ B_{i+1}(s; \lambda) = \frac{1}{j \omega_{i+1}(\lambda)} (A_{i+1}(s; \lambda)B_i(s; \lambda)) \]  \hspace{1cm} (15b)

Moreover, it follows from (14) that if there exists a \( \lambda^* \in [0, 1] \) such that \( A_i(s; \lambda^*) = 0 \) (resp. \( B_i(s; \lambda^*) = 0 \)) and \( B_{i-1}(s; \lambda^*) \neq 0 \) (resp. \( A_{i-1}(s; \lambda^*) \neq 0 \)) then the polynomials \( A_0(s; \lambda) \) and \( B_0(s; \lambda) \) have the greatest common factor \( B_{i-1}(s; \lambda^*) \) (resp. \( A_{i-1}(s; \lambda^*) \)).

Let us write

\[ A_i(s; \lambda) = a_i(\lambda) + a_{i,2}(\lambda)s + a_{i,3}(\lambda)s^2 + ja_{i,3}(\lambda)s^3 \]  \hspace{1cm} + ... + \left\{ a_{i,n-i}(\lambda)s^{n-i}, \text{ if } n-i \text{ is even} \right\} \hspace{1cm} (16a)
\[ B_i(s; \lambda) = b_i(\lambda) + b_{i,2}(\lambda)s + b_{i,3}(\lambda)s^2 + jb_{i,3}(\lambda)s^3 \]  \hspace{1cm} + ... + \left\{ b_{i,n-i}(\lambda)s^{n-i}, \text{ if } n-i \text{ is even} \right\} \hspace{1cm} (16b)

Then substituting the above two expressions in (15) and equating the coefficients of like powers, we obtain the following recursive formulas for computing the coefficients \( a_{i+k,0}(\lambda) \) and \( b_{i+k,0}(\lambda) \):

\[ a_{i+1,k}(\lambda) = \begin{cases} (-1)^k \frac{d_{i+k,0}(\lambda)}{d_{i+1,0}(\lambda)}, & \text{if } \lambda \neq 0 \\ -a_{0,0}(\lambda)b_{i+1,1}(\lambda), & \text{if } \lambda = 0 \end{cases} \]  \hspace{1cm} (17a)
\[ b_{i+1,k}(\lambda) = \begin{cases} (-1)^k \frac{e_{i+k,0}(\lambda)}{e_{i+1,0}(\lambda)}, & \text{if } \lambda \neq 0 \\ -b_{0,0}(\lambda)a_{i+1,1}(\lambda), & \text{if } \lambda = 0 \end{cases} \]  \hspace{1cm} (17b)

where \( i = 0, \ldots, n-1; k = 0, \ldots, n-i \), and

\[ e_i(\lambda) = \begin{cases} 1, & \text{if } l < 2 \\ a_{l-1,0}(\lambda), & \text{if } l \geq 2 \end{cases} \]  \hspace{1cm} (17d)

In constructing Table 1 for the parametric polynomial \( A(s; \lambda) \) by (17), the first two rows are filled by

\[ a_{0,k}(\lambda) = \begin{cases} a_k(\lambda), & \text{if } k \text{ is even} \\ b_k(\lambda), & \text{if } k \text{ is odd} \end{cases} \]  \hspace{1cm} (18a)
\[ b_{0,k}(\lambda) = \begin{cases} b_k(\lambda), & \text{if } k \text{ is even} \\ -a_k(\lambda), & \text{if } k \text{ is odd} \end{cases} \]  \hspace{1cm} (18b)

which are first-degree real-coefficient polynomials in \( \lambda \). For convenience, we call the rows with entries \( a_{i,k} \) and \( b_{i,k} \) the \( i \)th A-row and \( i \)th B-row, respectively. With this setting, we have the following lemma which can be easily proved with induction method:

**Lemma 2:** The entries of the Routh-like array of the form given by Table 1 constructed by (17) for the parametric polynomial \( A(s; \lambda) \) are polynomials in \( \lambda \).

Let the degrees of the polynomials \( a_i(s; \lambda) \) and \( b_i(s; \lambda) \) be denoted by \( n_{a,i} \) and \( n_{b,i} \), respectively. We have
where \( n_{a,0} = n_{b,0} = 1 \) and it is easy to verify that for \( i = 1, 2, \ldots, n \),

\[
\begin{align*}
n_{a,i} &= 2i \\
n_{b,i} &= 2i + 1
\end{align*}
\] (19a, 19b)

Two singular cases may occur in constructing the polynomial array in Table 1. The first singular case occurs when each entry of a row in the array vanishes, i.e., \( a_{i,k}(\lambda) = 0 \) or \( b_{i,k}(\lambda) = 0 \), for \( 0 \leq k \leq n - i \). As mentioned above this means that \( A_0(s; \lambda) \) and \( B_0(s; \lambda) \) have common factors and, hence, from (13) and Theorem 1, \( A(s; \lambda) \) is not Hurwitz stable. The second singular case occurs when some of the leading entries of a row vanish, i.e., for \( 0 \leq k < n - i \) and \( k < k' \leq n - i \), \( a_{i,k}(\lambda) = 0 \) and \( a_{i,k'}(\lambda) \neq 0 \) or \( b_{i,k}(\lambda) = 0 \) and \( b_{i,k'}(\lambda) \neq 0 \). In this case, following the method of dealing with the singular case of Routh's algorithm for the Hurwitz stability test of a real polynomial (Barnett and Šljšak, 1977), we can first multiply \( A(s; \lambda) \) by \((s-c)\), where \( c \) is a complex constant lying on the left-hand side of the complex plane and can always be chosen so that the elements of the first column of Table 1 do not vanish, and then construct the polynomial array in Table 1 associated with the new parametric polynomial \((s-c)A(s; \lambda)\). In the sequel we consider only the regular case, i.e., all the elements of the first column of Table 1 do not vanish.

When the fraction-free Routh-like array for the parametric polynomial \( A(s; \lambda) \) described by (8) is constructed, the robust Hurwitz stability of the segment polynomial family \( A(s; [0, 1]) \) can be checked with the following theorem.

**Theorem 2:** The segment polynomial family \( A(s; \lambda) \) described by (8) is robust Hurwitz if and only if the following conditions hold, provided that for \( i = 0, 1, \ldots, n, a_{i,0}(\lambda) \) and \( b_{i,0}(\lambda) \) in Table 1 do not vanish

(i) Both the end-point polynomials \( p_0(s) \) and \( p_1(s) \) are Hurwitz.

(ii) \( a_{n,0}(\lambda) b_{n-1}(\lambda) + b_{n,0}(\lambda) b_{n-1}(\lambda) > 0 \) for all \( \lambda \in (0, 1) \).

(iii) The array element \( a_{n,0}(\lambda) \neq 0 \) for all \( \lambda \in (0, 1) \).

Condition (i) is obvious and condition (ii) follows from Theorem 1. For proving condition (iii), we need the following lemma which can be easily proved by simple manipulation of determinant:

**Lemma 3:** Provided that for \( i = 0, 1, \ldots, n, a_{i,0}(\lambda) \) and \( b_{i,0}(\lambda) \) in Table 1 do not vanish, then

\[
a_{n,0}(\lambda) = j^{-n^2} \det(H) \tag{20}
\]

where

\[
H = \begin{bmatrix}
    b_{0,0}(\lambda) & j b_{0,1}(\lambda) & b_{0,2}(\lambda) & j b_{0,3}(\lambda) & \cdots \\
    a_{0,0}(\lambda) & a_{0,2}(\lambda) & a_{0,3}(\lambda) & \cdots \\
    b_{0,0}(\lambda) & j b_{0,1}(\lambda) & b_{0,2}(\lambda) & j b_{0,3}(\lambda) & \cdots \\
    a_{0,0}(\lambda) & a_{0,2}(\lambda) & a_{0,3}(\lambda) & \cdots \\
    \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots
\end{bmatrix}_{(2n \times 2n)}
\]

and for \( k > n, a_{k,0}(\lambda) = b_{k,0}(\lambda) = 0 \).

**Proof of condition (iii) in Theorem 2:** We prove condition (iii) by showing that \( a_{n,0}(\lambda) \neq 0, \lambda \in (0, 1) \) if and only if \( A_0(s; \lambda) \) and \( B_0(s; \lambda) \) defined in (13) have no common factors in the open interval \( \lambda \in (0, 1) \). This is accomplished by proving that \( a_{n,0}(\lambda) \) is equal to the resultant for \( A_0(s; \lambda) \) and \( B_0(s; \lambda) \) up to a constant.

We first consider the case of even \( n \). Using the relations in (18) to represent the coefficients of \( A_0(s; \lambda) \) and \( B_0(s; \lambda) \), the resultant matrix \( R(A_0, B_0) \) for \( A_0(s; \lambda) \) and \( B_0(s; \lambda) \) is (Kailath, 1980)

\[
R(A_0, B_0) = \begin{bmatrix}
    a_{0,n}(\lambda) & ja_{0,n-1}(\lambda) & a_{0,n-2}(\lambda) & \cdots \\
    0 & a_{0,n}(\lambda) & ja_{0,n-1}(\lambda) & \cdots \\
    b_{0,n}(\lambda) & ja_{0,n-1}(\lambda) & b_{0,n-2}(\lambda) & \cdots \\
    0 & b_{0,n}(\lambda) & ja_{0,n-1}(\lambda) & \cdots \\
    \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots
\end{bmatrix}_{(2n \times 2n)}
\]

where \( 0_m = [0 \cdots 0]_{1 \times m} \). The resultant \( r(A_0, B_0) \) for \( A_0(s; \lambda) \) and \( B_0(s; \lambda) \) is defined as the determinant of the resultant matrix \( R(A_0, B_0) \), i.e.,

\[
r(A_0, B_0) = \det(R(A_0, B_0)) \tag{23}
\]

If \( r(A_0, B_0) = 0 \), \( A_0(s; \lambda) \) and \( B_0(s; \lambda) \) have common factors, otherwise they are coprime (Kailath, 1980).
Let

$$J = \begin{bmatrix}
0 & 0 & \cdots & 0 & 1 \\
0 & 0 & \cdots & 1 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 1 & \cdots & 0 & 0 \\
1 & 0 & \cdots & 0 & 0
\end{bmatrix}_{2n \times 2n}$$

(24a)

Then it is easy to verify that

$$H = LR(A_0, B_0)J$$

(25a)

$$\det(LJ) = 1$$

(25b)

From Lemma 3, we have

$$a_n(\lambda) = j^{-n^2} \det(H) = j^{-n^2} \det(LR(A_0, B_0)J) = j^{-n^2} \det(LJ)r(A_0, B_0) = j^{-n^2} r(A_0, B_0)$$

(26)

From (26), we know that $A_0(s; \lambda)$ and $B_0(s; \lambda)$ have no any common factor if and only if $a_n(\lambda) \neq 0$ and therefore the proof follows from (13) and condition (iii) in Theorem 1. For the case of odd $n$, the proof is similar and is omitted here.

It is noted that condition (ii) in Theorem 2 is a positivity test for a real polynomial of degree two since $a_n(\lambda)$, $a_{n-1}(\lambda)$, $b_n(\lambda)$, and $b_{n-1}(\lambda)$ are all real polynomials in $\lambda$ of degree one. Also noted is that in constructing Table 1 for $A(s; \lambda)$ by (17), the first two rows are filled by real polynomials $a_k(\lambda)$ and $b_k(\lambda)$. Therefore, from Lemma 2 and (19), $a_n(\lambda)$ is a real-coefficient polynomial of degree $2n$ and, hence, condition (iii) in Theorem 2 can be tested by checking the absence of zeros of the $2n$th-degree real polynomial $a_n(\lambda)$ in the open interval $(0, 1)$. As a result, conditions (ii) and (iii) in Theorem 2 can be tested by generating Sturm sequences and, therefore, no root-finding procedure is required.

3. A NUMERICAL EXAMPLE

To illustrate the proposed algorithm, we consider the complex-coefficient segment polynomial $A(s; \lambda)$ given by Bollepalli and Pujara (1994):

$$A(s; \lambda) = (1 - \lambda)p_0(s) + \lambda p_1(s)$$

$$p_0(s) = s^3 + (4.2 + j)s^2 + (5.8 - 1.6j)s + (2.6 - 2.6j)$$

$$p_1(s) = s^3 + (6.1 - 7j)s^2 - (1.4 + 38.2j)s - (39.2 + 40.8j)$$

where $p_0(s)$ and $p_1(s)$ are Hurwitz stable. It is desired to check if $A(s; \lambda)$ is Hurwitz stable for $\lambda \in (0, 1)$.

To test condition (ii) in Theorem 2 we formulate $A(s; \lambda)$ as

$$A(s; \lambda) = s^3 + [(4.2 + 1.9\lambda) + j(1 - 8\lambda)]s^2 + [(5.8 - 7.2\lambda) + j(-1.6 - 36.6\lambda)]s + [(2.6 - 41.8\lambda) + j(-2.6 - 38.2\lambda)]$$

Clearly,

$$a_n(\lambda)a_{n-1}(\lambda) + b_n(\lambda)b_{n-1}(\lambda) = 4.2 + 1.9\lambda > 0, \quad \text{for } \lambda \in [0, 1]$$

Now we test condition (iii) in Theorem 2. With the algorithm in (17), the entry of the 3rd B-row of the Routh-like array for $A(s; \lambda)$ is computed to be:

$$b_{3,0}(\lambda) = 586367.52948\lambda^5 - 440778.25896\lambda^4$$

$$-152714.92876\lambda^3 + 43980.14722\lambda^2$$

$$-19476.32896\lambda + 563.3472$$

To check the absence of zeros of $a_{3,0}(\lambda)$ in $(0, 1)$ we construct the Sturm sequence associated with $a_{3,0}(\lambda)$. From the Sturm sequence it is found that $a_{3,0}(\lambda)$ has two zeros in the open interval $(0, 1)$. Therefore, by theorem 2, we conclude that $A(s; \lambda)$ is unstable.

4. CONCLUSIONS

We have proposed an efficient procedure of using rational arithmetic operations to test the robust Hurwitz stability of a convex combination of two $n$th-degree stable complex polynomials. The procedure is essentially based on constructing a fraction-free parametric parametric Routh-like array and invoking Sturm’s theorem for checking the positivity of a real polynomial of degree two and the absence of real zeros of a real polynomial of degree $2n$ in the open interval $(0, 1)$. We have established the connection between an entry of the parametric Routh-array and resultant of two complex parametric polynomials. Hence, the proposed procedure can also be viewed as an algorithmic implementation of resultant approach given by Bose for testing the robust Hurwitz stability of a segment of complex polynomials.
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6. REFERENCES


Table 1. The fraction-free Routh-like array

| \( a_{0,0} \) | \( a_{0,1} \) | \( a_{0,2} \) | \( \cdots \) | \( a_{0,n-1} \) | \( a_{0,n} \) |
| \( b_{0,0} \) | \( b_{0,1} \) | \( b_{0,2} \) | \( \cdots \) | \( b_{0,n-1} \) | \( b_{0,n} \) |
| \( a_{1,0} \) | \( a_{1,1} \) | \( a_{1,2} \) | \( \cdots \) | \( a_{1,n-1} \) | \( a_{1,n} \) |
| \( b_{1,0} \) | \( b_{1,1} \) | \( b_{1,2} \) | \( \cdots \) | \( b_{1,n-1} \) | \( b_{1,n} \) |
| \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \ddots \) | \( \vdots \) | \( \vdots \) |
| \( a_{n-1,0} \) | \( a_{n-1,1} \) | \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) |
| \( b_{n-1,0} \) | \( b_{n-1,1} \) | \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) |
| \( a_{n,0} \) | \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) |
| \( b_{n,0} \) |