Abstract: In this paper we present a predictive control strategy for the supervision of a formation of autonomous vehicles subject to coordination constraints. Such a system paradigm, referred hereafter to as constrained dynamic network, is characterized by a set of spatially distributed dynamic systems, connected via communication channels, with possibly dynamical coupling and constraints among them which need to be controlled and coordinated in order to accomplish their overall objective. The significance of the method is that it is capable of ensuring no constraints violation and loss of stability regardless of any, possibly unbounded, time-delay occurrence. An application to the coordination of two autonomous vehicles under input-saturation and formation accuracy constraints is presented.

Keywords: Networked Systems, Large-scale systems, Vehicle formation control, Constrained control.

1. INTRODUCTION

The advent of new wireless communication networks allows the conceivability of new challenging control applications, as those of accomplishing coordinated dynamic tasks amongst a network of remotely located dynamic systems connected via the Internet or other communication networks as depicted in Fig. 1. There, the master station is in charge of supervising and coordinating the slave systems. In particular, $r_i$, $w_i$, $x_i$, $y_i$ and $c_i$ represent respectively: the nominal references, the feasible references, the states, the performance-related outputs and the coordination-related outputs of the slave systems. In such a context, the supervision task can be expressed as the requirement of satisfying some tracking performance, viz. $y_i \approx r_i$, whereas the coordination task consists of enforcing some constraints $c_i \in \mathbb{C}_i$ and/or $f(c_1, c_2, ..., c_N) \in \mathbb{C}$ on each slave system and/or on the overall network. To this end, the supervisor is in charge of modifying the nominal references into the feasible ones, when the tracking of the nominal path would produce constraints violation.

Examples of constrained spatial networks which would require advanced coordination ability include unmanned flight formations (Chicka et al., 1999) and satellite constellations (Nakasuka and Motohashi, 1998); fault tolerant control systems for intelligent vehicle highway (Varaya, 1998), electric power grids (Amin, 1998) and telerobotics (Conway et al., 1990). See also (Speyer, 2000), (Girard et al., 2001) and references therein for a comprehensive and up-to-date discussion on the theoretical and applicative challenges on the topic.

The effectiveness of the proposed method will be demonstrated by considering the coordination of the planar motion of two autonomous vehicles, simply modelled as two masses, subject to a dynamic coupling due to elastic and viscous forces between them. See also (Nakasuka and Motohashi,
In (1) \( t \in \mathbb{Z}_+ := \{0, 1, \ldots \}; x(t) \in \mathbb{R}^n \) represents an augmented state containing the plant and pre-compensator states; \( w(t) \in \mathbb{R}^p \) the manipulable input which it would be \( w(t) = r(t) \) if the constraints were not present, \( r(t) \) being the reference sequence for the tracking output \( y(t) \in \mathbb{R}^p \); finally \( c(t) \in \mathbb{R}^{n_c} \) is the constraint vector which has to satisfy the following set-membership constraint
\[
c(t) \in \mathcal{C}, \quad \forall t \in \mathbb{Z}_+
\]
where \( \mathcal{C} \subset \mathbb{R}^{n_c} \) is a compact and convex set. It is assumed that:

1. **(Stability)** The system (1) is asymptotically stable;
2. **(Offset-free)** \( H_p(I_n - \Phi)^{-1} G = I_p \), viz. the system features zero tracking error in steady-state for constant inputs;

Let
\[
\begin{align*}
x_w :&= (I - \Phi)^{-1} G w \\
y_w :&= H_y x_w = w \\
c_w :&= H_c x_w + Lw = \left[ H_c(I - \Phi)^{-1} G + L \right] w
\end{align*}
\]
denote the steady-state equilibrium values for the system’s variables under a constant input \( w(t) = w \). Then, for an arbitrarily small \( \delta > 0 \), consider the following sets:
\[
\begin{align*}
\mathcal{B}_\delta(c) :&= \{ p \in \mathbb{R}^n_c \mid \| p-c \| \leq \delta \} \\
\mathcal{C}_\delta :&= \{ c \in \mathcal{C} \mid \mathcal{B}_\delta(c) \subseteq \mathcal{C} \} \\
\mathcal{V}_\delta :&= \{ w \in \mathbb{R}^p \mid c(k, w) \in \mathcal{C}_\delta, \forall k \in \mathbb{Z}_+ \} \\
\mathcal{V}(x(t)) :&= \{ w \in \mathcal{V}_\delta \mid c(k, x(t), w) \in \mathcal{C}, \forall k \in \mathbb{Z}_+ \} \\
c(k, x(t), w) :&= H_c \left( \Phi^k x(t) + \sum_{i=0}^{k-1} \Phi^{k-1-i} G w \right) + L w
\end{align*}
\]
The set \( \mathcal{C}_\delta \) represents a contraction of \( \mathcal{C} \); \( \mathcal{V}_\delta \) the set of all constant commands \( w \) such that the corresponding equilibrium \( c_w \) satisfy the constraints with margin \( \delta \); finally \( \mathcal{V}(x(t)) \subseteq \mathcal{V}_\delta \) the subset of constant commands \( w \in \mathcal{V}_\delta \) which, if applied to the system from the state \( x(t) \), produce admissible trajectories \( c(k, x(t), w) \in \mathcal{C}, \forall k \in \mathbb{Z}_+ \). Then, the Command Governor is a nonlinear static device
\[
w(t) = F(x(t), r(t))
\]
which is in charge of modifying, whenever necessary, the reference \( r(t) \) in the command \( w(t) \) if the application of \( r(t) \) to the system from the state \( x(t) \) would lead to a violation of the prescribed constraints. If many choices are possible for \( w(t) \in \mathcal{V}(x(t)) \), then the best feasible approximation of \( r(t) \) is selected. Such a command is applied to the system, the new state is collected and the procedure is repeated at the next time instant \( t+1 \) on the basis of \( x(t+1) \) and \( r(t+1) \).

A possible strategy to select \( w(t) \) at each time \( t \) is that of resorting to the following convex optimization problem
\[
w(t) := \arg \min_{w \in \mathcal{V}(x(t))} \| w - r(t) \|_{\Psi_w}^2 \quad \text{(4)}
\]
with \( \| w \|_{\Psi_w}^2 := w^T \Psi_w w \). The above strategy enjoys the following properties (Bemporad et al., 1997):

- \( \mathcal{V}(x(t)) \) is finitely determined;
- \( \mathcal{V}(x(0)) \) non-empty implies \( \mathcal{V}(x(t)) \) non-empty for all \( t \), that is if the problem (4) is solvable at time 0, then it will be solvable at each future time instant \( t \);
- the sequence of \( w(t) \) is bounded, that is the overall system remains stable;
- if \( r(t) \equiv r \) then \( w(t) \rightarrow w_r \) in finite time, \( w_r \) denoting the best feasible approximation of \( r \), viz. \( w_r := \arg \min_{w \in \mathcal{V}_r} \| w - r \|_{\Psi_w}^2 \);
- \( c(t) \in \mathcal{C} \) for all \( t \).

3. THE DISTRIBUTED MASTER/SLAVES

In this section will be illustrated how to extend the basic CG approach to master/slaves distributed control structures in the presence of no-negligible
communication time-delay. For simplicity, the presentation will be limited to a single slave. The extension to the more general case of many-slaves is, mutatis mutandis, direct.

The typical master/slave system structure that we will consider for each remote side is depicted in Fig. 2 where \( \tau \) indicates generically time-delay.

![Fig. 2. Slave side.](image)

It consists of a primal compensated plant, described by the following state-space representation

\[
\begin{align*}
x(t + 1) &= \Phi x(t) + G g(t) \\
y(t) &= H_p x(t) \\
c(t) &= H_c x(t) + L g(t)
\end{align*}
\]  

(5)

and of the slave part of the command governor (CG) logic and buffering devices. In particular, \( x(t) \in \mathbb{R}^n \) is an enlarged state which may collect plant and compensator states; \( \hat{w}(t, t - \tau) \in \mathbb{R}^m \) the command received from the master side. It is to be understood as generated at time \( t - \tau \) for being applied exactly at time \( t \) or never; \( w(t) \in \mathbb{R}^m \) the command that the slave CG logic actually applies to the plant at time \( t \). It would be typically \( w(t) = \hat{w}(t, t - \tau) \) if a constant time-delay \( \tau \) were present. However, in the cases the latter would not be available, the slave CG logic is typically instructed to apply the previous applied command \( w(t - 1) \); \( r(t) \in \mathbb{R}^m \) is a reference sequence which the output \( y(t) \in \mathbb{R}^m \) is required to track. Typically \( \hat{w}(t, t - \tau) = r(t) \) if no constraints were present; and finally \( c(t) \in \mathbb{R}^n \) the prescribed constraint output vector, viz. \( c(t) \in \mathcal{C}, \forall t \in \mathbb{Z}_+ \), with \( \mathcal{C} \) a specified convex and compact set.

At the master side, we consider the system structure of Fig. 3. In this case, we have the models of the remote systems and their primal controllers along with a buffering data structure which allows predictions and real data fusion. In particular, all future state predictions are updated each time a new piece of information is received from the remote sites. All “hatted” variables in the figure have the same meaning of their slave side counterparts.

The basic idea here is that the master CG logic device acts as if the time-delay would not be present by modifying, whenever necessary, the reference \( r(t + \tau) \) into \( \hat{w}(t, t + \tau) \) so as to avoid possibly constraint violation. Because of random, possibly unbounded, time-delay there exists a certain amount of uncertainty at the master side on the actual sequence of commands which the slave CG unit will apply from \( t \) onward and, in turn, on the remote state. Therefore, an effective choice of \( \hat{w}(t, t + \tau) \) cannot be based only on the state prediction \( \hat{x}(t + \tau) \), which would correspond to the timely application of all subsequent commands generated by the master unit as it would result from a constant time-delay \( \tau \). On the contrary, it should be based on a discrete set of state predictions \( \hat{X}(t + \tau | t) \), consisting of all state predictions \( \hat{x}(t + \tau) \) based on the information available at time \( t \) and corresponding to all possible command sequences \( \{w(t), w(t + 1), ..., w(t + \tau - 1)\} \) potentially generated by the slave CG selection strategy as a consequence of all possible combinations of missing data in the given time interval. It is worth pointing out here that, because of constraints, \( \hat{w}(t, t + \tau) \) is guaranteed to be admissible only if applied at time \( t + \tau \) when, supposedly, the state of the remote plant will be one of the vectors contained in \( \hat{X}(t + \tau | t) \).

In order to make our discussion more precise, let \( \tau_f(t) \) and \( \tau_b(t) \) be the forward and, respectively, backward time-delays at each time instant \( t \), viz. \( \tau_f(t) \) is the delay from the master to the slave unit whereas \( \tau_b(t) \) is the delay in the opposite direction. We assume further that the following upper-bounds

\[
\tau_f(t) \leq \bar{\tau}_f \quad \text{and} \quad \tau_b(t) \leq \bar{\tau}_b, \quad \forall t \in \mathbb{Z}_+
\]  

(6)

are either known due to the nature of the communication channel or prescribed as operative limits within to ensure some level of tracking performance. In the latter case, we can distinguish two different operative modes

Normal case: (6) holds true;

(7)

Abnormal case: \( \exists t \text{ s.t.} \quad \tau_f(t) > \bar{\tau}_f \).  

(8)

Notice that the abnormal mode depends only on the forward time-delay. The previous assumptions on time-delay and the two different operative modes allow one to cover in a unified fashion most of communication channels of interest, e.g. different delays between the forward and backward directions, constant or random delay, bounded or possibly unbounded delay, etc.

At each time instant \( t \), let \( t_b \leq t \) and \( t_f \leq t \) denote respectively the most recent time instants in which the master has received a piece of information from the slave and vice versa. In the
normal case (7) it results that \( t \geq t_b \geq t - \bar{\tau}_b \) and \( t \geq t_f \geq t - \bar{\tau}_f \). On the contrary, \( t - t_f \) and \( t - t_b \) can be arbitrarily large in the Abnormal Case (8). Then, we consider the following family of master/slave CG strategies.

**Master GC:**
\[
\hat{w}(t, t + \bar{\tau}_f) := F_1 \left( r(t + \bar{\tau}_f), X(t + \bar{\tau}_f | t_b) \right), \\
\hat{w}(t, t + \bar{\tau}_f - 1) 
\]

**Slave GC:**
\[
w(t) := F_2 \left( w(t - 1), r(t), \hat{w}(\cdot, t) \right) 
\]

where \( F_1 \) and \( F_2 \) are memoryless functions which implement the master/slave CG logic. In particular, \( \hat{w}(t, t + \bar{\tau}_f) \) in (9) is the command computed at time \( t \) for being applied at time \( t + \bar{\tau}_f \) and \( \hat{w}(\cdot, t) \) in (10) is the command generated in the past to be applied at time instant \( t \). Notice that such a command may possibly not be available at time \( t \) at the slave side. In designing \( F_1 \) and \( F_2 \) we want to build up a distributed mechanism which consists in selecting, at each time \( t \), commands \( \hat{w}(t, t + \bar{\tau}_f) \) and \( w(t) \) in such a way that \( w(t) \) is the best approximation of \( r(t) \) at time \( t \), under the constraint \( c(t) \in C \), for all \( t \in \mathbb{Z}_+ \), and irrespective of all possible time-delays such as (7)-(8). Moreover, in the normal case (7) it is further required that: 1) \( w(t) \rightarrow w_r \), when \( r(t) \rightarrow r \), \( w_r \) being the best feasible approximation of \( r \) defined in Sect. 2; and 2) the overall master/slave CG logic has a finite time response, viz. \( w(t) = \hat{r} \) in finite time, whenever \( r(t) \equiv r \). It is worth commenting that in the abnormal case (8) the latter tracking performance cannot be satisfied and only stability and constraint satisfaction must be ensured.

A suitable generalization of the selection logic of Sect. 2 is given, for the master part, by

\[
\hat{w}(t, t + \bar{\tau}_f) = \begin{cases} 
\min_{w \in V(X(t + \bar{\tau}_f | t_b))} \|w - r(t + \bar{\tau}_f)\|_{\Psi}^2, & \text{if } V(X(t + \bar{\tau}_f | t_b)) \text{ is non-empty} \\
\hat{w}(t - 1, t + \bar{\tau}_f - 1), & \text{otherwise}
\end{cases}
\]

(11)

where \( \Psi = \Psi' > 0 \) is \( \|w\|_{\Psi}^2 := x^T \Psi x \). The rationale underlying the above strategy hinges upon the property of virtual command sequences that ensures that if \( w \) is an admissible command at time \( t \) from the state \( x \), it will be as such in all future time instants if constantly applied. Notice that the fact that \( V(X(t + \bar{\tau}_f | t_b)) \) may be empty means that the actual uncertainty on \( x(t + \bar{\tau}_f) \) is so large that we cannot use (11) to compute an admissible virtual command. However, the previously computed virtual command \( \hat{w} \) is still admissible and we are authorized to deliver it to the slave CG unit for being applied at time \( t \).

The slave part of the CG logic is far more simple and reduces to

\[
w(t) = \begin{cases} 
\hat{w}(\cdot, t), & \text{if available and } \|\hat{w}(\cdot, t) - r(t)\|_{\Psi}^2 < \|w(t - 1) - r(t)\|_{\Psi}^2, \\
w(t - 1), & \text{otherwise}
\end{cases}
\]

(12)

The basic properties of the above master/slave strategy have been detailed in (Casavola et al., 2001) and are here briefly condensed.

**Theorem** - Let the assumptions (1)-(2) of pag. 2 hold true. Consider the system (5) along with the GC master/slave (11)-(12) selection strategy. Then:

**Abnormal case** (possibly unbounded time-delay):

1. The cardinality of \( X(t + \bar{\tau}_f | t_b) \) may become unbounded;
2. The set \( V(X(t + \bar{\tau}_f | t_b)) \) is non-empty and bounded;
3. \( V(\bar{\tau}_f | t_b) \) non-empty need not imply that \( V(X(t + \bar{\tau}_f | t_b)) \) be non-empty in some future time-window;
4. \( c(t) \in C \) for all \( t \in \mathbb{Z}_+ \);
5. The overall system remains asymptotically stable but tracking performance may be lost, viz. \( w(t) \neq w_r \) as \( r(t) \equiv r \).

**Normal case** (bounded time-delay):

1. The cardinality of \( X(t + \bar{\tau}_f | t_b) \) is bounded;
2. If the set \( V(X(t + \bar{\tau}_f | t_b)) \) is empty at a certain time instant \( t \), it remains empty for a finite number of steps only;
3. If the set \( X(t + \bar{\tau}_f | t_b) \) consists of a single vector, then \( V(X(\bar{\tau}_f | t_b)) \) non-empty implies \( V(X(t + \bar{\tau}_f | t - \bar{\tau}_b)) \) non-empty for all \( t \in \mathbb{Z}_+ \);
4. \( c(t) \in C \) for all \( t \in \mathbb{Z}_+ \);
5. The overall system remains asymptotically stable and tracking performance are never lost. In particular, \( w(t) \rightarrow w_r \) as \( r(t) \equiv r \).

\( \square \)

The previous GC master/slave strategy has a lot of customizing possibilities that can be exploited in order to trade-off between tracking performance and robustness with respect to time-delay, which ultimately depends on the choice of the number of predictions to be contained in \( X(t + \bar{\tau}_f | t_b) \). Hereafter we will consider the extreme cases with respect to the cardinality of \( X(t + \bar{\tau}_f | t_b) \), denoted as Lowest-Data-Redundancy (LDR) e Highest-Data-Redundancy (HDR) schemes.

**LDR:** It contains only one prediction, viz. \( X(t + \bar{\tau}_f | t_b) = \{ \hat{x}(t + \bar{\tau}_f | t_b) \} \) and \( \hat{x}(t + \bar{\tau}_f | t_b) = \Phi^\tau \hat{x}(t | t_b) + \sum_{k=0}^{\tau} \hat{\Phi}^k \hat{G} \hat{w}(t_b) \). It is based on the optimistic assumption that data are never lost. It works well during normal phases but it may degrade remarkably during abnormal phases.

**HDR:** It is based on pessimistic assumption that data are always lost. Then, \( X(t + \bar{\tau}_f | t_b) \) has to contain all predictions corresponding to all
possible combinations of admissible commands application. Observe that at each time instant two possibilities arise: to apply the scheduled command (if available) or keep to apply the most recent applied command. For this strategy, the tracking performance are quite independent from the occurrences of normal or abnormal phases.

**Remark 1** - It is worth pointing out that the LDR strategy requires a data re-synchronization procedure each time the slave CG unit doesn’t timely receive a new command from the master. This procedure is depicted in Fig. 4. Specifically, as a consequence of a missing-data occurrence the slave CG logic keeps to apply the last applied command and sends a re-synchronization request to the master. When the master receives it, stops to send new commands, updates its predictions and starts again to generate new commands. Along with the first command sent after updating, it sends also an acknowledgment that ends the recovery mode and restores the normal tracking mode. Such a re-synchronization procedure is not required by the HDR scheme. Notice also that in multi-slaves applications, when a slave makes a re-synchronization request all other slaves have to enter in the recovery mode. Then, this request has to be communicated to all other slaves within a fixed but selectable amount of time e.g. by means of direct communication links amongst the slaves.

![Master/Slave Re-Synchronization](image)

**Fig. 4. Master/slave re-synchronization.**

### 4. Example: Supervision of Two Dynamically Coupled Autonomous Vehicles

In this example we want to coordinate the motion of two autonomous vehicles by using a communication channel subject to possibly unbounded time-delay. The LDR strategy will be used and a direct link between the two vehicles is assumed. The system is represented in Fig. 5. The state-space representation of the overall system is given by the following equations

\[
\begin{align*}
\dot{x}_1 &= -k(x_1 - x_2) - \beta(\dot{x}_1 - \dot{x}_2) + F_{ix}^x \\
\dot{y}_1 &= -k(y_1 - y_2) - \beta(\dot{y}_1 - \dot{y}_2) + F_{iy}^y \\
\dot{x}_2 &= -k(x_2 - x_1) - \beta(\dot{x}_2 - \dot{x}_1) + F_{ix}^x \\
\dot{y}_2 &= -k(y_2 - y_1) - \beta(\dot{y}_2 - \dot{y}_1) + F_{iy}^y
\end{align*}
\]

\(\text{Fig. 5. Dynamically coupled autonomous vehicles.}\)

where \(m_1\) e \(m_2\) are the two masses, \(k\) the spring constant, \(\beta\) the viscous coefficient of the damper and \(F_{ix}^x\) and \(F_{iy}^y\), \(i = 1, 2\), the forces acting as inputs. Each subsystem is locally pre-compensated by a suitable controller ensuring offset-free tracking error to constant set-points on positions.

The constraint set \(\mathcal{C}\) is described by

\[
\begin{align*}
|F_{ix}^x(t)| &< 1 \text{ [N]} \\
|y_i(t) - r_i(\alpha(t))| &< 0.05 \text{ [m]}
\end{align*}
\]

where \(\alpha(t)\) is a real time-depending real parameter in [0, 1] which will be used to parameterize the nominal path \(r(\alpha(t))\). It is possible to show that the previous CG scheme can be formulated in terms of \(\alpha(t)\) (in the place of \(\dot{w}(t)\)), which allows the achievement of the feasible paths by selecting the largest admissible increment \(\hat{\alpha}(t) := \alpha(t + 1) - \alpha(t)\) along the nominal path. In the simulation we have used an upper-bound of \(\tau_f = 25\) sampling steps on the time-delay. In all next figures the gray zones denote the re-synchronization procedure.

**Fig. 6. References and positions. The nominal path is a roto-translation at constant speed.**

**5. Conclusions**

In this paper we have presented a predictive control strategy for the constrained supervision and coordination of dynamic systems in spatial networks. All its relevant properties have been summarized and its effectiveness demonstrated by a simulative experiment.

This preliminary results are encouraging and stimulate further research in extending the proposed methodology to more general examples of constrained supervision and coordination problems on dynamic networks, also in the presence of disturbances and model uncertainty.

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6. REFERENCES


