THE PRODUCTION SCHEDULING OF
MULTI-ITEMS IN A STOCHASTIC
MANUFACTURING SYSTEM WITH PREEMPTIVE
DECISION MAKING

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Abstract: A scheduling problem in a manufacturing system that produces different
types of items is presented in this paper. The demands for the items are Poisson
processes, and simultaneous production of two or more items is not allowed. The
decisions on stopping the production can be made at any time, even if the item being
produced is unfinished. An instantaneous setup cost is paid when the production
is stopped or initialized, or when the production is switched from one item to
another. The scheduling problem is formulated as an Impulse Control of Piecewise
Deterministic Markov Processes. The value function is characterized and approxima-
tion methods for the solution of the scheduling problem are proposed.

Keywords: Manufacturing Systems, Scheduling Algorithms, Production Control,
Stochastic Control, Jump Processes, Markovain Decision Processes.

1. INTRODUCTION

This work studies a stochastic control problem in
a manufacturing system that produces different
types of items. The demands for the items arrive
during the running of production and they form
Poisson Processes. Simultaneous production of
two or more types of items is not allowed and the
type of item to be produced is chosen at the end of
the production interval of the previous item.
During the production interval, the interruption
in favour of the production of another type of
item is not allowed ever, one can decide to
bring the production to a half at any time, in a
preemptive decision policy. In returning to pro-
duction, the unfinished item should be completed.
An instantaneous setup cost is paid when the
production is stopped or initialized, or when the
production is switched from one item to another.
The decision maker has to decide between pro-
ducing or stopping, and when the plant is active,
the type of items and production scheduling, in
order to minimize the expected cost associated with
setups, production and storage/shortage of the
items.

The optimization problem is formulated as an Im-
pulse Control of Piecewise Deterministic Markov

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Processes (PMDP) as presented in (Davis, 1993). There are many interesting works dealing with the application of PDMP in the context of production and storage planning, e.g., see, (Akella and Kumar, 1986), (Boukas and Haurie, 1990) or (Jiang and Lou, 1994), where the production planning and the preventive maintenance in a flexible manufacturing system is studied as a continuous control problem. In (Yan and Zhang, 1997), the production planning and setup scheduling in a failure prone manufacturing system is considered as a combination of continuous and intervention control problems. This scheduling model has some resemblances with the model we propose; however, the major difference is that it presents deterministic demand and instantaneous production. In those works the type of preemptive policy mentioned is never considered, although it is of clear interest when the production of each item is not instantaneous, and the time elapsed has to be taken account. Previous results on the problem of production & storage that assembles identical items with preemptive policy appears in (do Val and Salles, 1999) and (Salles and do Val, 2001).

This paper is organized as follows: in section 2 the production scheduling problem is formulated as a stochastic control problem of PMDP and in section 3 is presented two methods of solution for this problem. The first (see Proposition 3.2) is adapted from the general results presented in (Davis, 1993) for the Impulse Control of PMDP, and the second method (see Theorem 3.1) is derived from the results of (Salles and do Val, 2001), which explores the state space structure of the production scheduling model. Finally, the conclusions of this work are shown in section 4.

2. THE PRODUCTION SCHEDULING PROBLEM

The manufacturing system presented here produces $J$ different types of item, with the restriction that only one item is produced at each time. The demand of each type of item forms a Poisson process with rate $\lambda^j$, $j = 1, 2, \ldots J$ and the item of type $j$ requested by the demand until time $t$ is indicated by $d^j_t$. Let $\omega^j_t$ the number of items of type $j$ produced until time $t$; thus, the number of items of type $j$ on storage or in backlog is indicated by the process:

$$n^j_t := \omega^j_t - d^j_t, \quad t \geq 0,$$

where $n^j_t \in \mathbb{Z}$.

The production rate of each type of item is a constant given by $v \in \mathbb{R}^+$; the progression on production of each item is indicated by the process $t \to \xi_t$ and it satisfies $v = \frac{\partial}{\partial t} \xi_t$. The process $\xi_t$ assumes values on interval $[0, \Gamma^j]$; thus the production of an item $j$ is initialized if $\xi_t = 0$ and it is completed when $\xi_t = \Gamma^j$.

Let $z_t := (n^1_t, \ldots, n^J_t, \xi_t, j)$ the state of the production process. When the item $j$ is being produced, $t \to z_t$ lies in the subset $S^j_0 = \{0, \Gamma^j\}$. Let $\partial^j_0 = \mathbb{Z}^J \times \{\Gamma^j\}$ the boundary of $S^j_0$. When the production of item $j$ is stopped and no one item is produced, $t \to z_t$ lies in the subset $S^j_0 = \mathbb{Z}^J \times [0, \Gamma^j]$. We denote $\partial_\sigma = \bigcup_j \partial^j_0$, $S^j = \cup_j S^j_0$, $S'' = \cup_j S''_0$ and $S = S' \cup S''$. For any $z = z_0 \in S$, let us define the drift function by:

$$\varphi(t, z) =$$

$$\begin{cases}
(n^1, \ldots, n^J, \xi + v \cdot t, j), & 0 \leq t \leq t^*(z), \ z \in S^j_0, \\
(n^1, \ldots, n^J, \xi, j), & t \geq 0, \ z \in S''_0.
\end{cases}$$

where

$$t^*(z) := \inf\{t \geq 0 : \varphi(t, z) \in \partial_\sigma\}$$

With the convention $\inf \emptyset = +\infty$, where $\partial_\sigma$ is a subset in the active boundary $\partial_\sigma$, defined by:

$$\partial_\sigma := \{z \in \partial_\sigma : t > 0, z_0 \in S \text{ such that } (2)\}$$

Observe that $t^* : \partial_\sigma \to \mathbb{R}_+$ in (1) is continuous, and denote

$$\Lambda(t, z) := \int_0^t \lambda(\varphi(s, z))ds$$

for any $z \in S$ and $t \leq t^*(z)$. Let $u : \partial_\sigma \to U_0 = \{1, \ldots, J\}$, a control in the boundary of the state space such that, for $z \in \partial^j_0, j \in \{1, \ldots, J\}$, if $u(z) = i$, for $i \in \{1, \ldots, J\}$, then we must start the production of an item of type $i$ after the production of an item $j$ has been finished. The process $z_t$ as defined belongs to the class of PDMP (Davis, 1993) with state space $S = S' \cup S''$, and is characterized by the following elements:

(i) A deterministic drift $t \to \varphi(t, z), t \geq 0, \varphi(0, z) = z$, which is bounded in $S$. For each $z \in S''$ it satisfies $\varphi(t, z) = z, t \geq 0$, i.e. $\varphi$ has null velocity at $S''$. The deterministic drift defines the active boundary $\partial_\sigma$, as in (2).

(ii) A jump rate $\lambda \in C_b(S \cup \partial_\sigma)$, where $C_b(S \cup \partial_\sigma)$ is the space of real continuous and bounded functions on $S \cup \partial_\sigma$;

(iii) A probability of transition $\mu(\cdot, z, u)$ inside the state space. In the boundary $\partial_\sigma$, the transition is controlled and is denoted by $\mu_\sigma$. When $z = (n^1, \ldots, n^J, j) \in \partial^j_0$ and $u(z) = i$ then $\mu_\sigma(y, z, u) = 1$ if $y =
\[ P_z(T_1 > t) = \begin{cases} e^{-\Lambda(t, z)} & t < \tau^*(z) \\ 0 & t \geq \tau^*(z) \end{cases} \] (4)

and \( z_t = \varphi(t, z) \) for \( t < T_1 \), \( P_2 \)-a.s. Let \( F_t := \sigma(x_s : s \leq t) \) be the filtration of the process, and consider \( F_{T_1}^- \). We have that:

\[ P_z(z_T \in A | F_{T_1}^-) = \mu(A, \varphi(T_1, z))1_{\{T_1 < \tau^*(z)\}} + \mu_0(A, \varphi(T_1, z))1_{\{T_1 = \tau^*(z)\}} \] (5)

for any Borel measurable set \( A \in S \). After the jump time \( T_1 \) the process starts afresh at \( z_{T_1} \) following the drift \( \varphi(t, z_{T_1}) \) on the interval \( T_1 \leq t \leq T_2 \), and similar stochastic definitions in (4) and (5) apply to \( T_2 \) and \( z_{T_2} \), respectively.

Let us consider the sequences of intervention times denoted by \( \pi = \{\tau_1, \tau_2, \ldots\} \in \Pi \), and \( \theta = \{\xi_1, \xi_2, \ldots\} \in \Theta \) where \( \Pi \) and \( \Theta \) are classes of stopping times with respect to the filtration \( F_t \). These classes contain the intervention times associated with transitions between the subsets \( S'_j \) and \( S''_j \) and vice versa, and the completion times of each finished item. Notice that the intervention control \( \pi \in \Pi \) is associated to the decision on stopping or starting the production, whereas at the times \( \theta \in \Theta \) a decision \( u(z) \in U_0 \), for \( z \in \partial S'_j \), is associated to the switching of the type of item to be produced.

It is assumed that this manufacturing system is characterized by the following costs by unit of time: \( L_j : Z \to \mathbb{R}^+ \), the storage costs \( (n^l_j > 0) \) or the backlog costs \( (n^l_j < 0) \) of each item \( j; p : S \to \mathbb{R}^+ \), the cost associated to the production rate of each item; \( h : \partial S' \cup U_0 \to \mathbb{R}^+ \), that is the setup cost associated to the choice of the type of item to be produced; finally, \( g : S \to \mathbb{R}^+ \) is the cost associated to the stoppage or initialization of the production of each item.

Let us define the functions \( L = \sum_j L_j \), and \( f(z) = L(z) + p(z) \) and denote the descent rate by the constant \( \alpha \). Assume that \( f(z), h(z) \) and \( g(z) \) belong to the class of continuous function \( C_b(S \cup \partial S) \). For any control policies \( \pi \in \Pi \) and \( u \in U_0 \), let us associate the following cost

\[ J^\pi u(z) = E^\pi_u \left\{ \int_0^{\tau^*_1} e^{-\alpha s} f(z_s) \, ds \right. \\
+ \sum_{i=1}^{\infty} e^{-\alpha \tau^*_i} h(z_{\tau^*_i})1_{\{z_{\tau^*_i} \in \partial S\}} + \left. e^{-\alpha \tau^*_i} g(z_{\tau^*_i})1_{\{z_{\tau^*_i} \in \partial S\}} \right\} \]

where above expectation is get with respect to the process \( t \rightarrow z_t \) with \( z_0 = z \). In this scheduling problem, one has to determine a sequence of setups in order to minimize the operational cost given in (6). When it exists, such strategies is optimal and the value for the problem, \( V : S \cup \partial S \to \mathbb{R} \), defined by:

\[ V(z) = \inf_{\pi \in \Pi, u \in U_0} J^\pi u(z) \] (7)

3. SOLUTIONS FOR THE PRODUCTION SCHEDULING PROBLEM

The solution of problem (7) is obtained from successive approximation methods similar to that present in (Bertsekas and Shreve, 1978). A dynamic programming operator, denoted “the one-jump-or-one-intervention operator”, with control on the boundary, generate a sequence of functions that converge to the value function of the scheduling problem. This section presents two approximation schemes which differ in the way that the end cost of the one-jump-or-intervention operator is updated at each iteration. One method has a simple convergence and it requires a restriction in the initial function of the procedure, whereas the other presents an uniform convergence and is initialized by any continuous function.

3.1 The one Jump-or-Intervention Operator

For a point \( z \in S(\in S^0) \), let us denote the correspondent point in the copy \( S'(z) \) by \( \tilde{z} \). Similar notation applies for the process \( t \rightarrow z_t \), and we indicate by \( \tilde{z}_t \) the correspondent point in \( S'(z) \) or \( S''(z) \) to which the process is transferred whenever an intervention occurs at time \( t \).

Let us denote \( a \land b \) for \( \min\{a, b\} \) and \( \tilde{\lambda} := \alpha + \lambda(z) \). For \( z \in S \) and any \( \phi, \psi \in C_b(S \cup \partial S) \), let us define the following operators:

\[ \mathcal{M}[\phi](z) := \phi(z) + h(z, u(z))1_{\{z \in \partial S\}} \]

\[ \mathcal{R}_t[\phi, \psi](z) := \inf_{u \in U_0} \left\{ E^\pi_u \left\{ \int_0^{\tau^*_1} e^{-\alpha s} f(z_s) \, ds \right. \\
+ \sum_{i=1}^{\infty} e^{-\alpha \tau^*_i} \mathcal{M}[\phi](z_{\tau^*_i})1_{\{z_{\tau^*_i} \in \partial S\}} + \left. e^{-\alpha \tau^*_i} \mathcal{R}_t[\psi](z_{\tau^*_i})1_{\{z_{\tau^*_i} \in \partial S\}} \right\} \]

(8)
\[ \mathcal{R}_t[\phi, \psi](z) := \inf_{0 \leq t \leq t^*(z)} \mathcal{R}_t[\phi, \psi](z) \]

(9)

\[ Q_S[\phi](z) := \int_S \phi(y) \mu(dy, z), \]

(10)

\[ Q_0^u[\phi](z) := \int_{\partial^* \mathbb{S}} \phi(y) \mu(dy, z, u), \]

(11)

\[ N[\phi](z) := \frac{1}{\lambda(z)} \left( f(z) + \lambda(z) Q_S[\phi](z) \right), \]

(12)

**Remark 1.** The operator (9), denoted the “one-jump-or-Intervention operator,” characterizes an stopping time problem with end costs \( N[\phi] \) or \( \psi + g \), depending on the stopping time being the first jump or the first intervention, respectively.

We assume that:

\( (H_1) \ g(z) \geq g_0 > 0 \) for \( g_0 \in \mathbb{R}^+ \).

\( (H_2) \ \phi, \psi \) and \( h \) are in \( C_b(S) \).

The assumptions \((H_1)\) and \((H_2)\) guarantee that there only exists isolated interventions epochs in any candidate to optimal policy in the scheduling problem (7). The details to comply with standard conditions as in \( (\text{Davis}, 1993) \) are presented in appendix \( A \) of \( (\text{Salles and do Val, 2001}) \).

The value function for the one-jump-or-intervention problem, \( \nu : S \cup \partial S \rightarrow \mathbb{R} \), is defined by:

\[ \nu(z) := \mathcal{R}[\phi, \psi](z) \]

(13)

Let us denote:

\[ \hat{\lambda}(t, z) := \int_{0}^{t} \left( \alpha + \lambda(\varphi(s, z)) \right) ds = \int_{0}^{t} \hat{\lambda}(\varphi(s, z)) ds \]

**Lemma 2.** If \( H_2 \) holds and \( \phi, \psi \in C_b(S \cup \partial^* \mathbb{S}) \), then \( \nu \in C_b(S \cup \partial^* \mathbb{S}) \).

Proof: Let us define the following operator:

\[ \mathcal{R}_t[\phi, \psi](z) := \inf_{u \in U_0} \mathbb{E}_z^u \left\{ \int_{0}^{t \wedge T_1} e^{-\alpha t } f(z_s) ds + e^{-\alpha T_1} \phi(z_{T_1}) 1_{\{t \geq T_1\}} + e^{-\alpha t} (\psi(z_t) + g(z_t)) 1_{\{t < T_1\}} \right\} \]

(14)

\[ \mathcal{R}_t[\phi, \psi](z) = \mathcal{R}_t'[\phi, \psi](z) + \]

\[ \inf_{u \in U_0} \mathbb{E}_z^u \left\{ e^{-\alpha T_1} h(z_{T_1}, u(z_{T_1})) 1_{\{z_{T_1} \in \partial^* \mathbb{S}\}} \right\} \]

In view of the first jump time \( T_1 \) has the exponential distribution given by (4), we have that:

\[ \inf_{u \in U_0} \mathbb{E}_z^u \left\{ e^{-\alpha T_1} \{ h(s, u(z_s)) 1_{\{s \leq T_1\}} \} \right\} = \inf_{u} \left\{ e^{-\Lambda(\varphi(t^*, z))} h(\varphi(t^*, z), u(\varphi(t^*, z))) \right\} \]

(15)

Since \( t^*(z) \in C_b(S \cup \partial^* \mathbb{S}) \) we conclude that the above expression is in \( C_b(S \cup \partial^* \mathbb{S}) \). In addition

\[ \mathcal{R}_t[\phi, \psi](z) \in C_b(S \cup \partial^* \mathbb{S}) \text{ from Lemma (53.38), p. 224 in (Davis, 1993). Therefore, from (14), we conclude that} \]

\[ \mathcal{R}_t[\phi, \psi](z) \in C_b(S \cup \partial^* \mathbb{S}). \]

\( \Box \)

Let \( B \) denote the vector field associated to the deterministic drift \( \varphi \). For \( \phi \in C_b(S \cup \partial^* \mathbb{S}) \), and for points where \( \phi(z) \) is differentiable, we denote

\[ B[\phi](z) := \lim_{t \to 0} t^{-1} [\phi(\varphi(t, z)) - \phi(z)]. \]

Smoothness of the vector field \( B \) is required, as in \( (\text{Gaterek, 1992}) \):

\( (H_3) \) There exists \( a = (a_1 \ldots a_d) \) with \( a_j \in C_b(S), j = 1, \ldots, d \) such that \( B[\phi](z) < a(z), \nabla \phi(z) > \), for any \( \phi \in C_b(S) \) which is differentiable at \( z \in S \).

**Lemma 3.** Assume that \( H_2 \) is valid. Then \( \nu \) in (13) is a viscosity solution of the following variational equations

\[ \{ B[\nu](z) - \tilde{\lambda}(z) \nu(z) + \lambda(z) Q_S[\phi](z) + f(z) \} \wedge \{ \psi(z) + g(z) - \nu(z) \} = 0, \quad z \in S', \]

(16)

\[ \nu(z) = \{ \psi(z) + g(z) \} \wedge N[\phi](z), \quad z \in S'' \]

(17)

with the boundary condition for \( z \in \partial^* \mathbb{S} \):

\[ \nu(z) = \inf_{u \in U_0} \{ Q_0^u[\phi](z) + h(z, u) \} \wedge (\psi(z) + g(z)), \]

(18)

Conversely, if \( \nu \) satisfies (16)–(18) for each \( z \in S \), then \( \nu(z) = \mathcal{R}[\phi, \psi](z) \).

Proof: From the arguments in Lemma 6 of \( (\text{Salles and do Val, 2001}) \), we conclude that

\[ \mathcal{R}[\phi, \psi](z) = \nu(z) = \inf_{u \in U_0} \mathbb{E}_z^u \left\{ \int_{0}^{t \wedge T_1} e^{-\alpha t } f(z_s) ds + e^{-\alpha T_1} (M[\phi](z_{T_1})) 1_{\{t \leq T_1\}} + e^{-\alpha t} (\psi(z_t) + g(z_t)) 1_{\{t < T_1\}} \right\} \]

\[ - \int_{0}^{t} e^{-\tilde{\lambda}(s, z)} \Omega(\varphi(s, z)) ds \]

\[ + e^{-\tilde{\lambda}(t, z)} \Omega(\varphi(t, z)) 1_{\{t < t^*(z)\}} + \inf_{u} \Gamma_2[\varphi(t, z)] 1_{\{t < t^*(z)\}} \]

(19)

where, we set

\[ \Omega(z) := f(z) + \lambda(z) Q_S[\phi](z) - \tilde{\lambda}(z) \nu(z) \]

\[ + B[\nu](z) \quad \text{for } z \in S \]

\[ \Gamma_1(z) := \psi(z) - \nu(z), \quad \text{for } z \in S \]

\[ \Gamma_2(z) := Q_0^u[M[\phi]](z) - \nu(z) = Q_0^u[M[\phi]](z) + h(z, u) - \nu(z), \quad \text{for } z \in \partial^* \mathbb{S}. \]

(19)
By optimality, \( \Omega(z) \geq 0 \), \( \Gamma_1(z) \geq 0 \), and \( \Gamma_2(z) \geq 0 \), and the minimum is achieved if and only if in (19) \( \Omega(\varphi(s, z)) = 0 \), \( 0 \leq s \leq t \) and \( \Gamma_1(\varphi(t, z)) = 0 \), \( t < t^*(z) \), \( \inf_u \Gamma^u_1(\varphi(t, z)) = 0 \), \( t = t^*(z) \). Therefore, \( t = \arg \min \{ R_u[\phi, \psi](z) : 0 \leq s \leq t^*(z) \} \), and \( \tau_1 = t \) is an optimal stopping time, respectively. It follows from the above that \( \forall z \in \partial \mathcal{S}_0 \),

\[
\Omega(z) \land \Gamma_1(z) = 0, \forall z \in \mathcal{S} \text{ and } \Gamma_1(z) \land \inf_u \Gamma^u_1(\varphi(z)),
\]

(20)
holds, expressed for \( z \in \mathcal{S}' \) by (16) with the boundary condition (18). For \( z \in \mathcal{S}' \) notice that \( B[z] = 0 \), and from (12), \( \Omega(z) = f(z) + \lambda(z)Q_0 \phi(z) - \hat{\lambda}(z) \rho(z) = \hat{\lambda}(z)(\lambda' \phi(z) - \rho(z)) \). Writing (20) in this situation we get (17) equivalently.

Finally, we verify that \( \nu \) is indeed a viscosity solution by following identical steps to those in the proof of Proposition 5 of (Gatarek, 1992).

3.2 The First Approximation Method

For \( \phi \in \mathcal{C}_b(\mathcal{S}) \) let us define the following operator:

\[
\mathcal{L}[\phi](z) := \mathcal{R}[\phi, \phi](z),
\]

(21)
Let us define the sequence of functions \( V_i, i = 0, 1, 2, \ldots \), by:

\[
V_i(z) := \mathcal{L}[V_{i-1}](z), \quad \forall z \in \mathcal{S}
\]

(22)

**Definition 3.1.** Let the sequence of jump times \( \Sigma = \{ T_i : i = 0, 1, \ldots \} \) and the sequence of intervention times \( \Pi = \{ \tau_i : i = 0, 1, \ldots \} \). Let the sequence of intervention or jump times \( \Upsilon = \{ \xi_i : i = 0, 1, \ldots \} \), defined by:

\[
\xi_0 := 0, \quad \xi_i := \min \{ t > \xi_{i-1} : z_t \neq z_{i-1} \},
\]

where \( (z_{i-1} := \lim_{s \uparrow t} z_s) \).

These sequences are \( \{ F_i \} \)-stopping times, and in view of assumptions \( H_1 \) and \( H_2 \) we guarantee that \( \tau_i \to \infty P_{2-q}.c. \) and \( T_i \to \infty \ P_{2-q}.c. \), when \( i \to \infty \); thus \( \xi_i \to \infty P_{2-q}.c. \).

**Lemma 4.** For each \( z \in \mathcal{S} \) and \( i = 0, 1, \ldots \)

\[
V_i(z) = \inf_{\pi \in \Pi_i U_0} E^\pi_{z,u} \left\{ \int_0^t e^{-\alpha s} f(z_s) dt \right. \\
\left. + \sum_{j=1}^{i-1} e^{-\alpha T_j} g(z_{T_j})1_{\{T_j < T_i\}} + \\
\sum_{j=1}^{\xi_i} e^{-\alpha \xi_j} h(z_{\xi_j}), u(z_{\xi_j})1_{\{z_{\xi_j} \in \partial \mathcal{S}\}} + \\
\left. e^{-\alpha \xi_i} V_0(z_{\xi_i}) \right\},
\]

(23)

Proof: Using in (8) the fact that \( \xi_1 = \tau_1 \land T_1 \) for \( \tau_1 \in [0, \infty) \), we obtain from (3.1) the following representation:

\[
\mathcal{R}_1[\phi, \psi](z) := \inf_u \int_0^\infty e^{-\alpha s} f(z_s) ds \\
\left. + e^{-\alpha T_1} \left[ \phi(z_{T_1}) + h(z_{T_1}), u(z_{T_1}) \right] 1_{\{z_{T_1} \in \partial \mathcal{S}\}} + \\
e^{-\alpha \xi_1} \psi(z_{\xi_1})1_{\{z_{\xi_1} \in \partial \mathcal{S}\}} \right\}.
\]

(24)

By definition, \( V_i = \mathcal{L}[V_0] = \inf_{\tau_1} \mathcal{R}_1[\phi, \psi](z) \). Thus from (24) we show this result for \( i = 1 \). Suppose that (23) is satisfied for \( i > 1 \). Since \( \{z_t : t \geq 0\} \) is a strong Markov process, we conclude the following result from the dynamic programming arguments:

\[
V_{i+1}(z) = \inf_{\tau_1} \left\{ \int_0^\infty e^{-\alpha s} f(z_s) ds \\
\left. + \sum_{j=1}^{i} e^{-\alpha T_j} g(z_{T_j})1_{\{T_j < T_{i+1}\}} + \\
\sum_{j=1}^{\xi_i} e^{-\alpha \xi_j} h(z_{\xi_j}), u(z_{\xi_j})1_{\{z_{\xi_j} \in \partial \mathcal{S}\}} + \\
V_0(z_{\xi_i})1_{\{z_{\xi_i} \in \partial \mathcal{S}\}} \right\}.
\]

(25)

**Proposition 5.** Assume that \( H_1 \) and \( H_2 \) hold. Then \( V \) is the biggest Borel measurable solution of the system:

\[
\begin{cases}
w = \mathcal{L}[w] \\
w \leq h^u
\end{cases}
\]

for \( u \in U_0 \) with \( I : S \cup \partial S \to \mathbb{R} \) given by

\[
I(z) := E^u_{z} \left\{ \int_0^\infty e^{-\alpha s} f(z_s) ds \\
+ \sum_{j=1}^{\infty} e^{-\alpha T_j} h(z), u(z_{T_j})1_{\{z_{T_j} \in \partial \mathcal{S}\}} \right\}.
\]

(25)
Moreover, for a function $V_i \geq i$, the sequence $V_i \downarrow V$ as $i \to \infty$.

Proof: From theorem 54.23 in (Davis, 1993), $V_i(z) = \mathcal{V}[V_{i-1}](z), i = 1, 2, \ldots \text{for all } z \in S$ is a monotone decreasing sequence that converges to a continuous function $w$ such that $w = \mathcal{V}[w]$. In view of $\zeta_i \to \infty$ when $i \to \infty$, we conclude from Lemma 4 that $V_i \to V$ and therefore $V = w$. □

3.3 The Second Approximation Method

For $\phi \in C_b(S \cup \partial S^*)$ let us consider the following operator:

$$\mathcal{P}[\phi](z) :=
\begin{cases}
\mathcal{R}[\phi, \mathcal{N}\phi](z), & \text{for } z \in S', \\
(\mathcal{R}[\phi, \mathcal{N}\phi](z) + g(z)) \wedge \mathcal{N}[\phi](z), & \text{for } z \in S^u.
\end{cases}
$$

(26)

Theorem 3.1. Suppose that $H_1-H_3$ hold, then for any $W_0 \in C_b(S \cup \partial S^*)$, each function $W_i, i = 1, 2, \ldots$ defined by:

$$W_i(z) := \mathcal{P}[W_{i-1}](z), \forall z \in S,$$

(27)
is a viscosity solution of the variational inequalities:

$$\begin{align*}
&\{B[W_i](z) - \lambda(z)W_i(z) + \lambda(z)\mathcal{Q}[W_{i-1}](z) + f(z)\} \\
&\wedge \mathcal{N}[W_{i-1}](z) + g(z) - W_i(z) = 0 \text{ for } z \in S',
\end{align*}
$$

(28)

$$W_i(z) = [W_i(\bar{z}) + g(z)] \wedge \mathcal{N}[W_{i-1}](z), \forall z \in S^u,$$

(29)

with the boundary condition for $z \in \partial S^*$:

$$W_i(z) = \inf_{u \in U(z)} \{Q^u_0[W_{i-1}](z) + h(z, u)\} \wedge \mathcal{N}[W_{i-1}](z)$$

(30)

In addition, $W_i \to V$ uniformly, as $i \to \infty$.

Proof: From Lemma 3 and the arguments presented in the proof of Theorems 3 and 4 in (Sales and do Val, 2001).

4. CONCLUSIONS

A production scheduling problem in a stochastic manufacturing system that produces different items has been formulated as an Impulse Control of Piecewise Deterministic Markov Processes. The production scheduling determines a sequence of setups in the middle of the production interval, that represent the decisions on starting or stopping the production of an item, and a sequence of setups in the boundary of the state pace, that represent the decision on switching the production from one item to another. Two successive approximation methods that furnish the solution of the scheduling problem have been obtained: the first requires a restriction in the initial function of the procedure, and the second presents an uniform convergence and is initialized by any continuous function. The last procedure explores the fact that the state space of the production process has general drift in one copy of the state space, and in the other copy the deterministic drift is null. At the moment, these methods are being implemented, and numerical results will be presented at the conference.

5. REFERENCES


