SENSITIVITY ANALYSIS IN SIMULTANEOUS STATE/PARAMETER ESTIMATION FOR INDUCTION MOTORS

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Abstract: In this paper, a sensitivity analysis is carried out for the problem of simultaneous estimation of induction motor’s state and parameters. This is done using separable least squares formulation. It comes out that even in the presence of persistent excitations, the above problem is very sensitive to noises and/or uncertainties especially for one of the four possibly identifiable combinations of parameters. Numerical experiments are conducted that confirm the a priori sensitivity-based predictions.

Keywords: Sensitivity analysis, induction machine, parameter estimation, state estimation.

1. INTRODUCTION

In many application areas, induction motor stands out because of inherent qualities such as robustness, low cost and simplicity, while initially recognized to be difficult to control because of nonlinearities and the need for state estimation, induction motor presently takes advantage of many high quality estimation-based control schemes (see among other works (Chiasson, 1998; Hu et al., 1996; Balloul and Alamir, 2000; Lu and Chen, 1995; Glielmo et al., 1994; Espinosa and Ortega, 1994; Glumineau et al., 1993; Besançon et al., 1996; Ahmed-Ali et al., 1999; Espinosa-Perez et al., 1997)).

In nowadays industrial requirements, however, good control under nominal (or even, a slightly perturbed) model is no more sufficient. Monitoring parameter variations becomes unavoidable to maintain control performance level and, mainly, to enable a real-time diagnosis to be performed. Therefore, a simultaneous state/parameter estimation schemes need to be considered (Alonge et al., 2001; Sdid and Benbouzid, 2000; Besancon, 2001; de Kock-JA et al., 1994; Nilsen and Kazmierkowski, 1989; Marino et al., 1999). In all these works, the nominal feasibility of the above task is studied but no sensitivity analysis has been explicitly and rigorously addressed. This is however crucial in a realistic industrial context characterized by unavoidable sensor noises and model’s uncertainties.

The aim of the present paper is to address this problem, namely, sensitivity analysis in the context of induction motor’s simultaneous state and parameters estimation. This is done according to the following steps. First the motor’s equations are recalled and rewritten (section 2) in a convenient form in order to put the estimation problem in a somehow standard scaled separable least squares form (Bruls et al., 1999); then a sensitivity criterion is defined and justified (section 3). The later is then computed for several standard configurations under torque/flux control. Some numerical experiments are then conducted to validate the sensitivity-based predictions. Finally conclusions are dressed in section 5.
2. EQUATION REWRITING AND PRELIMINARY RESULTS

Using \( R, L \) and \( M \) to denote resistance, inductance and mutual inductance respectively, using \( x = (I_{\alpha s}, I_{\beta s}, \Phi_{\alpha s}, \Phi_{\beta s}) \) to designate state vector and indices \( r \) and \( s \) to refer to rotor and stator components respectively, the induction motor’s dynamical model is given by

\[
\dot{x} = \begin{pmatrix} p_1 I + 2\Omega J & p_2 I - 2p_3 J \\ 0_{2\times2} & 0_2 \end{pmatrix} x + \begin{pmatrix} p_1 I \\ 0_{2\times1} \end{pmatrix} u
\]

(1)

where \( p_1 := -(R_s + \frac{R_r}{\sigma L_r}) ; \ p_2 := \frac{R_r}{\sigma L_r} ; \ p_3 = \frac{1}{\sigma L_r} \) and \( q_1 = -R_s \) (\( \sigma := 1 - \frac{M^2}{L_r L_s} \), \( \Omega \) stands for the mechanical speed, \( I \) the identity in \( \mathbb{R}^{2\times2} \) and \( J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \)).

It can readily be inferred from (1) that whatever \( \tau \) can be hopefully estimated (Besançon and Mannseth, 2000). The sensitivity analysis can be performed by using (2) at instants \( t_1, t_1 + \tau_s, t_1 + 2\tau_s, \ldots, t_1 + N_n \tau_s \), namely, with obvious notations

\[
\begin{bmatrix} \bar{T}(t_1, \theta) \end{bmatrix} \hat{p} = \hat{F}(t_1) \quad (9)
\]

with \( \bar{T}(t_1, \theta) \in \mathbb{R}^{2N_n \times 3} ; \ \hat{F}(t_1) \in \mathbb{R}^{2N_n} \).

Remark 2. It is worth noting that although the least squares problem (9) is not equivalent to the identity (4) being imposed at “each” \( t \in [t, t + N_n \tau_s] \), “practical sensitivity analysis” on the later can be inferred from the former provided that \( \tau_s \) is sufficiently high. This is because, whatever is the dynamical estimation scheme being used, such unavoidable finite-precision-like limitations are to be accounted with (finite precision integration schemes for dynamical observers equation, sampled measurement acquisition, etc.)

3. A SENSITIVITY ANALYSIS

3.1 Definition of a sensitivity criterion

It is a well known fact (Golub and Pereyra, 1973; Bruls et al., 1999) that the solution of (9) is given by

\[
\theta_{opt} := \min_{\theta} \left\| \bar{I} - \bar{T}(t_1, \theta) \bar{F}(t_1, \theta) \right\|^2 \quad (10)
\]

\[
p_{opt} := \bar{T}(t_1, \theta_{opt}) \bar{F}(t_1) \quad (11)
\]

where \( \bar{I} \) in (10) stands for the identity matrix in \( \mathbb{R}^{(2N_n)\times(2N_n)} \) while \( \bar{T}(t_1, \theta) \) is the pseudo-inverse of \( \bar{T}(t_1, \theta) \).
In order to define a consistent sensitivity criteria, let us investigate what happens when measurement errors occur in the acquisition process together with approximation errors in the computation of the optimal solution of (10). This result in the use of erroneous measures \( \tilde{F}_{true}(t_1) + \delta \tilde{F}(t_1) \) in the estimation process (10)-(11) rather than the true vector \( F_{true}(t_1) \). In this case, the resulting \( \theta \), say \( \theta_e \), is given by

\[
\theta_e = \min_{\theta} \left\{ (I - \tilde{T}(t_1, \theta)\tilde{T}^T(t_1, \theta)) \left[ \tilde{F}_{true}(t_1) + \delta \tilde{F}(t_1) \right] \right\}^2 + \delta \theta_{pp}^2
\]

where \( \delta \theta_{pp}^2 \) is used to denote errors in \( \theta_e \) due to measurement errors \( \delta \tilde{F}(t_1) \) while \( \delta \theta_{pp}^2 \) denotes approximation errors in solving the non convex optimization problem (10) in the decision variable \( \theta \). Using (11), the computed \( p \), say \( p_e \), may be given by (using \( \delta \theta = \delta \theta_{pp}^2 + \delta \theta_{pp}^2 \))

\[
p_e = \tilde{T}^T(t_1, \theta_{true} + \delta \theta)\tilde{F}_{true}(t_1) + \delta \tilde{F}(t_1)
\]

\[
= p_{true} + \tilde{T}^T(t_1, \theta_{true})\delta \tilde{F}(t_1) + \frac{3}{\partial \theta_i}(\tilde{F}_{true}(t_1))\delta \theta_i + O(\|\delta \tilde{F}(t_1)\|^2)
\]

(12)

Now, provided that persistent excited conditions hold, that the relative measurement error \( \|\delta \tilde{F}\|/\|\tilde{F}\| \) is sufficiently small and that \( N_a \) is big enough, the second term in the r.h.s of (12), namely \( \tilde{T}^T(t_1, \theta_{true})\delta \tilde{F}(t_1) \), may lead to relative errors in \( p_e \) that are of the same order of magnitude than the relative measurement errors \( \|\delta \tilde{F}\|/\|\tilde{F}\| \) [see (11)]. No such easy arguments can be exhibited about the third term. This is the reason why, in this paper, attention is focused on the sensitivity gains defined for \( i = 1, 2, 3 \)

\[
S_i := \left[ \frac{\partial \tilde{T}^T_i}{\partial \theta_i}(t_1, \theta_{true})\tilde{F}(t_1) \right] \in \mathbb{R}^3
\]

(13)

Another way of justifying the interest carried in these sensitivity gains is to notice that one may write according to (12) \( i = 1, 2, 3 \)

\[
S_i := \left. \frac{\partial (p_e - p_{true})}{\partial \theta_i} \right|_{\text{perfect measurements}}
\]

that is, even under perfect measurements, \( S_i \) represents the sensitivity of the estimation error \( p_e - p_{true} \) to errors in the estimation of \( \theta_i \). These errors may be viewed for instance as transient errors under asymptotically convergent nonlinear state and parameters observer schemes, or even, persistent errors due to unavoidable finite, non vanishing, integration step time used in the integration of such simultaneous observers. In other words, \( S_i \) may be viewed as a sensitivity indicator that is intimately linked (or even originated from) the very simultaneous aspect of state/parameter estimation for induction motors.

3.2 Computational issues

Equation (13) shows that the key issue in the computation of the sensitivity indicators \( S_i \) defined in the preceding section is the computation of the gradient of the pseudo-inverse \( \tilde{T}^T(\theta) \) of \( \tilde{T}(\theta) \) with respect to the parameter \( \theta \). It has been shown in (Golub and Pereyra, 1973) that, provided that the gradient \( \tilde{T}_e(\theta) \) can be analytically given by the following expression :

\[
\tilde{T}_e(\theta) = -\tilde{T}^T[\tilde{T}_e(\theta)]\tilde{T}^T + \tilde{T}^T[\tilde{T}_e(\theta)]^T[\tilde{T}_e(\theta)]T\tilde{T}^T[\tilde{T}_e(\theta)] + \tilde{T}^T[\tilde{T}_e(\theta)]^T[\tilde{T}_e(\theta)]^T[\tilde{T}_e(\theta)]^T
\]

(15)

where \( I_n \) stands for the identity matrix in \( \mathbb{R}^{n \times n} \).

Note that from (5)-(6), it can be inferred that \( \tilde{T} \) is an affine function in the arguments \( \xi_1 \) and \( \xi_1 \) and hence in \( \theta \). Therefore, \( \tilde{T}_e \) are quite easy-to-compute matrices that are independent of \( \theta \).

As in any nonlinear problem, the sensitivity indicators \( S_i \) depend on the current configuration under consideration. More concretely, they depend on the least squares problem (9) which is defined by the state and control trajectories over the time interval \( [t_1, t_1 + N_a \tau] \). Unless going through all the configuration space, any result one may present necessarily represents some particular case. It is worth noting, however, that many changes in initial states have been tried without dramatic changes in the main qualitative conclusions concerning the sensitivity analysis.

In what follows, the conditions under which several least squares problems (9) are constructed (matrices \( \tilde{T}(t_1, \theta) \) and \( \tilde{F}(t_1) \)) are clearly explained, then, the resulting sensitivity indicators \( S_i \) are computed. From this, some qualitative predictions are made. The later are finally confirmed through further numerical investigations.

From a control point of view, it is well known (Besançon et al., 1996; Espinosa-Perez et al., 1997) that the torque/flux controlled induction machine has a relative degree 1 (Isidori, 1989) and a state feedback tracking control law may be easily obtained under the form \( \bar{u} := K(t, \tilde{y}, \tilde{\xi}, \tilde{p}, \tilde{q}) \). Injecting this feedback law in the system equations (2)-(3) adding possibly an excitation term :

\[
\bar{u} = K(t, \tilde{y}, \tilde{\xi}, \tilde{p}, \tilde{q}) + v
\]

(16)
yields an autonomous system that may be integrated starting from some initial state $\bar{x}_0 := (\bar{y}_0, \bar{\xi}_0)^T$ over $[0, t_f]$ (see Figure 1). Now taking several values for $t_1$ in $[0, t_f - N_a\tau_s]$ and the corresponding $\theta = (\xi(t_1), \dot{q}_1)$, several configurations can be used to build the least squares problem and compute the corresponding sensitivity indicators $S_t$.

4. NUMERICAL RESULTS AND VALIDATION

The basic simulation corresponds to the scaled initial state

$$\bar{y}_0 = (0, 1)^T ; \bar{\xi} = (0.6, 0.8)^T$$

The tracking problem is given by a set-point change in the scaled torque/flux references given by $z^r(\bar{t}) = z^r_{1\bar{t}}$ for $\bar{t} \leq 5$ and :

$$z^r(\bar{t}) := z^r_{1\bar{t}} + (z^r_{2\bar{t}} - z^r_{2\bar{t}})e^{-\mu(\bar{t}-5)}$$

where $z^r_{1\bar{t}} := (1, 1)$ ; $z^r_{2\bar{t}} = (0.8, 1.2)$ and $z^r := (\Gamma^r, \Psi^r)^T$ in which $\Gamma^r$ and $\Psi^r$ are scaled references on torque and squared norm of flux vector respectively (scaling values are 100 N.m and 1.0 Wb respectively), $\mu := 1$ (for a time reference value for scaling $t_r = 1$ ms). The induction machine’s parameter values are given on table 1. The mechanical speed $\Omega$ in (1) is constant and equals $\Omega = 900$ tr/min.

Reference values used in equations scaling are given on table 2.

Acquisition parameters are $N_a = 100$ and $\tau_s = 10\mu s$. This corresponds to a window’s width of 1 ms which is quite compatible with system’s dynamics. It is worth noting that when the induction machine is regulated around constant torque and flux, there is no steady state associated to these constant values. Currents and flux vector continuously oscillate which offer natural persistent excitation.

Table 3 shows the values of the sensitivity indicators $S_t \in \mathbb{R}^3$, $i = 1, 2, 3$ for three different values of $t_1 = 5, 8$ and 14 ms. This enables to study transient ($t_1 = 5$ ms), near constant regulated variables (but not measured outputs) ($t_1 = 8$ ms) and in completely constant regulated variables ($t_1 = 14$ ms).

Table 4 shows the sensitivity of the same scenarios but under the additional excitation signal [see (16)] :

$$v(\bar{t}) := 0.2 \left( \sin(2\pi \bar{t} + \pi/5) \cos(2\pi \bar{t}) \right) \quad (17)$$

From the examination of tables 3 and 4, the following qualitative facts can be inferred :

- The second parameter’s combination $p_2 := \frac{R_{L}}{\pi L_c \ell}$ is very sensitive to estimation errors on $\theta$, especially those errors affecting the estimation of $\theta_1$ and $\theta_2$, that is, the flux vector. Relative errors on $\theta_1$ and $\theta_2$ may be multiplied by 3 orders of magnitude to yields relative errors on $p_2$.

- Under persistent excitations, relative estimation errors on $p_3$ are at most of the same order of magnitude than those on $\theta$ (see table 4).

- Relative estimation errors on $p_1$ are 2 orders of magnitude greater than those on $\theta_1$ and $\theta_2$.

- Some particularly singular configurations (like the one corresponding to $t_1 = 14$ ms in the last line of table 3 with relative error amplification of $10^4 \ldots$) may be "improved" by adding persistent excitation. However, this does not change the qualitative facts cited above. This is because, as mentioned earlier in this paper, even when the regulated variable are constant, the state $x = (I_{sa}, I_{sb}, \Phi_{sa}, \Phi_{sb})$ periodically time-varying and hence, persistent excitation naturally holds.

The results shown on table 5 validate the qualitative facts mentioned above. Indeed, on this table are shown the predicted $p$ computed from
| \(t_1\) | \(|S_1| := |\frac{\partial p}{\partial \theta_1}|\) | \(|S_2| := |\frac{\partial p}{\partial \theta_2}|\) | \(|S_3| := |\frac{\partial p}{\partial \theta_3}|\) |
|---|---|---|---|
| 5 ms | (14, 3572, 0.46) | (39, 4587, 1.3) | (0.3, 56, 0.03) |
| 8 ms | (29, 5557, 2.9) | (17, 1822, 7.7) | (0.9, 86.8, 0.47) |
| 14 ms | (767, 30563, 686) | (324, 15025, 288) | (16, 547, 16.2) |

Table 3. Sensitivity results for \(v = 0\)

| \(t_1\) | \(|S_1| := |\frac{\partial p}{\partial \theta_1}|\) | \(|S_2| := |\frac{\partial p}{\partial \theta_2}|\) | \(|S_3| := |\frac{\partial p}{\partial \theta_3}|\) |
|---|---|---|---|
| 5 ms | (13.6, 3442, 0.45) | (37.5, 4250, 1.0) | (0.28, 46, 0.02) |
| 8 ms | (32.7, 5821, 0.12) | (27, 2540, 0.32) | (0.24, 40, 0.03) |
| 14 ms | (38.5, 5262, 0.02) | (18, 4387, 0.4) | (0.27, 48, 0.03) |

Table 4. Sensitivity results for \(v\) given by (17)

<table>
<thead>
<tr>
<th>(t_1)</th>
<th>(v)</th>
<th>(p(t_1, \theta(t_1)))</th>
<th>(p_{LS}(t_1, 0.995 \times \theta(t_1)))</th>
<th>(p_{CLS}(t_1, 0.995 \times \theta(t_1)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ms</td>
<td>0</td>
<td>(1.00, 1.00, 1.000)</td>
<td>(0.79, 28.11, 0.993)</td>
<td>(0.93, 1.30, 1.014)</td>
</tr>
<tr>
<td>8 ms</td>
<td>0</td>
<td>(1.00, 1.00, 1.000)</td>
<td>(0.81, 31.48, 1.038)</td>
<td>(1.18, 1.30, 1.300)</td>
</tr>
<tr>
<td>14 ms</td>
<td>0</td>
<td>(1.00, 1.00, 1.000)</td>
<td>(0.10, 37.88, 0.200)</td>
<td>(1.22, 1.30, 1.300)</td>
</tr>
<tr>
<td>5 ms</td>
<td>Eq. (17)</td>
<td>(1.00, 1.00, 1.000)</td>
<td>(0.80, 26.36, 0.995)</td>
<td>(0.95, 1.30, 1.015)</td>
</tr>
<tr>
<td>8 ms</td>
<td>Eq. (17)</td>
<td>(1.00, 1.00, 1.000)</td>
<td>(0.78, 33.85, 0.999)</td>
<td>(0.99, 1.30, 1.015)</td>
</tr>
<tr>
<td>14 ms</td>
<td>Eq. (17)</td>
<td>(1.00, 1.00, 1.000)</td>
<td>(0.77, 36.84, 1.000)</td>
<td>(0.98, 1.30, 1.000)</td>
</tr>
</tbody>
</table>

Table 5. Validation of the sensitivity-based predictions

A slightly erroneous \(\theta = 0.995 \times \theta(t_1)\) according to (11) in which \(\theta_{opt}\) is replaced by either the true \(\theta(t_1)\) or the above erroneous value, namely \(\theta = 0.995 \times \theta(t_1)\). The resulted \(p\) are shown with the notations \(p_{LS}\) on table 5. The last column in table 5 shows the estimated \(p\) if a constrained least squares solver is used in order to force the scaled \(p\) to meet the following constraints

\[
p_{CLS} \in [0.5, 1.3] \times [0.5, 1.3] \times [0.5, 1.3]
\]

Note the particular sensitivity of \(p_2\) and the relative robustness of \(p_3\) (under unconstrained least squares). Note also the extreme sensitivity of all the \(p_i\)’s in the case \(t_1 = 14\) ms and \(v = 0\) that can be predicted from the last line of table 3. Finally, it may be important to note that the values (1.000) in the column denoted by \(p(t_1, \theta(t_1))\) on table 5 are effectively computed values using either constrained or unconstrained least squares. This proves that \(p\) is theoretically identifiable if the exact value of \(\theta\) is known, the errors in the computation of \(p_{LS}\) and \(p_{CLS}\) in table 5 really reflects sensitivity problems.

6. REFERENCES


5. CONCLUSION

In this paper, sensitivity analysis is carried for the problem of simultaneous estimation of state and parameter for induction machine controlled in torque and flux. This is done through the definition of sensitivity indicators appearing when putting the problem under standard separable least squares form. The qualitative results show extreme sensitivity of certain parameters to estimation error of the state going as far as multiplying relative errors by three orders of magnitude. This should incite to the caution in the analysis of works carrying the simultaneous estimation of induction machine’s state and parameters. These should explicitly handle this sensitivity problem by studying the performance of the proposed scheme under different sorts of errors (measurement errors, finite precision integration schemes, actuator uncertainties). Validations based on nominal and free-errors simulation are to be avoided.
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