Integrated Design and Control of Flexure-Based Nanopositioning Systems — Part II: Application Case Study

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Abstract: Flexure-based mechanisms contain slender beam-like modules that undergo linear elastic deflections over small ranges of motion at nanoscale resolutions. They are hence often used as bearing elements in nanopositioning systems, along with precision actuator and sensing subsystems. An integrated design and control methodology proposed in Part I of this paper proposed varying design topology and controller order for meeting performance requirements of the closed-loop controlled system. A detailed set of steps was given for meeting requirements such as a desired static or dynamic load-capacity, bandwidth, or range of motion. In this part of the paper, an application case study for a practical precision positioning and alignment system is worked out to illustrate the steps involved in using the proposed methodology. The details of optimization problem formulation and solutions for design and control are presented. The outcome of the exercise is a novel design topology, with it shape and size optimized, and a controller synthesized such that a desired control bandwidth and design requirements of strength and modal separation are met.

Keywords: Flexure-based mechanisms, Nanopositioning, Topology Generation, Synthesis.

1. INTRODUCTION

Precision positioning applications built around conventional bearings (such as sliding contact or rolling contact bearings) are often hindered by friction, backlash, hysteresis, and other motion non-linearities. Flexure-based mechanisms rely inherently on the beam-like material behavior, and can be designed to show linear elastic behavior free of such motion non-linearities, allowing for nanoscale resolutions over small ranges of motion on the order of a few millimeters.

In Part I of this paper, a novel methodology integrating design and control considerations was presented. The key distinction of this “co-design” approach is that the design is iterated over topologies and not just parameters within a selected topology. The topology generation is aimed as a valuable addition to the design toolkit, facilitating novel designs that could not have been conceived otherwise. The parameters within any particular topology could be adjusted at a subsequent phase through a detailed shape and size optimization. Further, a novel controller parameterization is used to vary the controller order while directly tuning the sensitivity function to a desired form.

In this Part II of the paper, we detail an application case study of a precision positioning and alignment system containing a flexure module driven by a precision actuator such as a piezoelectric actuator, so that a desired set of performance requirements are met. The rest of the paper is organized as follows. An overview of the problem is presented in Section 2. Application of detailed steps of the integrated design and control methodology is described in Section 3. Simple lumped parameter models used for deriving parametric relations between performance requirements and parameters of topologies are discussed in Section 4. An optimization problem formulated for the system is presented in Section 5 and the results are discussed in Section 6, highlighting the demonstrated advantages of varying the topology and the controller order in the proposed “co-design” approach. The paper concludes with a summary of contributions and directions for future research in Section 7.

For simplicity of illustration of the methodology, a simple one-degree-of-freedom system is studied in this paper. Application of the presented methodology to generate multi-degree-of-freedom flexure-based nanopositioning systems is detailed in [9]. In this reference, we report a prototype flexure-based XYZ scanner has been developed, fabricated, and tested for rapid scanning in atomic force microscopy applications.

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2. PROBLEM OVERVIEW

In this section, we work out in simulation an example application to illustrate the integrated design and control methodology presented in Part I of this paper and highlighted briefly above. A simple, yet practical, positioning system example with a flexure-based mechanism driven by a piezoelectric actuator is considered. A broad overview of the rest of the section is as follows. We present a generic problem description, and then specify a set of critical performance requirements for the problem. The methodology is applied to first generate a set of design concept topologies. Based on design screen tests, a few topologies are ruled out. An optimization problem is formulated in terms of a desired cost function and a set of physical constraints. Design topologies passing the screening test are then input to the optimization problem. A MATLAB-based optimizer is used to fine-tune the shape and size of topology candidates. If the design or control cost function cannot be physically obtained, an optimal solution cannot be obtained and the topology is discarded.

2.1 Description

Consider the precision positioning problem shown schematically in Fig. 1. The objective is controlling a relative separation \( z \) between a moving surface and a fixed surface to form a controllable gap. Such a gap can be used to study physical phenomena at sub-micron scales, such as radiative heat transfer or force interactions such as Casimir forces that occur between metals [2]. Other example applications include size-based filtration for macromolecular separations [1] and characterization of electrochemical properties of gas and liquid molecules [9].

A piezoelectric stack actuator with a lever amplification mechanism is suggested for generating a large displacement range on the order of 100 \( \mu m \) required for the gap \( z \). A schematic diagram showing the concept of a lever mechanism with piezoelectric actuator is presented in Fig. 2. The piezoelectric stack, shown as generating an input displacement \( y_{in} \), pushes a lever at a distance \( L_s \) away from its pivot. The gap is formed at the distal end, a distance \( L_a \) away from the pivot, where the lever output displacement \( y_{out} \) is sensed with a laser interferometer. For small-angle motions of the lever about the pivot, the output displacement \( y_{out} \) for a input displacement \( y_{in} \) is given as

\[
y_{out} = y_{in} \frac{L_s}{L_a}
\]

Fig. 1. Schematic diagram showing a positioning system example. The goal is to vary the gap \( z \) over a large range of motion and control bandwidth.

Our goal here is to illustrate the design and control methodology for the positioning system conceptualized in Fig. 2, using flexure-based mechanisms in the design to for the pivot. Unlike friction-based bearings, flexure-based bearings are ideal candidates for the pivot owing to their smooth elastic motion and minimal nonlinearities such as backlash or hysteresis.

Before we proceed any further, we need to make a few assumptions for the relevant parameters. First, we assume the piezoelectric stack actuator has a blocking force of \( F_{max} = 850 \) N and free deflection \( y_{piezo,max} \) of 18 \( \mu m \). The static force-deflection characteristic of the actuator is shown in Fig. 3. In our design, we use two piezoelectric stack actuators held mechanically in series, so that their displacements add up to cause the net displacement input. For a maximum displacement \( y_{in} \) of 18 \( \times \) 2 = 36 \( \mu m \), we need to meet a target of 100 \( \mu m \) at the output. Let us assume reasonable values for the distance of the sensor from the pivot \( L_s = 2 \) in, and distance from actuator to pivot \( L_a = 0.5 \) in. This results in an amplification ratio of 4, and a resultant maximum output displacement \( y_{out} \) of 144 \( \mu m \), which satisfies our target displacement of \( y_{desired} = 100 \mu m \). Basing on the stiffness of the structure, the applied force may vary, and the net displacement input can be smaller, so the extra buffer of 44 \( \mu m \) is desirable.

Note that the simple model of Fig. 2 also depicts the simplified dynamics of a disk drive actuator subsystem example given in [4]. As we will be discussing towards the end of this chapter, in the example of [4], the geometry of a design is altered to improve control performance. Therein, changing the geometry involves changing the parameters within a selected topology. Here, as an alternative approach, we explore the option of varying the design topology to improve on the control performance. The case in which parameters within a design topology are varied is covered in our broad methodology.

2.2 Problem Statement

The problem statement for applying the proposed integrated design and control methodology to the example of the positioning system of Fig. 2 is as follows:

Given a lever amplification mechanism of Fig. 2 with the following parameters:

(i) output displacement \( y_{out} \) measured at a distance \( L_s = 2 \) in from the pivot.

2 As explained in Section 4, where we discuss the dynamics of a few designs represented by the simple model of Fig. 2, we motivate the need for altering the design topologies so as to move the nonminimum phase zero outside the range of frequencies of interest. Our approach of integrated design and control is implemented for achieving this feature. In the example of disk drive actuator system given in [4], altering geometry of the given topology eliminates nonminimum phase zeros. In a actual multi-DOF system, given many constraints on geometry, and design requirements, both (i) varying parameters within a topology and (ii) varying the topology (and parameters within each topology) should be explored. As we have seen earlier, the integrated design and control methodology applied in Section 3 covers both these cases.
Fig. 3. Typical static force-deflection characteristic curve of a piezoelectric stack actuator. The piezoelectric stack actuator we select in this application has a maximum force capacity (blocking force) $F_{\text{max}} = 850 \text{ N}$ and free deflection $y_{\text{max}}$ of 18 $\mu\text{m}$.

(iii) input displacement $y_{\text{in}}$ provided at a distance $L_a = 0.25 \text{ in}$ from the pivot.

(iv) a piezoelectric stack actuator with a blocking force $F_{\text{max}} = 850 \text{ N}$ and free deflection $y_{\text{piezo,max}}$ of 18 $\mu\text{m}$.

**Design a flexure-based pivot that meets the performance requirements given in Table 1.**

### 3. IMPLEMENTATION OF METHODOLOGY

Given the above parameters for the lever and the piezoelectric stack actuator, we examine the topology, shape-size optimization and control performance of the system when a flexure-based mechanism is used as a pivot for the lever. We now follow the steps of the methodology as presented in Section 3 for the integrated design and control of the flexure-based pivot. For simplicity, we restrict our interest to planar implementations, which can be manufactured relatively easily on an abrasive waterjet or a wire-EDM.

Table 1. Specifications for 1DOF flexure-based positioning system example.

<table>
<thead>
<tr>
<th>Desired Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range of motion, max ($y_{\text{out}}$)</td>
<td>$&gt; 50 \mu\text{m}$</td>
</tr>
<tr>
<td>Control bandwidth</td>
<td>$&gt; 1 \text{ kHz}$</td>
</tr>
<tr>
<td>$z_{\text{parasitic,max}}$</td>
<td>$&lt; 1 \mu\text{m}$</td>
</tr>
<tr>
<td>Fatigue Performance</td>
<td>Infinite life, i.e. $\approx 10^7 - 10^9$ cycles or more</td>
</tr>
</tbody>
</table>

Step 1: Performance specifications: The specifications for the positioning are as given in Table 1.

Step 2: Design Topology Library Generation: A set of topology concepts derived for flexural pivots are shown in Fig. 5. The idea is to use one of these pivots in the amplification mechanism shown in Fig. 2. One design topology using the flexure-based mechanism in Fig. 5 (c) as a pivot is shown in Fig. 4.

![Fig. 4. Design topology showing a flexure-based mechanism as a pivot for the lever amplification mechanism.](image)

The candidate topologies of Fig. 5 were generated as follows. The concepts shown in Fig. 5 (a) and (b) are simple examples of a rotational joint achieved with a lumped rotational compliance. While the notch flexure joint in Fig. 5(a) has a localized compliance around its neck, the beam flexure of Fig. 5(b) has a compliance distributed over its length.

The rest of the design topologies shown in Figs. 5(c)-(j) are obtained as follows. First, we start with a beam flexure as a primitive used to suspend a mass. This primitive is shown in Fig. 6(a). To improve on the load-capacity and fatigue performance of the primitive, we add a beam flexure on the other side of the mass. This enhanced primitive is shown in Fig. 6(b).

Note, however, that there is a second-order effect of over-constraint (the beams fighting with each other), which can be minimized with suitable geometry (for example, longer beam length). An improvement which eliminates the over-constraint is a parallelogram flexure in a folded back configuration [8]. For simplicity, this alternate primitive is not considered in this example.

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$^3$ Both of these are high strength requirements. A large range of motion can be obtained for the same load-capacity with redundant replication of the flexure constraint, instead of reinforcing a single constraint.
Further, note also that the primitive flexure choice is not unique to a problem. We converged at the design topologies presented in Fig. 5 (c)-(j) starting with a beam flexure of Fig. 6(a) as the primitive flexure. An alternate primitive flexure that can also be considered is a notch flexure. An example double sided notched flexure equivalent of Fig. 6 (b) is shown in Fig. 7. Unlike the beam flexure which has continuous distribution of compliance, the notch flexure has a localized compliance at the thin necks of the notch.

The concept topology shown in Fig. 5(c) is the double-sided beam flexure primitive of Fig. 6(b). With a force applied at an offset from the center of the mass, a rotational motion can be imparted. The pivoting action achieved with this topology is schematically shown in Fig. 4.

The concept of topology shown in Fig. 5(d) has the same mass now suspended on a parallel stacking of two sets of beam flexures on either side. On similar lines, the concept of topology shown in Fig. 5(e) and (f) have the same mass now suspended on a parallel stacking of three and four sets, respectively, on either side of the mass.

The concept topology shown in Fig. 5(g) is similar to that of Fig. 5c, but with a rod flexure pinning down the mass at its center of rotation, and hence curbing the trampoline-like $z$-mode. Since the rod flexure has a large rotational compliance compared to axial compliance, this topology corresponds to a large modal separation $S_{Z,\theta}$ between the rotational and vertical DOFs.

Step 3: Design Topology Screening: Screening criterion for topologies can be decided according to the needs of the particular application under consideration. There is no unique way to select a screening criterion. Of many possible screening criteria to select the ideal topologies from the candidate topologies of Fig. 5, we select the following criterion that targets the specification of minimal lateral motion errors:

$$S_{X,Z} \gg 1$$

where $S_{X,Z}$ specifies the modal separation of the fundamental $Z$ vibration mode from the $X$ (lateral) vibration mode. We define the modal separation index between any two modes as a ratio of natural frequencies of the modes; a judicious choice of flexure constraints is implemented to maximize the modal separation to minimize parasitic motion errors. The design topologies in Fig. 5(a) and Fig. 5(b) are more compliant in lateral direction than the vertical direction and hence do not satisfy this criterion.

In other words, for a slight vertical misalignment of the piezoelectric stack actuator, there would be a horizontal force component that will likely cause a large lateral error owing to the small lateral stiffness.
of the design topologies in Fig. 5(a) and Fig. 5(b). Hence, these two design topologies are eliminated.

In contrast, the high lateral stiffness of the beam flexures in Fig. 5(c)-(j) result in a high modal separation with the lateral DOF occurring at much higher frequencies than the vertical DOF. Hence, these ten candidates are passed to the subsequent shape and size optimization.

Step 4: Controller Selection/Screening: Of all possible stabilizing controllers, we screen for those that ensure both (i) good command following over frequencies up to $1 \text{kHz}$ and (ii) steady state error of zero for a step input. As will be shown later in Section 4, the plants corresponding to the five nominal design topologies, or plants, have no free integrators. Hence, it is imperative for the controller to have a free integrator in order to satisfy the screening criteria. Many nominal controllers can be constructed to satisfy this screening criterion, such as an integral controller, a proportional-integral controller, a lag controller, and other higher order controllers that have at least one free integrator. In this example, for simplicity, we select a simple integral controller as the nominal controller as given below:

$$C_0(s) = \frac{k}{s} \quad (3)$$

where $k$ is a nominal gain selected for the given plant to ensure stability of the nominal closed-loop system.

Step 5: Optimization: We follow a sequential approach with the design optimized first, and the optimized design passed to the controller optimization routine. The formulation of the optimization and discussion of optimization results are presented in Sections 5-6. Before we proceed any further, we need to derive lumped parameter models and extract parametric relations needed for the optimization problem formulation.

4. LUMPED PARAMETER MODELING

A lumped parameter model for the design topologies using flexure-based pivots of Figs. 5(c)-(f) is shown in Fig. 8. In this model, the flexure-based pivot is shown as a lumped mass suspended on its either side by flexures having a lumped linear stiffness $k_y$ and a rotational stiffness $k_\theta$. The pivoting point, or the instantaneous center of rotation of the system is the center of the mass, denoted by $C$ in the figure. For a downward deflection $y$ of the center $C$ and an angle of rotation $\theta$ of the mass and the lever (about an axis $Z$ perpendicular to the page and passing through $C$), the equations of motion for the system are:

$$m\ddot{y} = f - 2b_y \dot{y} - 2k_y y \quad (4)$$
$$J\ddot{\theta} = \tau - 2b_\theta \dot{\theta} - 2k_\theta \theta \quad (5)$$
$$f = f_{max} - \frac{f_{max}}{y_{max}} y_{in} \quad (6)$$

where $m$, $J$ are the total mass and moment of inertia about $Z$ axis passing through $C$, $\tau = fL_a$ is the moment applied by the force $f$ applied by the piezoelectric stack actuator. Assuming lightly damped harmonics, damping factors $b_y$ and $b_\theta$ denoting small damping in the flexures are used.

From the kinematics, as shown in Fig. 9, the output displacement $y_{out}$ in terms of the downward deflection $y$ of the instantaneous center of rotation and the angle of rotation $\theta$ is given as below:

$$y_{out} = -y + L_a \theta \quad (7)$$

Fig. 8. Lumped parameter model for depicting dynamic behavior of topology concepts using flexure-based mechanisms of Fig. 5(c)-(f) as pivots in the 1-DOF positioning system.

Fig. 9. Schematic diagram showing kinematic relation between the output displacement $y_{out}$, the downward displacement $y$ of at the center of rotation $C$ and an angle of rotation $\theta$. For small angle motions, since $y_{out} + y = L_a \theta$, we have $y_{out} = L_a \theta - y$.

The transfer function $\frac{Y_{out}(s)}{F(s)}$ between the applied force input $F(s)$ from the piezoelectric stack actuator to the output displacement $Y_{out}(s)$ is given by:

$$Y_{out}(s) = \left\{ \begin{array}{l} \frac{1}{ms^2 + 2b_y s + 2k_y} L_a \\ \frac{L_s}{Js^3 + 2b_\theta s + 2k_\theta} \end{array} \right.$$  

The first term in Eq. (8) corresponds to the contribution of the fundamental vertical ($y$) mode of the flexural pivot as seen at the output displacement measurement. Similarly, the second term corresponds to the contribution of the fundamental rotational ($\theta$)
mode of the flexural pivot. Note the negative sign premultiplying the vertical mode. This means that, at the output, the difference of these two modes is being measured.

A lumped parameter model for the design topologies using flexure-based pivots of Figs. 5(g)-(j) is shown in Fig. 10. This model is the same as the model of Fig. 8 except for the enhanced lumped stiffness components at the center A. The rod flexure adds a high vertical stiffness \( k_{0y} \), and a mild rotational stiffness \( k_{0\theta} \). The equations of motion for the enhanced system are:

\[
\begin{align*}
    m\ddot{y} &= f - b'_y \dot{y} - k'_y y \\
    J\ddot{\theta} &= \tau - b'_\theta \dot{\theta} - k'_\theta \theta \\
    k'_y &= k_y + k_{0y} \\
    k'_\theta &= k_\theta + k_{0\theta} \\
    y_{out} &= -y + L g \theta \\
    f &= f_{max} - \frac{f_{max}}{y_{in}} \theta
\end{align*}
\]

After applying Laplace transforms to the equations of motion, the transfer function \( \frac{Y_{out}(s)}{F(s)} \) between the applied force input \( F(s) \) from the piezoelectric stack actuator to the output displacement \( Y_{out}(s) \) is given by:

\[
\frac{Y_{out}(s)}{F(s)} = \left\{ \frac{1}{ms^2 + b'_y s + k'_y} \right\} + L g \left\{ \frac{L_o}{Js^2 + b'_\theta s + k'_\theta} \right\}
\]

5. OPTIMIZATION: PROBLEM FORMULATION

Optimization parameters

For the flexure-based pivots of Figs. 5(c)-(f), the optimization parameters are selected as the length \( l \) and thickness \( h \) of the beam flexures. For the case of Fig. 5(g)-(j), another variable, the length \( l_0 \) of the rod flexure is also considered.

Constraints

The geometry/dimensional bounds on the parameter for a given footprint of the flexure-based pivot include:

\[
0.25 \text{ in} \leq l < D_{max} - 2L_o = D_{max} - 1 \text{ in} \\
0.05 \text{ in} \leq h \leq 0.3 \text{ in} \\
0.25 \text{ in} \leq l_0 \leq 2 \text{ in}
\]

Let \( \sigma \) be the maximum stress in the beam flexures, \( \sigma_{r} \) the maximum stress in the rod flexure, \( y_{in} \) the displacement input from the piezoelectric stack actuator, \( y_{out} \) is the output displacement, and \( x_{out} \) the lateral motion error. The constraints used in the optimization are:

\[
\text{Constraint} : \quad \sigma < \sigma_{max} \quad (16)
\]

\[
\sigma_{r} < \sigma_{max} \quad (17)
\]

\[
y_{desired} < y_{out} \quad (18)
\]

\[
y_{in} < y_{piezo,max} \quad (19)
\]

\[
x_{out} < x_{parasitic,max} \quad (20)
\]

where \( \sigma_{max} \) is the maximum allowed stress in the material, \( y_{desired} \) is the 100 \( \mu \text{m} \) output displacement requirement, \( y_{piezo,max} \) is the maximum piezoelectric stack deflection, and \( x_{parasitic,max} \) is the maximum allowed parasitic lateral displacement. In order that the mechanism can withstand an infinite number of stress cycles, the maximum stress \( \sigma_{max} \) is defined for maximizing the output displacement and minimizing the lateral error motion as follows:

\[
\text{Cost} : \quad 0.5 \frac{y_{out}}{y_{min}} - 0.5 \frac{x_{min}}{x_{out}} \quad (21)
\]

Using parametric relations derived using the models of Section 4, a constrained minimization problem was set up in MATLAB using \textit{fmincon} with an optimization parameter vector \( U = [l; \ h; \ l_0] \) with an initial guess \( U_0 = [0]; \ h^0; \ l_0^0] \), and the bounds \( U_{min} = [0.25 \text{ in}; \ 0.05 \text{ in}; \ 1 \text{ in}] \) and \( U_{max} = [D_{max} - 2L_o; \ D_{max} - 2L_o; \ 2 \text{ in}] \), the cost function given in Eq. (21), the constraint conditions given in Eq. (20) using the constrained minimization solvers of MATLAB.

The details of the controller optimization are as follows:

Control Parameter

A control parameter to tune in our optimization is the parameter transfer function \( Q(s) \).

Cost function

For good command following we choose a weight \( W_{c}(s) \), such that a norm, say the \( \infty \)-norm, of the weighted sensitivity transfer function \( W_{s}(s)S(s) \) is optimized as follows:

\[
\text{Cost} : \quad 0.5 \frac{\| W_{s}(s)S(s) \|_{\infty}}{W_{c}(s)S(s)} \quad (22)
\]
Table 2. Results of Optimization of Design and Control for the case of topologies of Fig. 5(g)-(j) used as flexure-based pivot.

<table>
<thead>
<tr>
<th>Topology</th>
<th>Parameter Values</th>
<th>Range (μm)</th>
<th>Control Bandwidth (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 5(g)</td>
<td>ℓ = 1.5 in, h = 1.25 in,</td>
<td>124.5</td>
<td>1194</td>
</tr>
<tr>
<td></td>
<td>t₀ = 1.00 in</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fig. 5(h)</td>
<td>ℓ = 1.5 in, h = 0.05 in,</td>
<td>141.1</td>
<td>1194</td>
</tr>
<tr>
<td></td>
<td>t₀ = 1.00 in</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fig. 5(i)</td>
<td>ℓ = 1.5 in, h = 0.05 in,</td>
<td>139.7</td>
<td>1194</td>
</tr>
<tr>
<td></td>
<td>t₀ = 1.00 in</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fig. 5(j)</td>
<td>ℓ = 1.5 in, h = 0.05 in,</td>
<td>138.4</td>
<td>1194</td>
</tr>
<tr>
<td></td>
<td>t₀ = 1.00 in</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
|W_s(s)|\infty \leq 1 \tag{22}
\]

The weight \(W_s(s)\) is chosen as [7]

\[
W_s(s) = \frac{(s + M\omega_d)(s + fM\omega_d)}{s(s + fM\omega_d)} \tag{23}
\]

with \(M = 1.5, f = 10,\) and \(\omega_d = 1.5\ kHz\) is the desired closed-loop system bandwidth. To make the controller \(C(s)\) obtained with this choice of \(Q(s)\) strictly proper, a filter with two first-order poles at \(200\times\omega_d\) is used to ensure a roll-off at high frequencies.

6. RESULTS AND DISCUSSION

The results of the optimization are presented in Table 2. An optimal solution was found for the case of flexure-based pivots of Fig. 5(g)-(j) for both the design and control optimization problems. A grid of 1000 points uniformly spread in the three-dimensional optimization parameter space \(U_{\text{min}} \leq U \leq U_{\text{min}}\) were each selected as an initial guess for the optimization. The values of the optimization parameters that gave the best optimal solution, i.e. lowest cost function value with no violation of constraints within a numerical tolerance

The optimization problem resulted in an infeasible solution in both design and control problems for the case of flexure-based pivots of Fig. 5(c)-(f). Note that the infeasible solution was an outcome of optimization at each of the 1000 grid points chosen in the three-dimensional optimization parameter space \(U_{\text{min}} \leq U \leq U_{\text{min}}\) for a maximum number 200 sequential quadratic programming iterations at every grid point.

The design topologies of Fig. 5(c)-(f) turn out to be infeasible, the reason for which is discussed as follows. First, it can be shown that these topologies are hindered in their control performance by the presence of non-minimum phase zeros.

Fig. 11 shows a typical pole-zero plot for the open-loop plant. Size or shape-optimization accomplished by varying flexure length \(f\) or thickness \(h\) cannot move the open-loop zeros out of the right half plane.

The presence of the right half-plane zero limits the bandwidth of these design topologies to about half the frequency of the zero, and hence the desired bandwidth of \(1\ kHz\) cannot be achieved [7]. The limitation on the bandwidth can be explained from a root-locus viewpoint: higher controller gains resulting from high bandwidth requirements will result in the closed-loop system poles moving toward the right-half-plane zero, and hence result in instability. To ensure stability, the gains have to be limited, and hence the bandwidth has to be limited.

The bound on the bandwidth for the topologies achieved with the flexure-based pivots of Figs. 5(c)-(f) can be derived as follows [7]. Let \(\omega_0\) denote the right half plane zero of the system. Since the system has no right half plane poles, the sensitivity transfer function \(S(s)\) should obey the following constraint:

\[
|W_s(\omega_0)S(\omega_0)| = |W_s(\omega_0)S(\omega_0)| < 1 \tag{26}
\]

Using the weighting filter \(W_s\) given in Eq. (23), the above inequality assumes the form given below:

\[
\left|\frac{(\omega_0 + M\omega_d)(\omega_0 + fM\omega_d)}{\omega_0(\omega_0 + fM\omega_d)}\right| < 1 \tag{26}
\]

with \(M = 1.5, f = 10,\) an upper bound on the achievable control bandwidth \(\omega_d\) is:
For the flexure-based pivots of Fig. 5(c)-(f) the maximum value for the frequency of the non-minimum phase zero was found to be \( \max(\omega) = 944.49 \) Hz over the optimization parameter space, which then results in a value for the maximum achievable bandwidth from Eq. (27) to be 251.73 Hz. This value is much lower than our target bandwidth of 1000 Hz and hence these design topologies cannot meet the control performance requirements. Further the design constraint of infinite fatigue life were found in the optimization to be too stringent on the beam flexures. Since the central rod flexure is not available in these topologies all the applied load is taken by the beam flexures, which have a limited stress handling capability owing to the material yield limit. Hence, the flexure-based pivots of Fig. 5(c)-(f) need to be discarded in our integrated design and control methodology.

One strategy to tackle the non-minimum phase zero is to move it far beyond the frequencies of interest.

In the topologies of Figs. 5(g)-(j) the rod flexure stiffening the trampoline-like y mode of the pivot allows for pushing the non-minimum phase zeros on to the imaginary axis. The resulting system is minimum-phase with no bandwidth limitations imposed by their presence.

Of all the designs, the design topology with four beams in Fig. 5(h) has the largest vertical range. The design topology of Fig. 5(j) has the lowest lateral displacement error since it has the largest stiffness in X direction. Surprisingly, the design topology of Fig. 5(g) does not have the largest vertical range. This result is not obvious, and the optimal parameter vector is different from the rest of the topologies. One possible reason is that since there are few beam flexures to carry the load, the stress in the material is a limiting factor. This in fact is reflected in the optimal width of \( h \approx 1.25 \) in for this topology, as against \( h \approx 0.65 \) in for the rest of the topologies.

All the design topologies of Fig. 5(g)-(j) meet the control performance requirement of 1000 Hz bandwidth. The control performance of the design topology with the the flexure-based pivot of Fig. 5(j) is shown in Fig. 12 in terms of the sensitivity transfer function. The nominal sensitivity transfer function resulting from a nominal controller \( C_0(s) = \frac{1000}{s} \) has a low bandwidth, while the desired sensitivity has a bandwidth of 1000 Hz, a rolloff of 40db/dec. Under a novel control parameterization approach, with a model-matching procedure, the controller parametric transfer function \( Q(s) \) was designed such that the sensitivity transfer function closely achieves the desired sensitivity transfer function of \( \frac{1}{W_r(s)} \). Hence,

\[
Q(s) = \frac{Q_{num}(s)}{Q_{den}(s)}
\]

\[
Q_{num}(s) = s^4 + 2.121 \times 10^5 s^3 + 1.73 \times 10^8 s^2 + 3.665 \times 10^{13} s + 3.716 \times 10^{10}
\]

\[
Q_{den}(s) = s^4 + 1.55 \times 10^5 s^3 + 2.172 \times 10^8 s^2 + 2.668 \times 10^{13} s + 3.445 \times 10^{17}
\]

To ensure a roll-off behavior for the resulting controller, a 2-pole low pass filter with coincident poles at \( s = -200 \omega_d \) was multiplied with the controller. The resulting sensitivity transfer function matches well with the desired sensitivity transfer function as shown in Fig. 12.

For comparison, a robust controller designed with a mixed-sensitivity criterion (allowing for tuning both sensitivity and complementary sensitivity) was also simulated, using mixsyn routine in MATLAB Robust Control Toolbox. The sensitivity transfer function obtained with this approach shows in Fig. 12 a flat profile at low frequencies and a roll-on of 40 dB/dec starting at about 0.1 rad/s.

The peak response of the sensitivity transfer function obtained with our method has about 2 dB taller peak than obtained from the mixed-sensitivity method, which implies a relatively poorer robustness to uncertainties in the positioning system. The closed-loop system sensitivity developed with our method shows a bandwidth of about 1194 Hz, while that of the mixed-sensitivity approach shows 1430 Hz. The mixed-sensitivity approach resulted in a controller of 8th order, and has a lower peak in the sensitivity transfer function, indicating better robustness, which can be owed to the higher order of this controller.

With performance comparable to a well-established routine like the mixed-synthesis controller of MATLAB Robust Control Toolbox, the controller parameterization introduced here allows for tuning directly the sensitivity transfer function, which plays an important role in addressing lightly damped harmonics of flexible structures. The details of the controller parameterization are not covered here and will be part of a future paper from our group.

In a nutshell, we have converged at a final design topology that meets the specified performance requirements of a bandwidth of greater than 1000 Hz and a range of motion exceeding 50 \( \mu m \), with infinite stress-cycle life. Further, we have a systematic procedure to develop the topologies, screen them for desired features, and optimize them while dealing with dimensional and material constraints. An outcome of this exercise is the apriori identification of non-minimum phase zeros in flexure-based mechanism designs. Non-minimum phase zeros occur whenever non-collocated actuator and sensor arrangements are implemented. Avoiding the non-minimum phase zero may require reconsidering where to measure relative to where we actuate the system. In our case, we chose not to vary the actuator or sensor location [6], or change the geometry.
of the design [4], but rather design the mechanism to be stiffened beyond the bandwidth of interest while still meeting the desired motion requirements. This was possible because of judicious design of topology in terms of flexure constraints, while satisfying strength and dynamic performance requirements.

7. SUMMARY

In this paper, we presented a flow chart for iterating on design (plant) and controller to achieve a desired closed-loop system specification. It is emphasized that iterating a design is not just about fine-tuning shape and size of a particular design configuration. Instead, we need to iterate over design topologies and controller order. An example of a flexure-based 1-DOF positioning system was worked out to show the integrated design and control methodology. Parametric relations were derived from lumped parameter models to formulate an optimization problem over the design space and the control performance space. The methodology was worked out step-by-step to cover (i) generation of design topologies (ii) screening of topologies for obvious design choices that cannot work for the given application, (iii) optimization formulation in terms of design parameters, cost functions, and equality and inequality constraints, and (iv) controller generation based on model-matching of a sensitivity transfer function. The infeasibility of a set of topologies was explained by the presence of non-minimum phase zeros that limit the achievable control bandwidth. Based on the intuition gained from this exercise, we also suggested a new screening guideline for checking for non-minimum phase zeros possible for a design topology along with an actuator and sensor placement. This example provides a practical 1-DOF positioning system application and provides a guided approach to converge at a novel design topology that meets all the requirements.

REFERENCES


Fig. 12. Comparison of magnitude response of desired and achieved sensitivity transfer function designed with a model-matching matching procedure. A corresponding response obtained for the case of mixed-sensitivity design is also shown.