Pass Balancing Switching Control of a Four-passes Furnace System

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Abstract: Pass balancing control of industrial multi-passes furnace systems, which are widely used in petroleum refineries, is very necessary to improve the product quality, economic efficiency, and plant safety. From considerations of reducing the time of the flowrate valves being regulated and hence prolonging their lifespan, a switching based control technique has been suggested to address the related problems for multi-passes furnace systems. This paper further studies the SDCT technique, with an emphasis on the balanceability of the switching control policy. The principle of the technique and a switching control policy are first introduced, the balanceability problems, which concerns whether or not a furnace system could be balancing controlled by a switching policy, are then elaborated, and finally a sufficient condition for the balanceability of the control policy for four-passes systems is obtained.

Keywords: Pass balancing control, Multi-passes furnace systems, Outlet temperature control, Algebraic constraint, Generalized switching control, Balanceability.

1. INTRODUCTION

An industrial furnace with multiple passes is commonly used in petroleum refineries to improve the heat transfer efficiency and to reduce the possibility of coke formation (Cheng et al., 1999, Friedman 1994, Garg 1999, Herzog 2004, Li et al. 1994, Wang & Zheng 2005, 2006, 2007, Zhang et al 2008). As shown in Fig. 1, the crude oil enters the furnace through N parallel passes, and the oil flowrates $f_i$ to $f_N$ come from the same source $F_T$, total oil inlet flowrate, and hence they satisfy the algebraic constraint

$$\sum_{i=1}^{N} f_i = F_T.$$  (1)

For the product quality, economic efficiency, and plant safety considerations, the control measures are needed to keep the outlet temperatures of multiple parallel passes as identical as possible. It can be found from (1) that, in order to obtain a uniform outlet temperature, regulating the inlet flowrate of certain pass must regulate that of some other pass or passes simultaneously to keep (1) satisfied, that is to say, control loops among the multiple passes are seriously coupled. The difference control technique (DCT) has been proposed to dynamically distribute the inlet flowrates among the passes to maintain the outlet temperatures uniformity (Li et al., 1994, Wang & Zheng, 2005) and applied to a furnace with four passes successfully (Wang & Zheng, 2005). The basic idea of the DCT technique is that, the difference of the two outlet temperatures is controlled to be zero, and the output of the single input single output (SISO) controller is the flowrate deviation, which is added to the flowrate setvalue of one pass whose outlet temperature is high and at the same time subtracted from that of another whose temperature is low. Thus, the two outlet temperatures can be controlled using just one SISO controller, and its output does not change the sum of the two flowrates, which resolves the flowrate algebraic constraint (1) effectively and conveniently.

The DCT technique transforms the pass balancing control problem for a furnace system with two parallel passes to a conventional single variable control problem. However, for a furnace with N parallel passes, there would need iteratively employing the DCT technique $N$ times to realize the pass balancing control, which is boring especially when $N$ is large. In addition, in some cases where $N=3$ or $N=3·2^n$, $n = 1, 2, …$, such iterative employment of the DCT technique might result in the problem that the inlet flowrate coupling cannot be decoupled properly. In order to overcome the mentioned shortcomings of the DCT technique, Wang & Zheng (2006, 2007) have proposed a generalized version of the difference control technique, called differences control technique (DsCT).

For both of the DCT and the DsCT techniques, all the N inlet flowrate valves are regulated through the whole time horizon, which is disadvantageous for the lifespan of the flowrate valves. Considering that, in recent years, the study of switched systems has obtained many research results and the related switching control techniques have been widely applied to engineering practices (Lenhartson et al 1996, Morse 1997, Engell et al 2007, Li & Guan 2001, El-Farra & Christofides 2003, Wang et al 2003, Jin & Huang 2010), a switching control technique, called SDCT technique, has been suggested to control the N parallel passes in a time sharing manner to reduce the regulation time of the flowrate valves and hence to help prolong their lifespan (Wang et al 2008). This paper further studies the SDCT technique, with an emphasis on the balanceability of the switching control policy for the four-passes systems.

The rest of the paper is organized as follows. In Section 2, the SDCT technique related problems are introduced. Section 3
formulates the balanceability problems for balancing control of furnace systems with multiple parallel passes, investigates the balanceability of a switching control policy, and obtains a sufficient condition for the policy being balanceable. Finally, some concluding remarks are presented in Section 4.

2. SDCT TECHNIQUE AND SWITCHING POLICY

2.1 SDCT Technique

Considering that the DCT technique can be employed for balancing control of the two parallel passes (Wang & Zheng 2005) and it is hoped to reduce the regulation time of the flowrate valves for prolonging their lifespan, a switching control scheme is suggested as follows: at any given instant, just some two passes are selected from the \( N \) passes for balancing control using the DCT technique, leaving the other passes in no control status temporarily. With time evolving and based on certain feedback policy, different passes are dynamically chosen for balancing control so that all the \( N \) passes can be controlled in a time sharing manner. Thus, at any given time interval there are two and only two passes being controlled, and hence the regulation times of all the \( N \) flowrate valves are reduced significantly as a whole. The related technique is here called the switching difference control technique (SDCT) (Wang et al 2008).

The SDCT control system is a switched system, where the basic switching unit is a subsystem, denoted as \( DCT \), that consists of two parallel passes and an SISO controller. A schematic diagram of the \( DCT \) subsystem that is composed of Passes 1 and 2, and controller \( C_{12} \) is given in Fig. 2. The controller in the subsystem can be called “server”, and when the subsystem consists of Passes \( i \) and \( j \), the location of the server is said to be \( \text{Loc}(i, j) \). Once one or two passes leave the subsystem, it is said that the location of the server switches, and hence the SDCT system is a generalized switched server (GSS) system (Wang & Zheng 2006, 2009, Wang 2008). An SDCT control system with the server location being \( \text{Loc}(1, 2) \) is shown in Fig. 3.

For a subsystem \( DCT \), the difference of the two outlet temperatures \( TD \) is controlled by a continuous time controller with the reference input being zero, and the control objective is to make the two temperatures identical. The output of the controller is added to and subtracted from the two input setvalues simultaneously, which makes the sum of the two inputs do not change with the controller output. Fig. 2 shows a subsystem \( DCT_{12} \), where the inputs of the two passes are given by

\[
\begin{align*}
  f_1 &= f_{s1} + \Delta f \\ 
  f_2 &= f_{s2} - \Delta f
\end{align*}
\]

(2)

2.2 Switching Control Policy

Wang & Zheng (2006, 2009) have proposed some switching control policies for the SDCT control systems. For convenience of narration for the balanceability analysis of the switching policy, the FRR (first balancing control for some two passes and then round-robin switching) policy is briefly reviewed as follows: the current subsystem switches when it reaches the pass balancing status, and the subsystem switches in a round-robin manner when it switches. That is, at initial time \( t = T_0 = 0 \), some two adjacent passes, say Passes 1 and 2, are selected for balancing control. At time \( T_n \), \( n = 1, 2, \ldots \), assume that the indices of the two parallel passes that are selected for control are \( i \) and \( i+1 \) respectively. From the time \( T_n \), the regulation run for the parallel passes \( i \) and \( i+1 \) continues to the time \( T_{n+1} \) when the subsystem reaches to the
pass balancing status. At the time $T_{n+1}$, the basic switching unit is switched to be the subsystem that consists of parallel passes $i+1$ and $i+2$ for $i = 1, 2, ..., N-2$; or parallel passes $N$ and 1 for $i = N-1$; or parallel passes 1 and 2. The regulation and the evolution procedures of the basic switching unit are repeated like this.

3. BALANCEABILITY PROBLEMS

3.1 Problem Formulation

As shown in Fig. 4, the balanceability problem is formulated as that, under a zero system input ($F_T = 0$), for any given initial outlet temperature $T_o = [T_{o1}, T_{o2}, ..., T_{oN}]$, where the $T_{oi}$ is the initial outlet temperature of the parallel pass $i$, $i = 1, 2, ..., N$, whether or not the given switching control policy could drive the $N$ parallel passes to have identical outlet temperatures. Denote

$$T(t) = \begin{bmatrix} T_1(t) \\ T_2(t) \\ \vdots \\ T_N(t) \end{bmatrix}$$

as the system load status at time $t$, where the $T_i(t)$ is the outlet temperature of the parallel pass $i$, $i = 1, 2, ..., N$.

**Definition 1:** A switching control policy is said to be balanceable, if the limit of (3)

$$\lim_{t \to \infty} T(t) = \begin{bmatrix} T_1(t) \\ T_2(t) \\ \vdots \\ T_N(t) \end{bmatrix} = U$$

for all $T_o = \begin{bmatrix} T_{o1} \\ T_{o2} \\ \vdots \\ T_{oN} \end{bmatrix}$, (4)

where the $U$ is a uniform vector whose components are all identical.

3.2 Constraint Decoupling

**Theorem 1:** For the FRR switching control policy, the system structural algebraic control input constraint can be decoupled, if it can be at the initial time $t_0$.

**Proof:** From (2), it can be found that, for the basic switching unit $DCT_{ij}$, the sum of the task inputs of the two parallel units is given by

$$f_i + f_j = f_{i1} + f_{i2},$$

and it does not change with the output of the continuous time controller. In addition, for the FRR policy, only some adjacent two parallel passes are selected for balancing control during any given time slot, keeping the other $N-2$ oil flowrate $f_i$ to $f_N$ be constant temporarily. Thus, the sum of the $N$ oil flowrate

$$\sum_{i=1}^{N} f_i$$

does not change with the time. So, if the value of (6) is set to be the total inlet flowrate $F_T$ at initial time $t_0$, Equation (1) holds, that is, the system structural algebraic control input constraint is decoupled.

3.3 Balanceability of FRR Switching Policy

Denote

$$DCT^{|k,k+1|}_T$$

as the basic switching unit subsystem which is composed of parallel passes $i$ and $j$, $i \neq j$, $i, j = 1, 2, ..., N$, during the time interval $(kh, (k+1)h)$, $k=0, 1, 2, ..., $, where $h$ is the control time slot. For a four-passes furnace system and FRR policy, the evolution of the basic switching unit (7), can be given by

$$\begin{align*}
DCT^{|4n,4n+1|}_{12}, & \ t \in (4nh,(4n+1)h) \\
DCT^{|4n+1,4n+2|}_{23}, & \ t \in ((4n+1)h,(4n+2)h) \\
DCT^{|4n+2,4n+3|}_{34}, & \ t \in ((4n+2)h,(4n+3)h) \\
DCT^{|4n+3,4n+4|}_{41}, & \ t \in ((4n+3)h,(4n+4)h)
\end{align*}$$

Fig. 4. A schematic diagram showing the balanceability problem formulation, where the $\phi_i$ is the oil inlet flowrate of parallel pass $i$, and the $\omega_i$ is the holding value of the $\phi_i$ when the switch takes place, $i = 1, 2, ..., N$.

Fig. 5. A schematic diagram for the performance analysis of the basic switching unit subsystem.
where \( n=0, 1, 2, \ldots \).

**Lemma 1:** For a basic switching subsystem with zero input, if the subsystem can be balancing controlled by its continuous time controller, then, the identical outlet temperatures of the two parallel passes would be given by

\[
T_1 = T_2 = \frac{T_{t_01} + T_{t_20}}{2},
\]

where the \( T_{t_01} \) and \( T_{t_20} \) are the two initial outlet temperatures.

**Proof:** Consider a subsystem with zero input (as shown in Fig. 5). From the condition, it is known that the two outlet temperatures \( T_1(t) \) and \( T_2(t) \) would be identical at and after the time \( t_s \) when the subsystem reaches to a pass balancing status, that is, \( T_1(t) = T_2(t) \) for time \( t > t_s \).

It can be found from Fig. 5 that \( f_i(t) = -f_i(t) \) and

\[
egin{align*}
T_1(t) &= y_1(t) + T_{t_01} \\
T_2(t) &= y_2(t) + T_{t_02}.
\end{align*}
\]

(10)

In addition, for time \( t > t_s \), the following holds.

\[
y_i(t) = K \cdot f_i(t),
\]

(11)

where the \( K \) is the steady gain of parallel passes \( i, i = 1, 2 \).

Considering (10), (11), and that \( T_1(t) = T_2(t) \), it can be found that

\[
T_i(t) = \frac{T_{t_01} + T_{t_20}}{2}
\]

(12)

for time \( t > t_s \). This completes the proof of the lemma. \( \square \)

**Theorem 2:** For a four-passes furnace system, if the basic switching unit subsystems can be balancing controlled by the continuous time controller during the control time slot \( h \), then, for the evolution procedures presented by (8), the system outlet temperature (3) at switching time \( t = kh \) \((k = 4n+1, 4n+2, 4n+3, 4n+4, \text{where } n = 0, 1, 2, \ldots)\)

\[
T(t)_{f=kh} = \begin{bmatrix}
T_1^{(k)} \\
T_2^{(k)} \\
T_3^{(k)} \\
T_4^{(k)}
\end{bmatrix}
\]

(13)

can be given respectively by

1) for \( n = 0 \),

\[
\begin{bmatrix}
T_1^{(1)} \\
T_2^{(1)} \\
T_3^{(1)} \\
T_4^{(1)}
\end{bmatrix} = \begin{bmatrix}
T_{t_01} / 2 + T_{t_02} / 2 \\
T_{t_1}^{(1)} \\
T_{t_03} \\
T_{t_04}
\end{bmatrix},
\]

(14-1)

\[
\begin{bmatrix}
T_1^{(2)} \\
T_2^{(2)} \\
T_3^{(2)} \\
T_4^{(2)}
\end{bmatrix} = \begin{bmatrix}
T_{t_01} / 2 + T_{t_02} / 2 \\
T_{t_01} / 2^2 + T_{t_02} / 2^2 + T_{t_03} / 2 \\
T_{t_01} / 2^2 + T_{t_02} / 2^2 + T_{t_03} / 2 \\
T_{t_04}
\end{bmatrix},
\]

(14-2)

\[
\begin{bmatrix}
T_1^{(3)} \\
T_2^{(3)} \\
T_3^{(3)} \\
T_4^{(3)}
\end{bmatrix} = \begin{bmatrix}
T_{t_01} / 2 + T_{t_02} / 2 \\
T_{t_01} / 2^2 + T_{t_02} / 2^2 + T_{t_03} / 2 \\
T_{t_01} / 2^2 + T_{t_02} / 2^2 + T_{t_03} / 2 + T_{t_04} / 2 \\
T_{t_04}
\end{bmatrix},
\]

(14-3)

\[
\begin{bmatrix}
T_1^{(4)} \\
T_2^{(4)} \\
T_3^{(4)} \\
T_4^{(4)}
\end{bmatrix} = \begin{bmatrix}
T_{t_01} / 2^2 + T_{t_02} / 2^2 + T_{t_02} / 2^2 + T_{t_03} / 2^2 + T_{t_03} / 2^2 + T_{t_04} / 2^2 \\
T_{t_02} / 2^2 + T_{t_02} / 2^2 + T_{t_03} / 2^2 + T_{t_04} / 2 \\
T_{t_01} / 2^3 + T_{t_02} / 2^3 + T_{t_03} / 2^3 + T_{t_04} / 2 \\
T_{t_04}
\end{bmatrix},
\]

(14-4)

or 2) for \( n \geq 1 \),

\[
\begin{bmatrix}
T_1^{(4n+1)} \\
T_2^{(4n+1)} \\
T_3^{(4n+1)} \\
T_4^{(4n+1)}
\end{bmatrix} = \begin{bmatrix}
(1 + f_{2n} / 3 + g_{2n} / c) / 4 T_{t_01} + (1 + f_{2n} / 3 + g_{2n} / c) / 4 T_{t_02} \\
(1 + f_{2n} / 3 + g_{2n} / c) / 4 T_{t_03} + (1 + f_{2n} / 3 + g_{2n} / c) / 4 T_{t_04} \\
(1 + f_{2n} / 3 + g_{2n} / c) / 4 T_{t_01} + (1 + f_{2n} / 3 + g_{2n} / c) / 4 T_{t_02} \\
(1 + f_{2n} / 3 + g_{2n} / c) / 4 T_{t_03} + (1 + f_{2n} / 3 + g_{2n} / c) / 4 T_{t_04}
\end{bmatrix}
\]

(15-1)

where \( c = \sqrt{2^n - 1} \),

\[
\begin{bmatrix}
f_n = \frac{1}{2^n} \cos(n \cdot \arctan c) \\
g_n = \frac{1}{2^n} \sin(n \cdot \arctan c)
\end{bmatrix}
\]
\[
T_1^{(4n+2)} = T_1^{(4n+1)} \\
T_2^{(4n+2)} = \frac{T_1^{(4n+1)} + T_4^{(4n+1)}}{2} \\
T_3^{(4n+2)} = T_2^{(4n+2)} \\
T_4^{(4n+2)} = T_4^{(4n+1)} \\
T_1^{(4n+3)} = T_1^{(4n+3)} \\
T_2^{(4n+3)} = T_2^{(4n+2)} \\
T_3^{(4n+3)} = \frac{T_1^{(4n+2)} + T_4^{(4n+2)}}{2} \\
T_4^{(4n+3)} = T_3^{(4n+3)} \\
T_1^{(4n+4)} = \frac{T_1^{(4n+3)} + T_4^{(4n+3)}}{2} \\
T_2^{(4n+4)} = T_2^{(4n+3)} \\
T_3^{(4n+4)} = T_3^{(4n+3)} \\
T_4^{(4n+4)} = T_1^{(4n+5)} \\
\]

(15-2) \quad T_1^{(k)} \square T_1^{(4n+2)} \\
T_2^{(k)} \square T_2^{(4n+2)} \\
T_3^{(k)} \square T_3^{(4n+2)} \\
T_4^{(k)} \square T_4^{(4n+2)} \\
(17-2)

Proof: The proof can be finished by using the mathematical induction approach, however, due to the page limit, the proof is omitted. ■

Theorem 3: For a four-passes furnace system, the FRR switching control policy is balanceable, if the basic switching unit subsystems can be balancing controlled by the continuous time controller during the given switching time slot \( h \).

Proof: Consider the series of the system outlet temperature (3) at the switching time \( t = kh, k = 0, 1, 2, \ldots \)

\[
T^{(k)} \square T(t) = \begin{bmatrix}
T_1^{(k)} \\
T_2^{(k)} \\
T_3^{(k)} \\
T_4^{(k)}
\end{bmatrix}
\]

(16)

From (8), Equation (16) can be completely disassembled into the following four sub-series.

\[
T_1^{(k)} \square T_1^{(4n+1)} \\
T_2^{(k)} \square T_2^{(4n+1)} \\
T_3^{(k)} \square T_3^{(4n+1)} \\
T_4^{(k)} \square T_4^{(4n+1)} \\
(17-1)

\]

Proof: The proof can be finished by using the mathematical induction approach, however, due to the page limit, the proof is omitted. ■

It can be found from (15-1) that sub-series (17-1) is convergent when the \( n \) approaches to infinity, and its limit is given by

\[
\lim_{n \to \infty} T^{(4n+1)} = \begin{bmatrix}
\lim_{n \to \infty} T_1^{(4n+1)} \\
\lim_{n \to \infty} T_2^{(4n+1)} \\
\lim_{n \to \infty} T_3^{(4n+1)} \\
\lim_{n \to \infty} T_4^{(4n+1)}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0
\end{bmatrix}
\]

(18)

where the \( \mu \) is a constant, and it is given by

\[
\mu = \frac{(T_{01} + T_{02} + T_{03} + T_{04})}{4}.
\]

Similarly, it can be easily verified from (15-2) to (15-4) that the sub-series (15-2) to (17-4) are also convergent, and they have the same limits as that of the sub-series (17-1). So, the system outlet temperature series (16) is convergent when the \( k \) goes to infinity, and its limit is given by

\[
\lim_{k \to \infty} T^{(k)} = \begin{bmatrix}
1 & 1 & 1 & 1
\end{bmatrix}
\]

(19)
where the $\mu$ is as in (18).

In addition, for a basic switching unit subsystem $DCT_{ij}$, the two outlet temperatures satisfy

$$T_i(t) = T_j(t), \quad (20)$$

for time $t \in (\kappa h, (\kappa + 1)h)$, if $T_i(t) = T_j(t)$ at switching time $t = \kappa h$, where $i \neq j$, $i, j = 1, 2, 3$, and $\kappa$ is some positive integer.

It can be found from (19) and (20) that

$$\lim_{t \to \infty} T(t) = \begin{bmatrix} T_1(t) \\ T_2(t) \\ T_3(t) \\ T_4(t) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \mu,$$

where the $\mu$ is as in (18).

Thus, from Definition 1, it is known that the FRR switching control policy is balanceable for a four-passes furnace system. This completes the proof. ■

4. CONCLUSIONS

This paper has investigated the balancing control problems for industrial multi-passes furnace systems, which is widely used in petroleum refineries. For considerations of reducing the time of the flowrate valves being regulated and hence prolonging their lifespan, a switching control based technique for the four-passes industrial furnace system. Through an analysis of the basic switching subsystem and the switching sequence, a sufficient condition for the balanceability of the FRR policy for the four-passes systems is obtained.

It should be pointed that further studies, e.g., how to extend the number of passes $N=4$ to an arbitrary $N$, the analysis for dynamical performances, etc., are needed to be done, and the corresponding problems are being researched.

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REFERENCES


